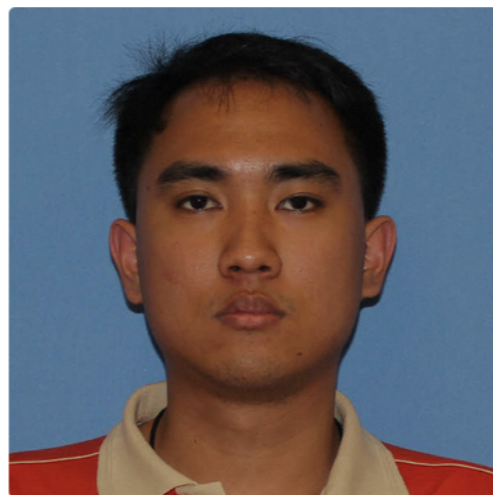


# Noether's Second Theorem and Strange Metals

Thanks to: NSF,DOE

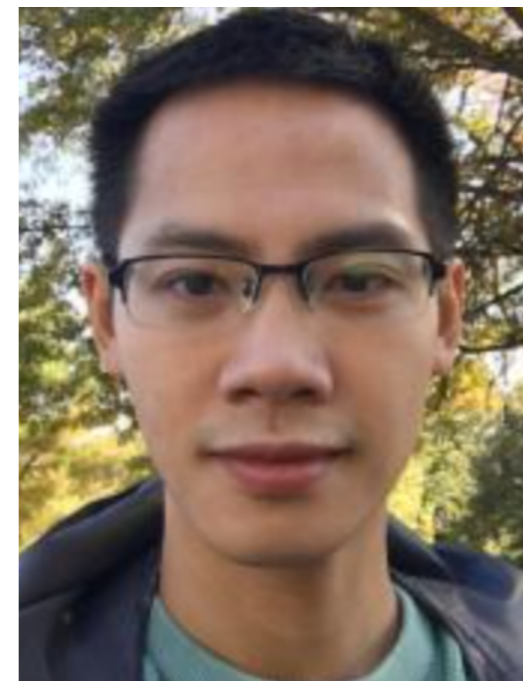
Rev. Mod. Phys. 2019  
CIMP 2019  
Adv. Th. Math. Phys.  
Science, xxx 2022

