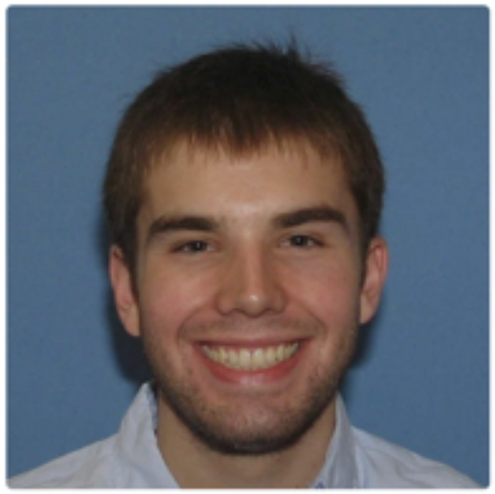
Are the Cuprate High-Temperature Superconductors full of Unparticles?

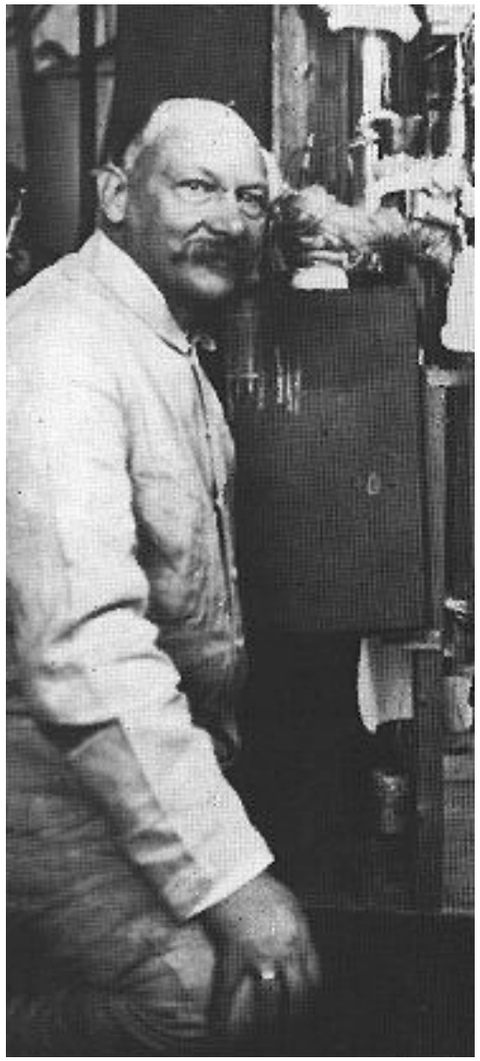
Thanks to: NSF, EFRC (DOE)

Brandon Langley



Garrett Vanacore

Kridsangaphong Limtragool



H. K. Onnes

Kammerlingh Onnes (Leiden)