Gabriele La Nave

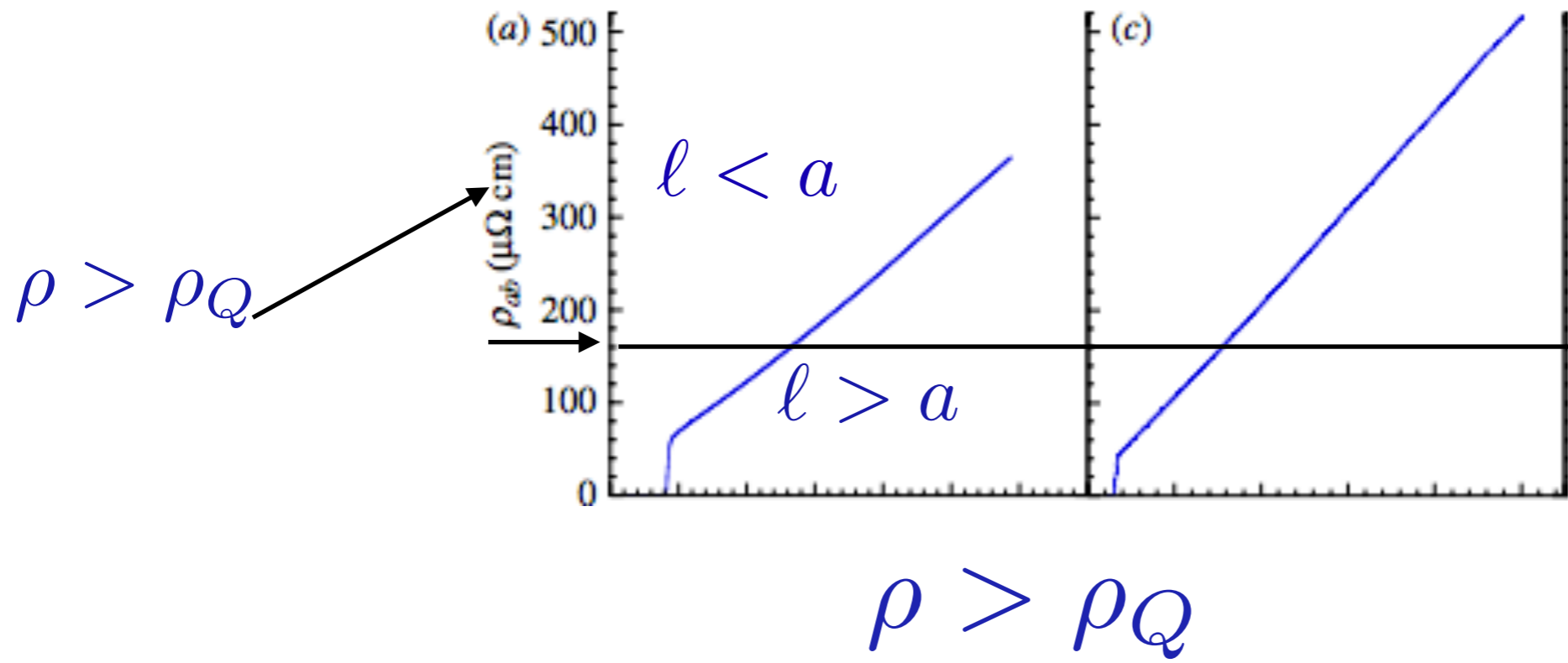


Kridsangaphong Limtragool

Peizhi Mai



# T-linear resistivity

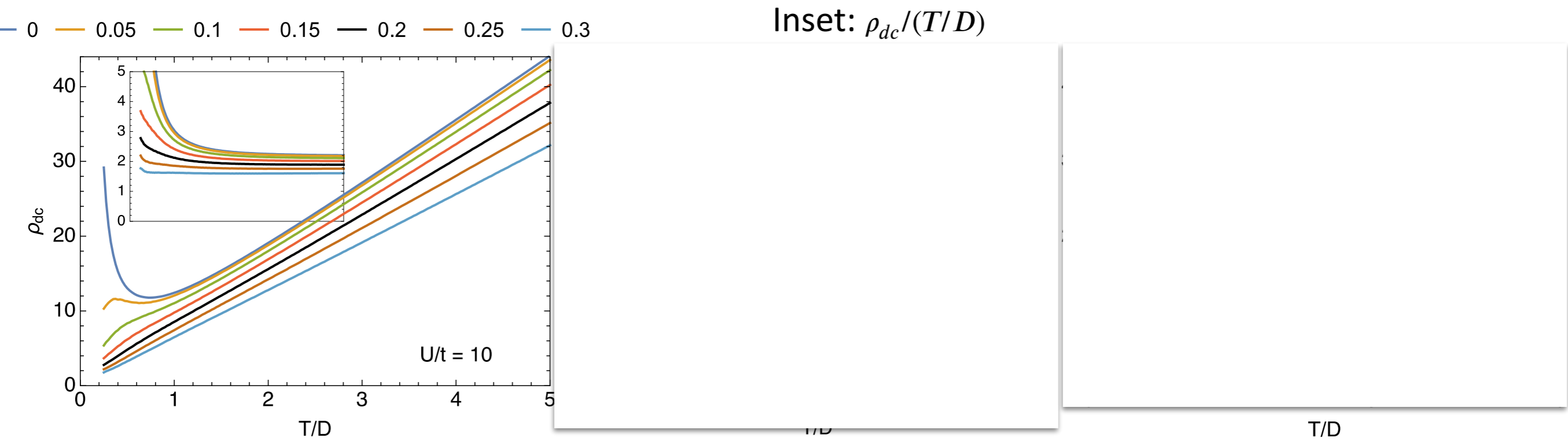


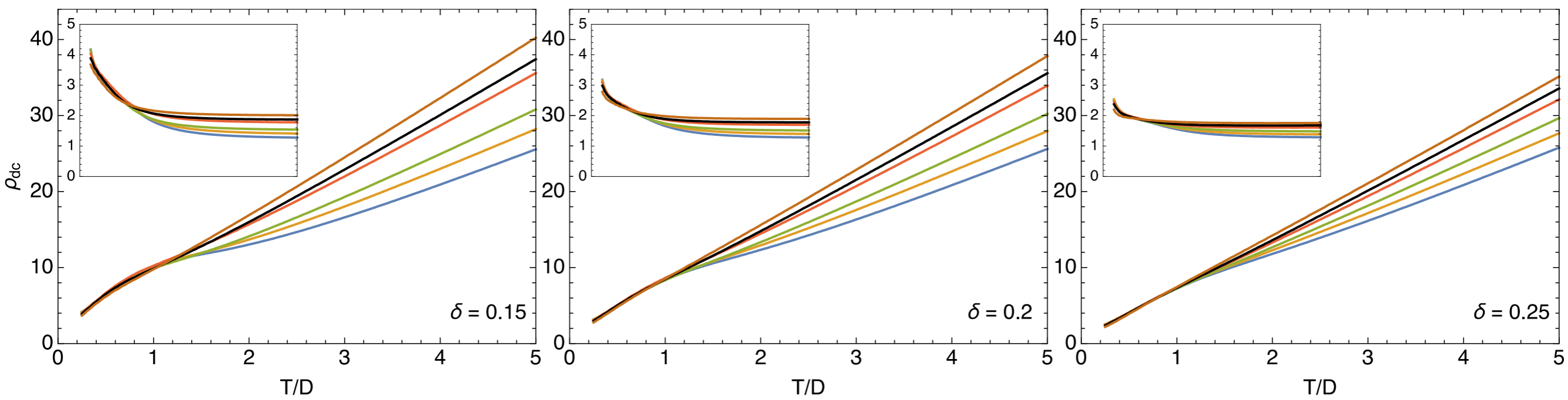
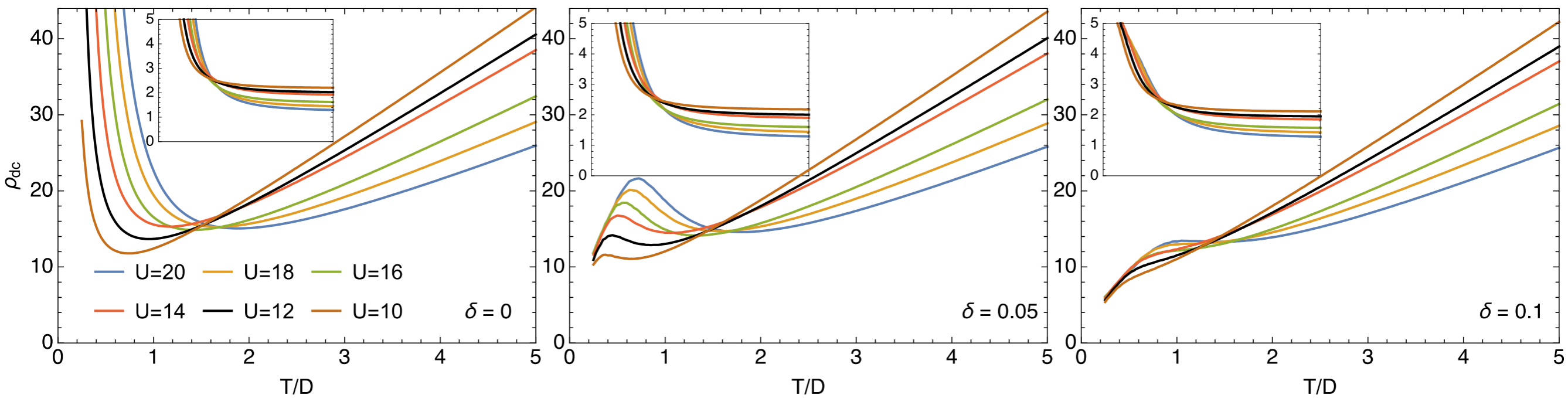
Violates MIR limit

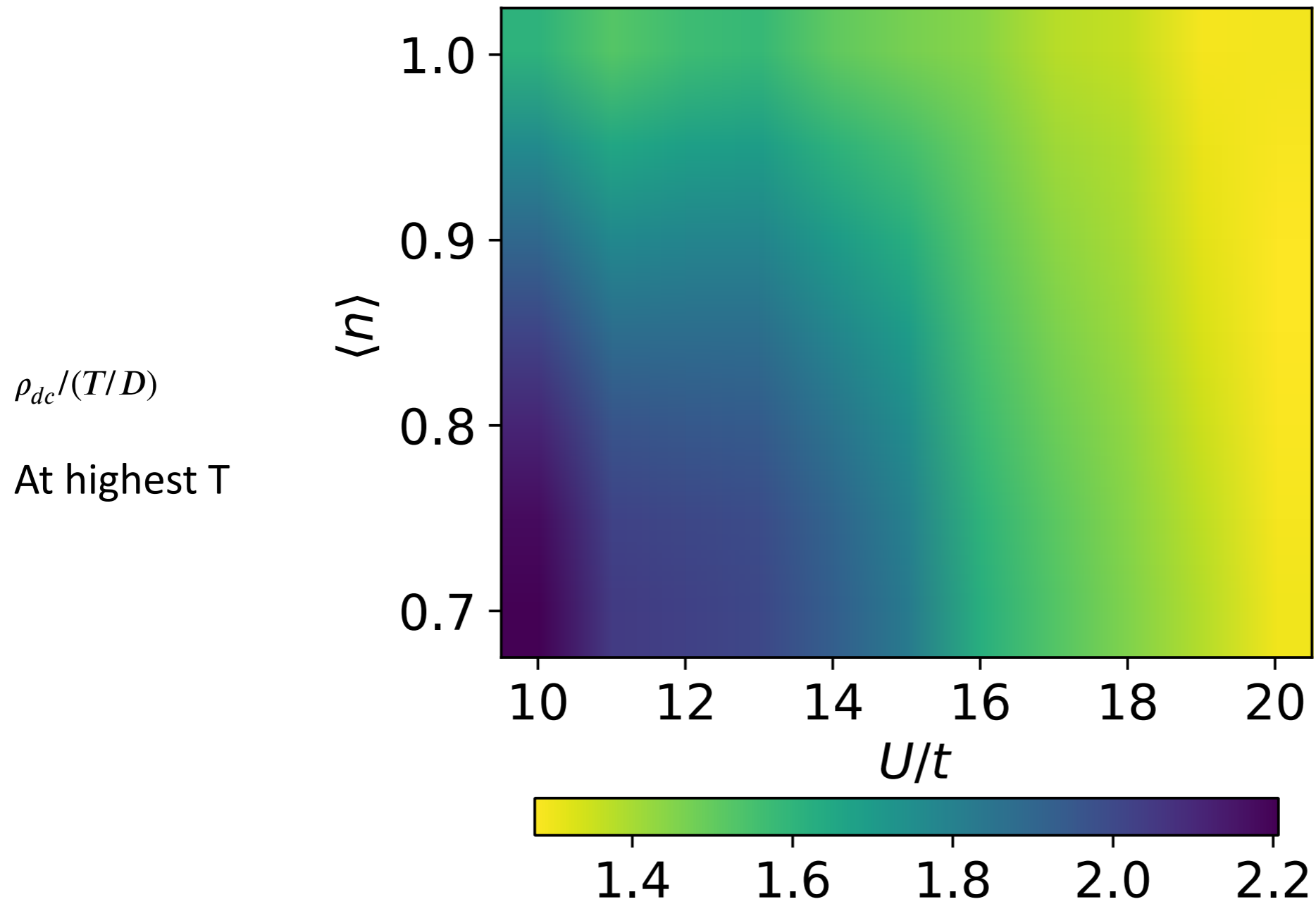
electrons not charge carriers

?

# Finite-T Lanczos on Hubbard Model

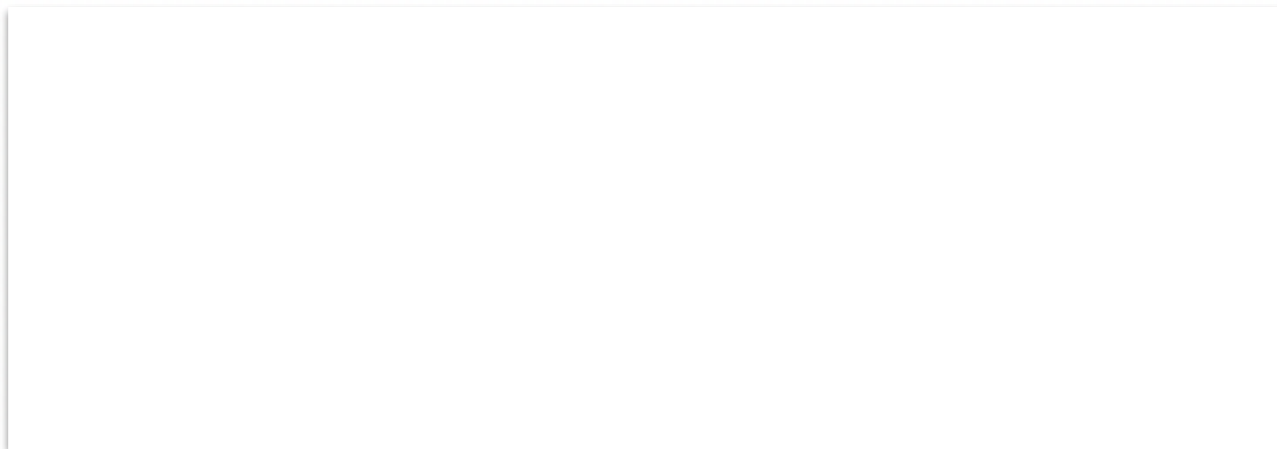
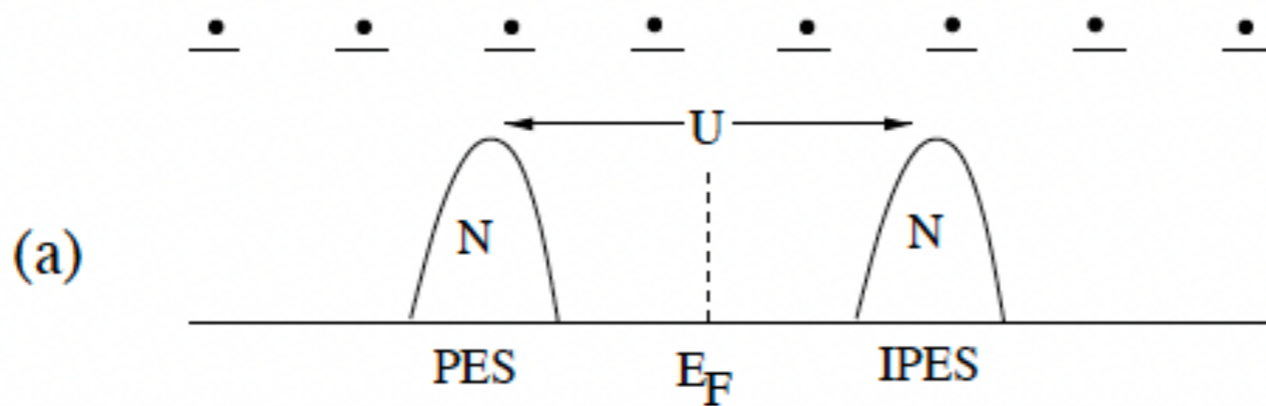






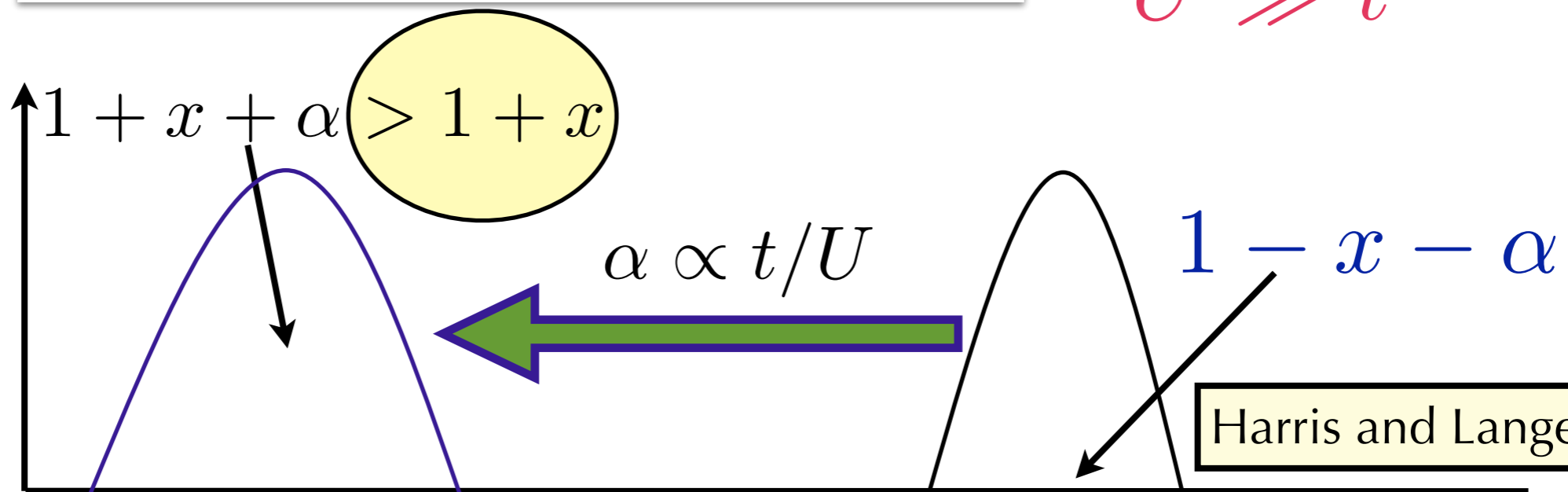
$$\frac{\rho_{dc}}{T} = \alpha(t/U)$$

# spectral function (dynamics)



$$U \gg t$$

density of states



Harris and Lange (1967)

not exhausted by counting electrons alone

Does anything  
local carry the charge?

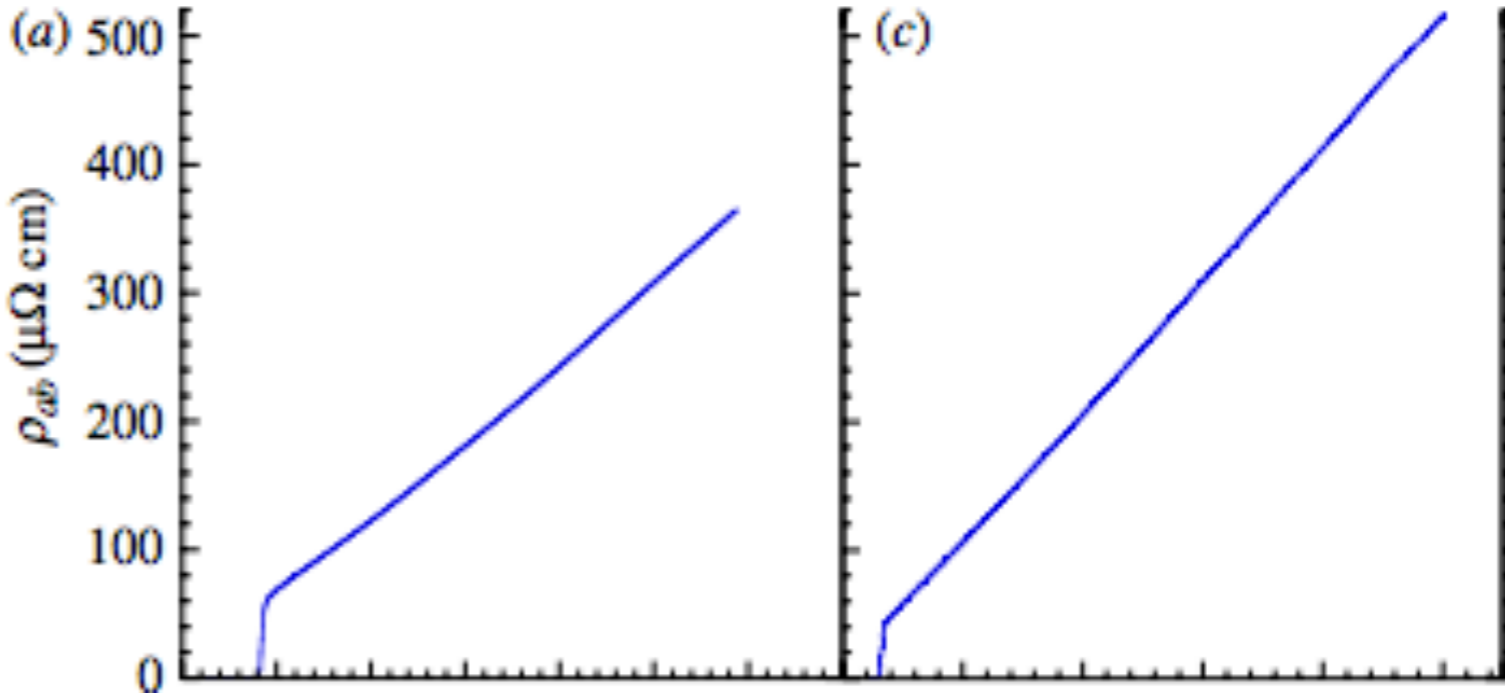
Is a new gauge  
principle operative?

# strange metal explained!

## Hall Angle

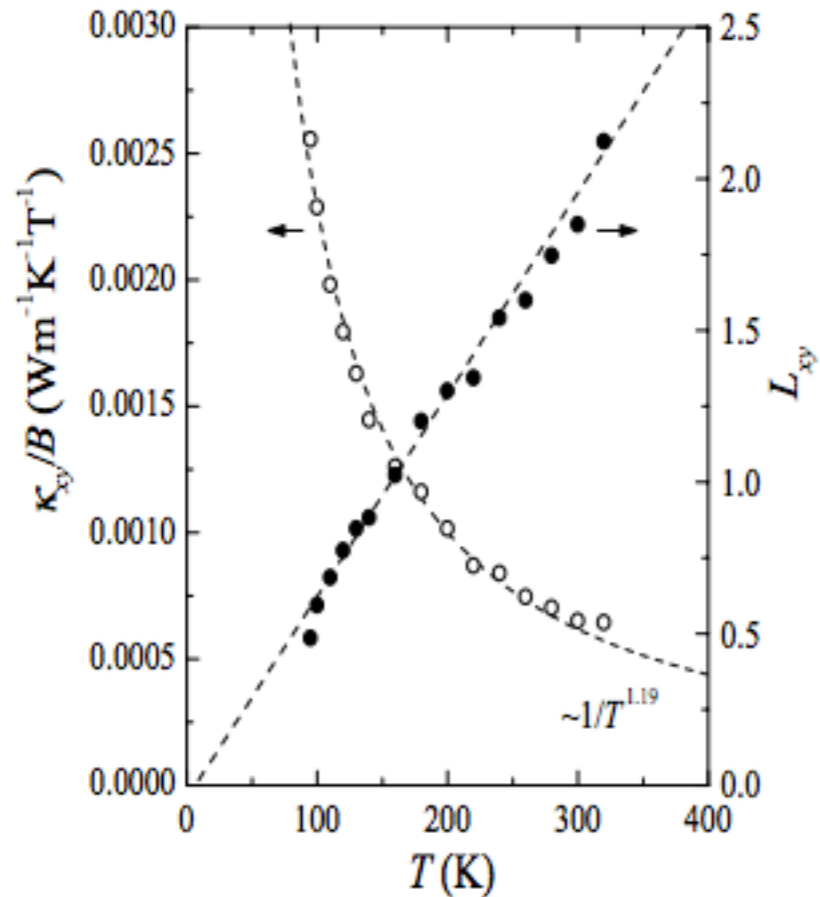
$$\cot \theta_H \equiv \frac{\sigma_{xx}}{\sigma_{xy}} \approx T^2$$

## T-linear resistivity



## Hall Lorenz ratio

$$L_{xy} = \kappa_{xy} / T \sigma_{xy} \neq \# \propto T$$



## all explained if

$$[J_\mu] = d - \theta + \Phi + z - 1$$

## Hartnoll/Karch

$$[A_\mu] = 1 - \Phi$$

$$\Phi = -2/3$$



# strange metal

$$[J_\mu] = d - \theta + \Phi + z - 1$$

$$[A_\mu] = 1 - \Phi$$
$$\Phi = -2/3$$
$$[E] = 1 + z - \Phi$$
$$[B] = 2 - \Phi$$



note  $\pi r^2 B \neq \text{flux}$

$$\oint A \cdot dl \notin h\mathbb{Z}$$

?

# Noether's First Theorem

$U(1) \longleftrightarrow qA \rightarrow qA - q\partial_\mu\Lambda$   
 $\psi' = e^{iq\Lambda}\psi$

$$S = \int d^d x (J_\mu A^\mu + \dots)$$

$$\begin{aligned}
 [q\Lambda] &= 0 \\
 [qA] &= 1 \\
 [A] &= 1
 \end{aligned}$$

fixes dimension of current

$$[d^d x J A] = 0$$

$$[J] = d - 1$$

$$S \rightarrow S + \int d^d x \cancel{J_\mu \partial \Lambda}$$

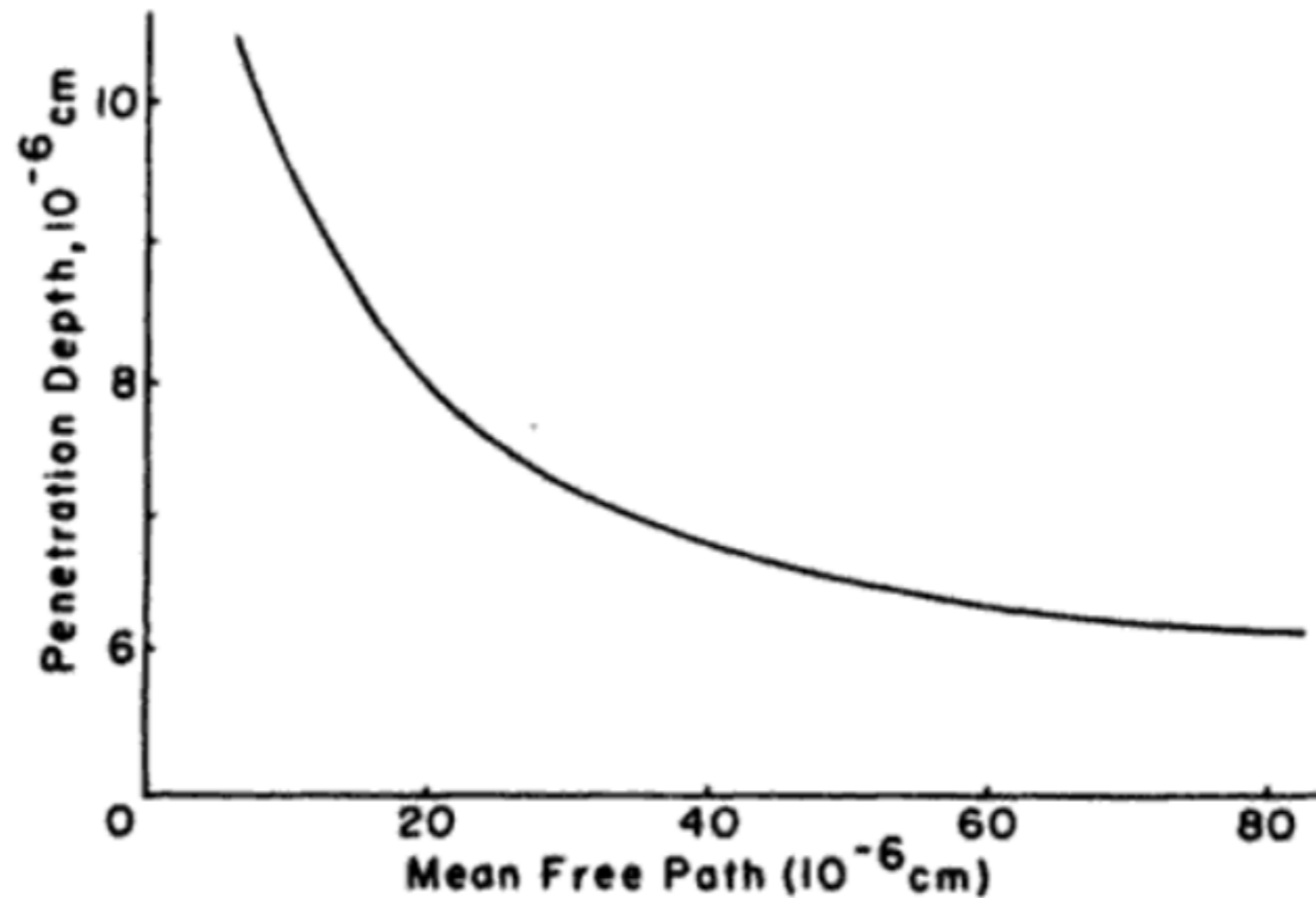
$$\partial_\mu J^\mu = 0$$

Noether's Thm. I

current conservation

Are there exceptions?

# Pippard's problem



$$J_s \neq \frac{-c}{4\pi\lambda^2} A$$

London Eq.

failure of local London relations

# Pippard Current

Pippard  
kernel

$$C(\vec{r}, \vec{r}') \cdot A(\vec{r}')$$

$$J_s = -\frac{3}{4\pi c \xi_0 \lambda} \int \frac{(\vec{r} - \vec{r}')((\vec{r} - \vec{r}') \cdot \vec{A}(\vec{r}')) e^{-(\vec{r} - \vec{r}')/\xi(\ell)}}{(\vec{r} - \vec{r}')^4} d^3 \vec{r}'$$

non-local

$$[J] = d - d_C - d_A$$

anomalous  
dimension

Are there other  
examples of  
currents with  
anomalous  
dimensions?

underlying  
electricity and  
magnetism?

is symmetry  
breaking  
necessary?

# possible gauge transformations

$$S = -\frac{1}{4} \int d^d x F^2$$



$$S = \frac{1}{2} \int \frac{d^d k}{2\pi^d} A_\mu(k) \underbrace{[k^2 \eta^{\mu\nu} - k^\mu k^\nu]}_{M_{\mu\nu} k^\nu = 0} A_\nu(k)$$

$$M_{\mu\nu} k^\nu = 0$$

zero eigenvector

$$ik_\mu \rightarrow \partial_\nu$$
$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$$

family of zero eigenvalues

$$M_{\mu\nu} \underbrace{f k^\nu} = 0$$

generator of gauge symmetry

- 1.) rotational invariance
- 2.)  $A$  is still a 1-form
- 3.)  $[f, k_\mu] = 0$



only choice

$$f \equiv f(k^2)$$

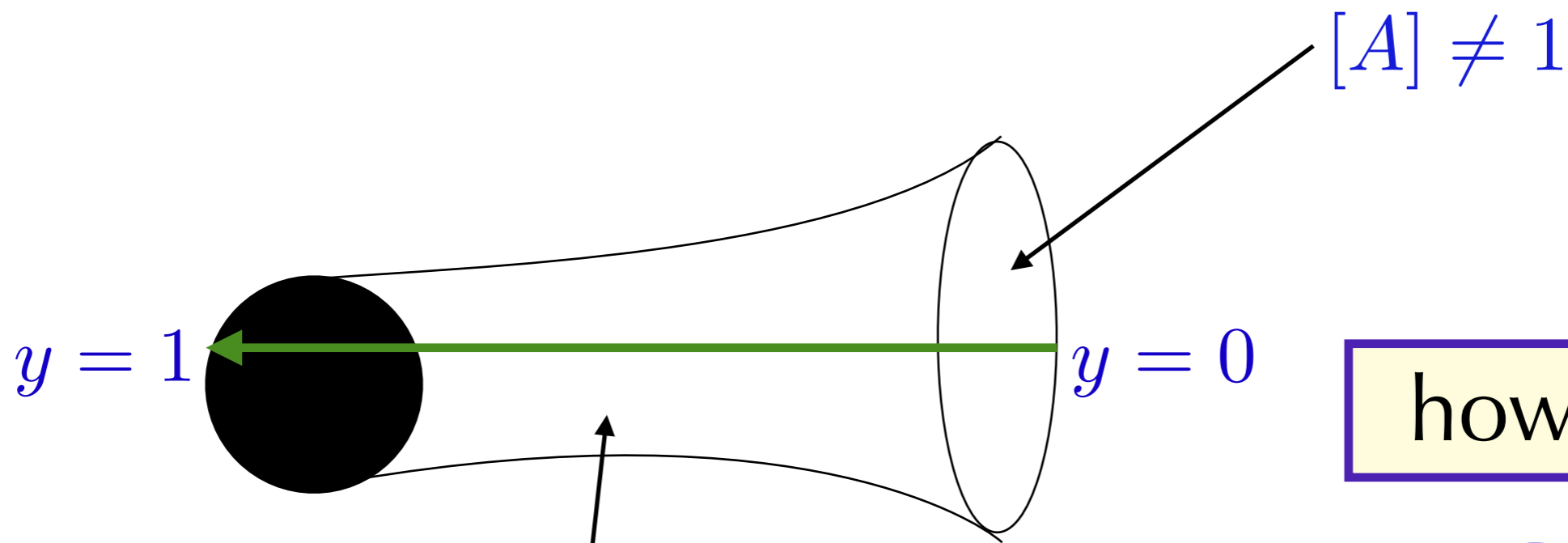


$$(-\Delta)^\gamma$$

$$A_\mu \rightarrow A_\mu + (-\Delta)^{\frac{\gamma-1}{2}} \partial_\mu \Lambda \quad [A_\mu] = \gamma$$

what kind of E&M has such  
gauge transformations?

claim: extra dimension



$$S = \int dV_d dy (y^a F^2 + \dots)$$

?

eom  $d(y^a \star dA) = 0$

Karch: 1405.2926  
Gouteraux: 1308.2084

if holography is RG then  
how can it lead to an  
anomalous dimension?

construct `boundary`  
theory explicitly

$$S = \int dV_d dy (y^a F^2 + \dots)$$



eom

$$d(y^a \star dA) = 0$$

Caffarelli-Silvestre  
extension theorem  
(2006)

$y$

$$g(x, y = 0) = f(x)$$

$$\Delta_x g + \frac{a}{y} g_y + g_{yy} = 0$$

$$\nabla \cdot (y^a \nabla g(x, y)) = 0$$

$$\lim_{y \rightarrow 0} y^a \partial_y g$$

?

$$C_{d,\gamma} (-\Delta)^\gamma f$$

$x$

fractional Laplacian

$$g(z = 0, x) = f(x)$$

$$\gamma = \frac{1 - a}{2}$$

closer look

$$\nabla \cdot (y^a \nabla u) = 0$$

scalar field  
(use CS theorem)

$$d(y^a \star dA) = 0$$

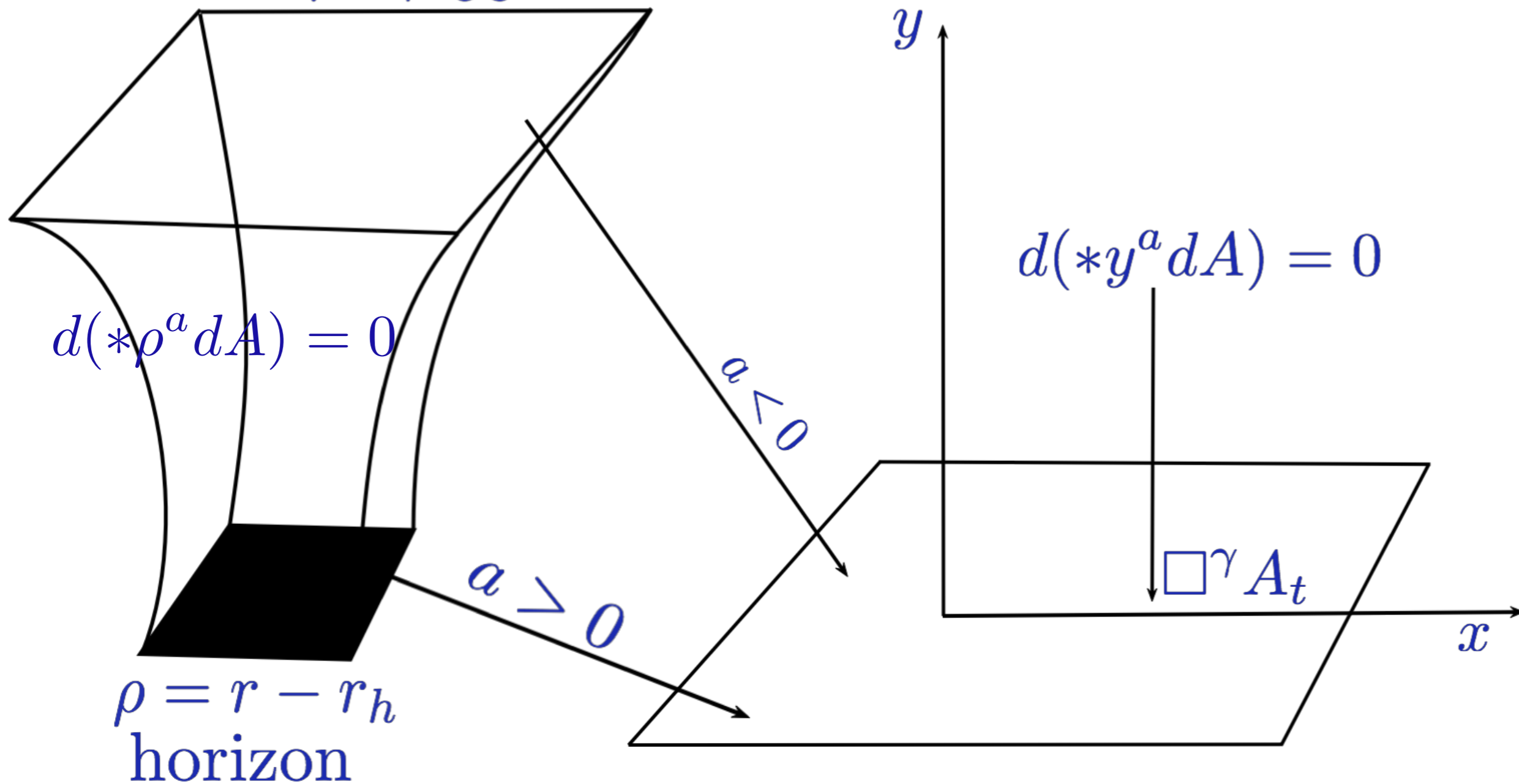
holography

similar equations

generalize CS theorem to  
p-forms

CIMP, 366, 199 (2019)

UV  
conformal boundary  
 $r \rightarrow \infty$



$\rho = r - r_h$   
horizon

IR

$$A \rightarrow A + d_\gamma \Lambda \equiv A'$$

$$d_\gamma \equiv \square^{\frac{\gamma-1}{2}} d$$



boundary action:  
fractional Maxwell  
equations

$$\square^\gamma A_\perp = J$$

boundary action has  
'anomalous dimension'  
(non-locality)

$$F \rightarrow d_\gamma A = \partial_\mu \square^{(\gamma-1)/2} A_\nu - \partial_\nu \square^{(\gamma-1)/2} A_\mu$$

if holography is RG then  
how can it lead to an  
anomalous dimension?

$$S = \int dV_d dy (y^a F^2 + \dots)$$



$$[A] = 1 - a/2$$

dimension of A is fixed by  
the bulk theory: not really  
anomalous dimension

# Ward identities

$$C^{ij}(k) \propto (k^2)^\gamma \left( \eta^{ij} - \frac{k^i k^j}{k^2} \right).$$

standard Ward  
identity

$$k_i C^{ij}(k) = 0 \quad \longrightarrow \quad \partial_i C^{ij}(k) = 0$$

but

$$k^{\gamma-1} k_\mu C^{\mu\nu} = 0 \quad \longrightarrow \quad \partial_\mu (-\Delta)^{\frac{\gamma-1}{2}} C^{\mu\nu} = 0$$

inherent ambiguity in E&M

# Noether's Second Theorem

$$\begin{aligned}
 & \sum \psi_{\mathbf{i}} \delta u_{\mathbf{i}} = \delta \mathcal{F} - \\
 & - \frac{d}{dx} \left\{ \sum \left[ \binom{1}{1} \frac{\partial \mathcal{F}}{\partial u_{\mathbf{i}}^{(1)}} \delta u_{\mathbf{i}} + \binom{2}{1} \frac{\partial \mathcal{F}}{\partial u_{\mathbf{i}}^{(2)}} \delta u_{\mathbf{i}}^{(1)} + \dots + \binom{\kappa}{1} \frac{\partial \mathcal{F}}{\partial u_{\mathbf{i}}^{(\kappa)}} \delta u_{\mathbf{i}}^{(\kappa-1)} \right] \right\} + \\
 & + \frac{d^2}{dx^2} \left\{ \sum \left[ \binom{2}{2} \frac{\partial \mathcal{F}}{\partial u_{\mathbf{i}}^{(2)}} \delta u_{\mathbf{i}} + \binom{3}{2} \frac{\partial \mathcal{F}}{\partial u_{\mathbf{i}}^{(3)}} \delta u_{\mathbf{i}}^{(1)} + \dots + \binom{\kappa}{2} \frac{\partial \mathcal{F}}{\partial u_{\mathbf{i}}^{(\kappa)}} \delta u_{\mathbf{i}}^{(\kappa-2)} \right] \right\} + \\
 & \vdots \\
 & + (-1)^\kappa \frac{d^\kappa}{dx^\kappa} \left\{ \sum \left[ \binom{\kappa}{\kappa} \frac{\partial \mathcal{F}}{\partial u_{\mathbf{i}}^{(\kappa)}} \delta u_{\mathbf{i}} \right] \right\}
 \end{aligned} \tag{6}$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda + \partial_\mu \partial_\nu G^\nu + \dots,$$

$$A \rightarrow A + d_\gamma \Lambda \equiv A'$$

$$d_\gamma \equiv (-\Delta)^{\frac{\gamma-1}{2}} d$$

$$\Delta \rightarrow \square$$

# Noether's Second Theorem and Ward Identities for Gauge Symmetries

Steven G. Avery<sup>a</sup>, Burkhard U. W. Schwab<sup>b</sup>

For simplicity, we focus on the case when the transformation may be written in the form<sup>6</sup>

$$\delta_\lambda \phi = f(\phi) \lambda + f^\mu(\phi) \partial_\mu \lambda, \quad (10)$$

but it is straightforward to consider transformations, as Noether did, involving arbitrarily high derivatives of  $\lambda$ . (Although, the authors know of no physically interesting examples.) Let us start with

arxiv:1510.07038

family of zero eigenvalues

$$M_{\mu\nu} f k^\nu = 0$$

most fundamental conservation law

$$\partial^\mu \underbrace{(-\nabla^2)^{(\gamma-1)/2} J_\mu}_{J'_\mu} = 0$$

is boundary non-locality a problem?

entanglement?

Li/Takayanagi (PRL 106, 141301 (2011))

$$I_B(\phi) = \int d^d x \phi (-\Delta)^\gamma \phi$$



$$S_B = \kappa_{d-2} \left( \frac{1}{\epsilon} \right)^{d-2}$$

Area

$$I_C(\phi) = \int d^d x \phi e^{(-\Delta)^\gamma} \phi$$

?

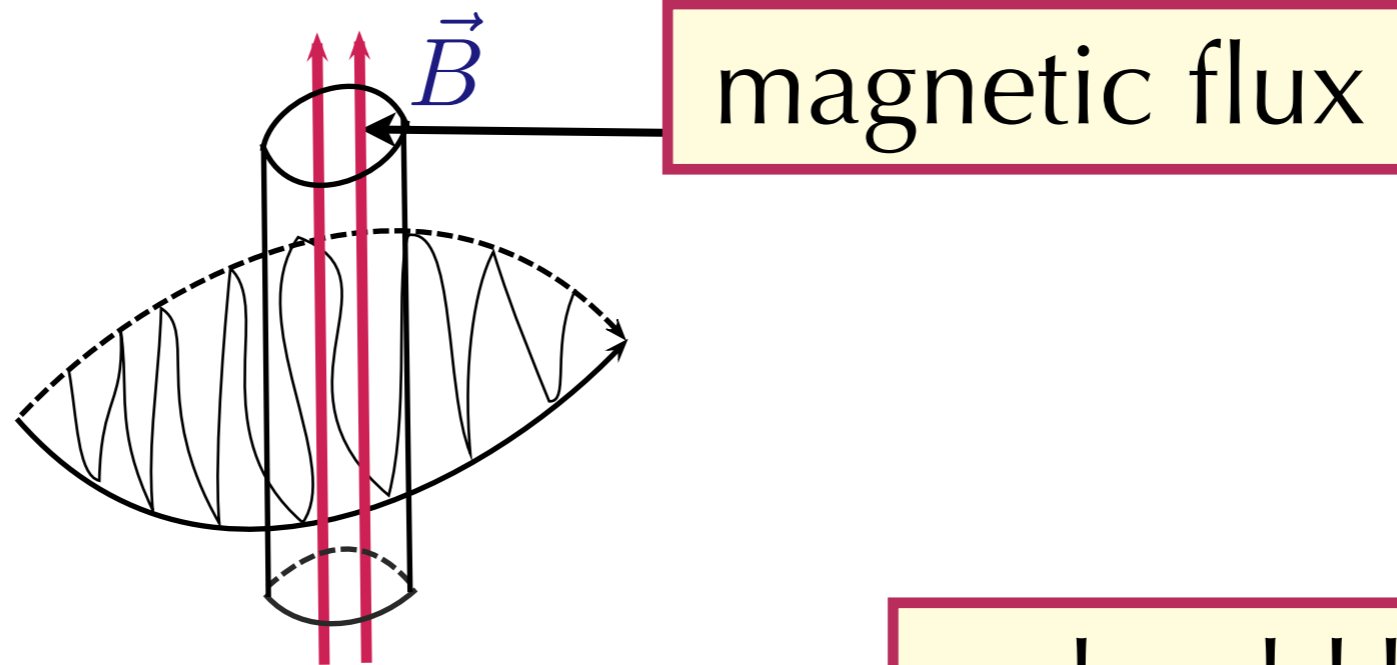


$$S_C \sim \kappa_{d-2} \left( \frac{1}{\epsilon} \right)^{d-2+2\gamma}$$

>Area

experiments?





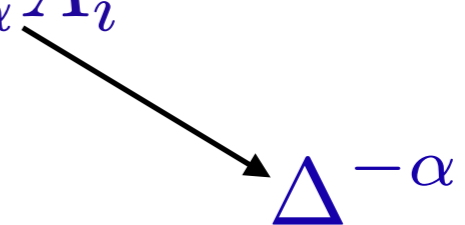
$$\pi r^2 B$$

should be  
dimensionless

$$[B] = 2 - \Phi = 2 + 2/3 \neq 2$$

what's the resolution?

correct dimensionless quantity

$$a_i \equiv [\partial_i, I_i^\alpha A_i] = \partial_i I_i^\alpha A_i$$


what's the relationship?

$$\oint_{\partial\Sigma} a \qquad \oint_{\partial\Sigma} A$$

$$\text{Norm} \oint_{\partial\Sigma} a = \frac{1}{\Gamma(3/2 - \gamma)} \oint_{\partial\Sigma} A$$

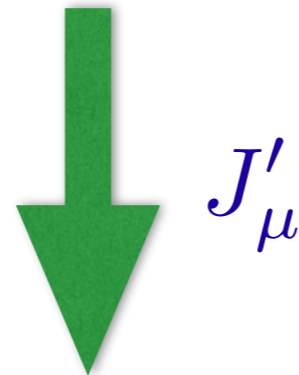
not an  
integer

# obstruction theorem to charge quantization (NST)

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda + \partial_\mu \partial_\nu G^\nu + \dots,$$

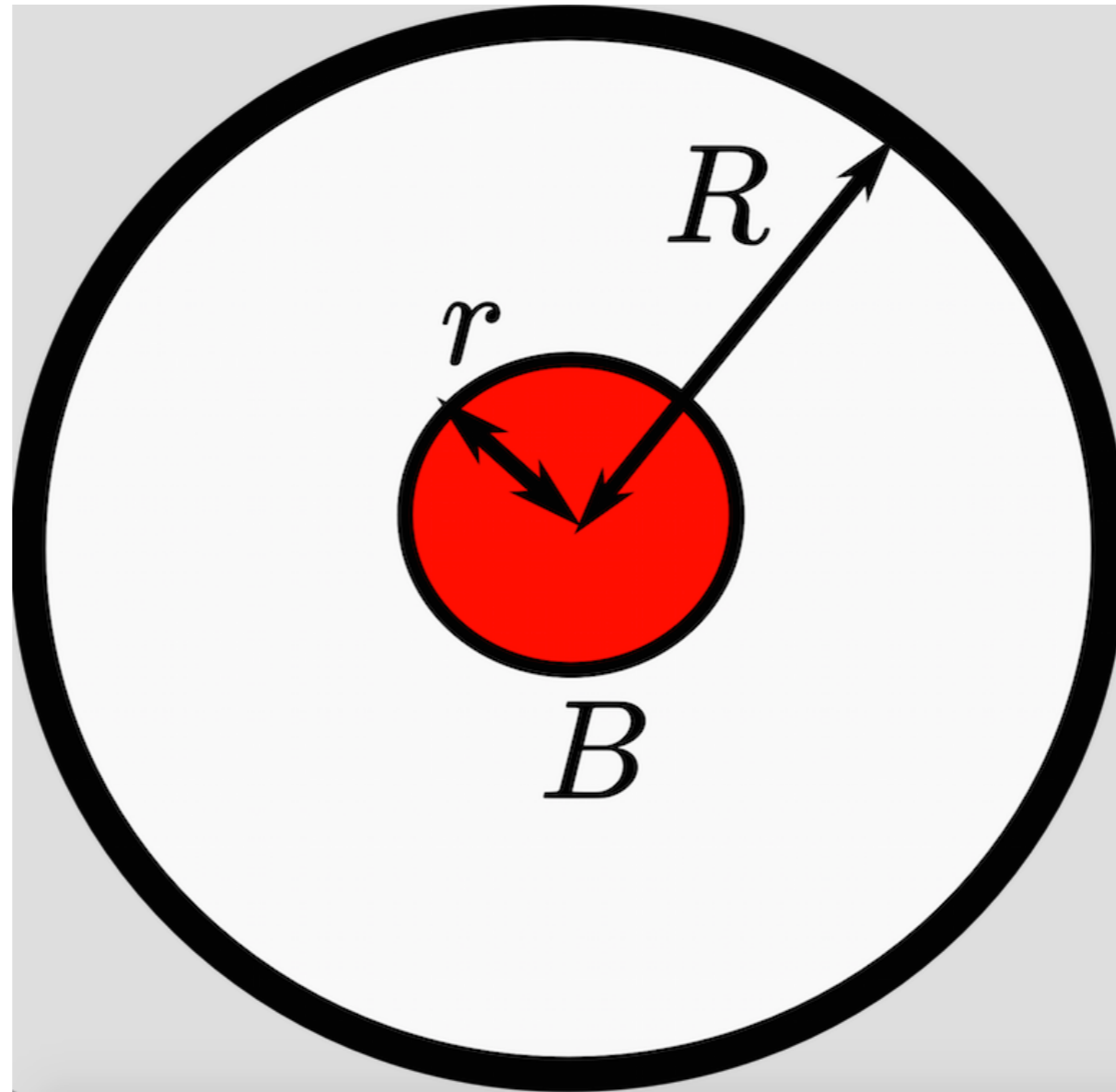
$$A \rightarrow A + d_\gamma \Lambda \equiv A'$$

$$d_\gamma \equiv (-\Delta)^{\frac{\gamma-1}{2}} d$$



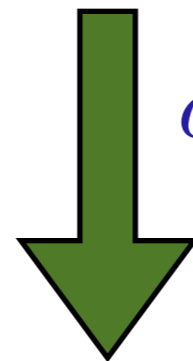
charge ill-defined (new  
landscape problem)

# New Aharonov-Bohm Effect



$$\Delta\phi_D = \frac{e}{\hbar} \pi r^2 B R^{2\alpha-2} \left( \frac{\sqrt{\pi} 2^{1-\alpha} \Gamma(2-\alpha) \Gamma(1-\frac{\alpha}{2})}{\Gamma(\alpha) \Gamma(\frac{3}{2}-\frac{\alpha}{2})} \sin^2 \frac{\pi\alpha}{2} {}_2F_1(1-\alpha, 2-\alpha; 2; \frac{r^2}{R^2}) \right)$$

is the correction large?



$$\alpha = 1 + 2/3 = 5/3$$

$$\Delta\Phi_R = \frac{eB\ell^2}{\hbar} L^{-5/3} / (0.43)^2$$

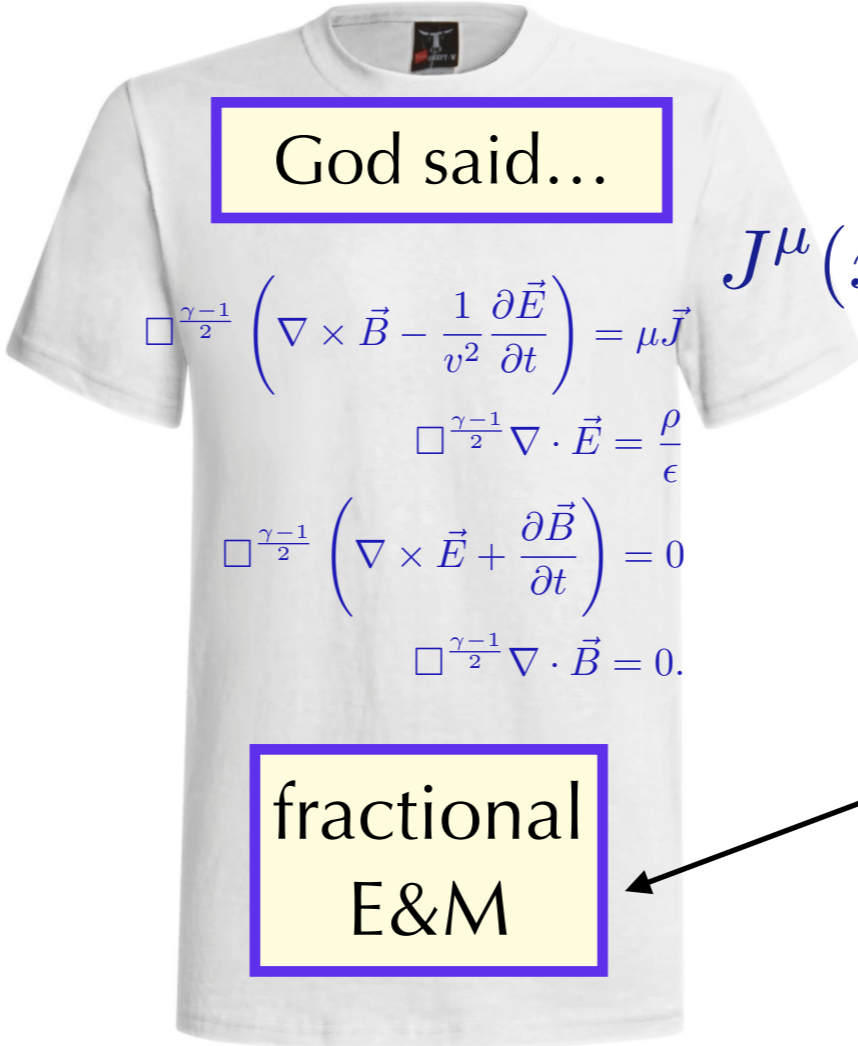
yes!

if in the strange metal



$$[A_\mu] = d_A \neq 1$$

Pippard Kernel



God said...

$$\square^{\frac{\gamma-1}{2}} \left( \nabla \times \vec{B} - \frac{1}{v^2} \frac{\partial \vec{E}}{\partial t} \right) = \mu \vec{J}$$

$$\square^{\frac{\gamma-1}{2}} \nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$

$$\square^{\frac{\gamma-1}{2}} \left( \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} \right) = 0$$

$$\square^{\frac{\gamma-1}{2}} \nabla \cdot \vec{B} = 0.$$

$$J^\mu(x) = - \int d^d x' C_{\mu\nu}(|x - x'|) A^\nu$$

$$[J] \neq d - 1$$

$$[A] \neq 1$$

fractional E&M

in SC!

$$\omega = ck$$