- LIQUIFIES HELIUM 1908
- DISCOVERS SUPERCONDUCTIVITY in MERCURY 1911
- Receives NOBEL PRIZE 1913

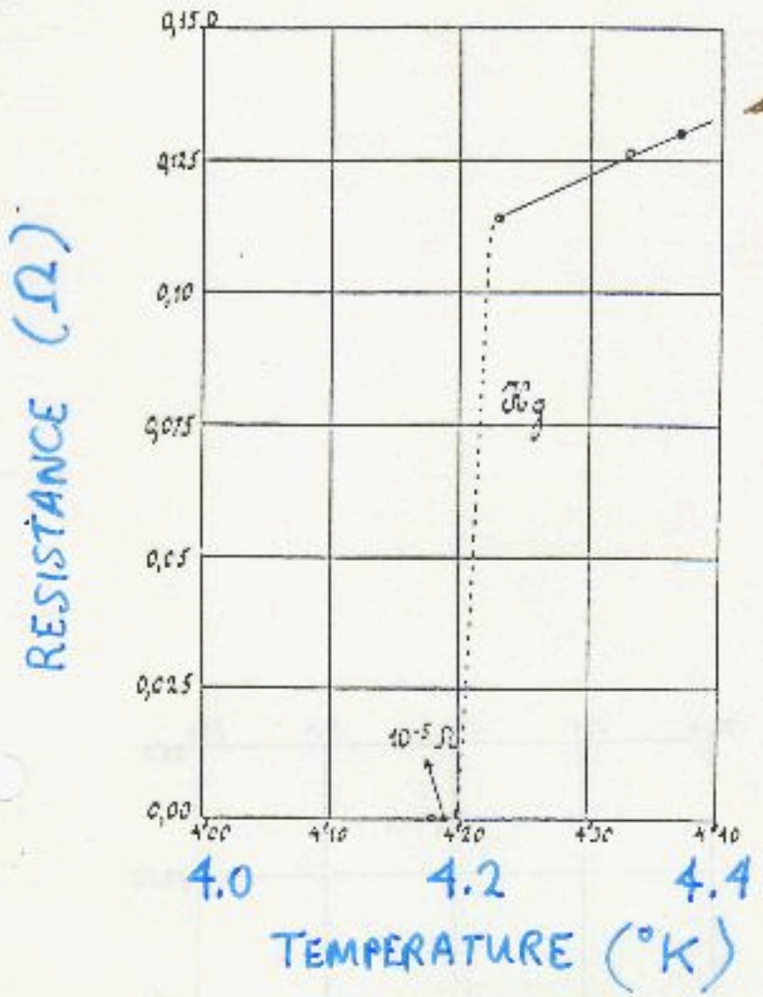


Figure 1 Resistance in ohms of a specimen of mercury versus absolute temperature. This plot by Kamerlingh Onnes marked the discovery of superconductivity.


High
Ba

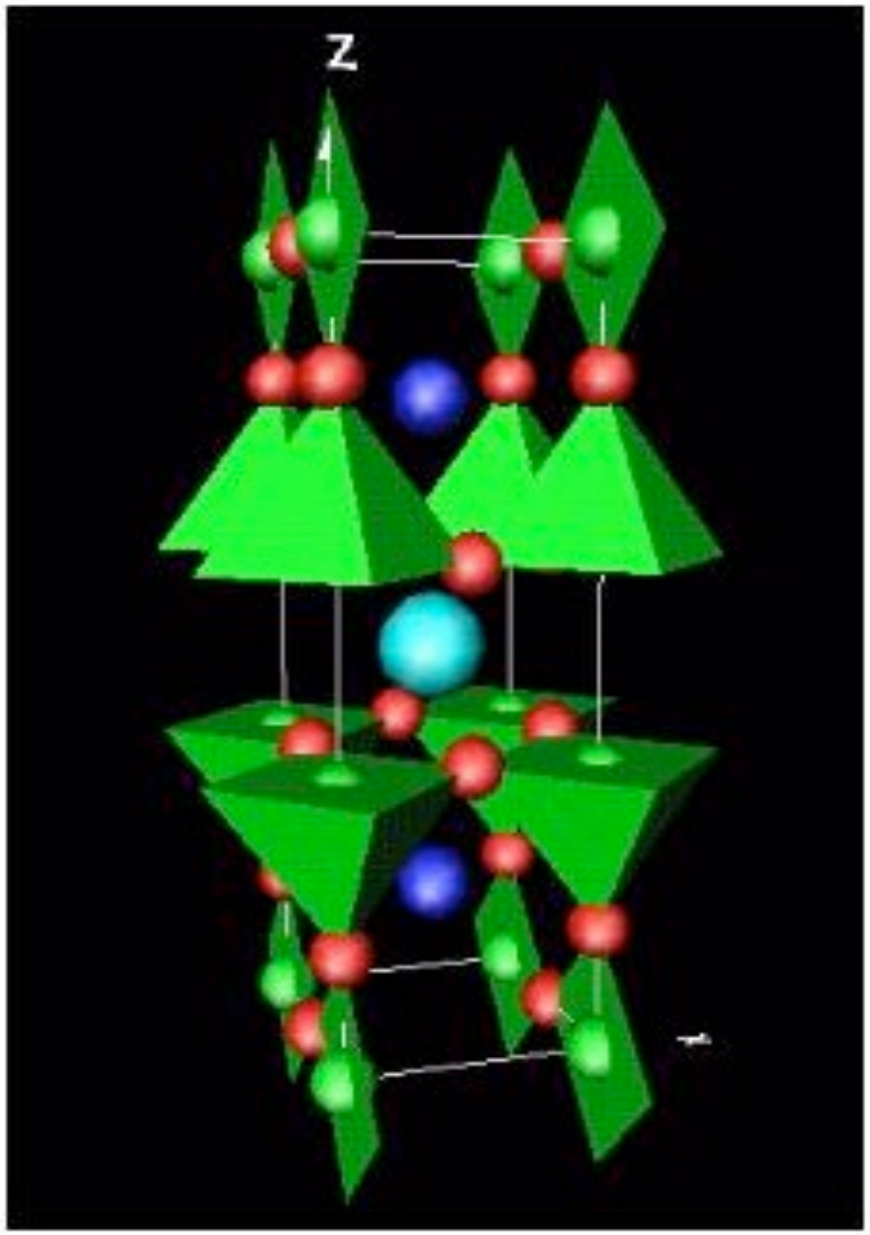
London Equation

$$J \propto A$$

London Equation

$$J \propto A$$

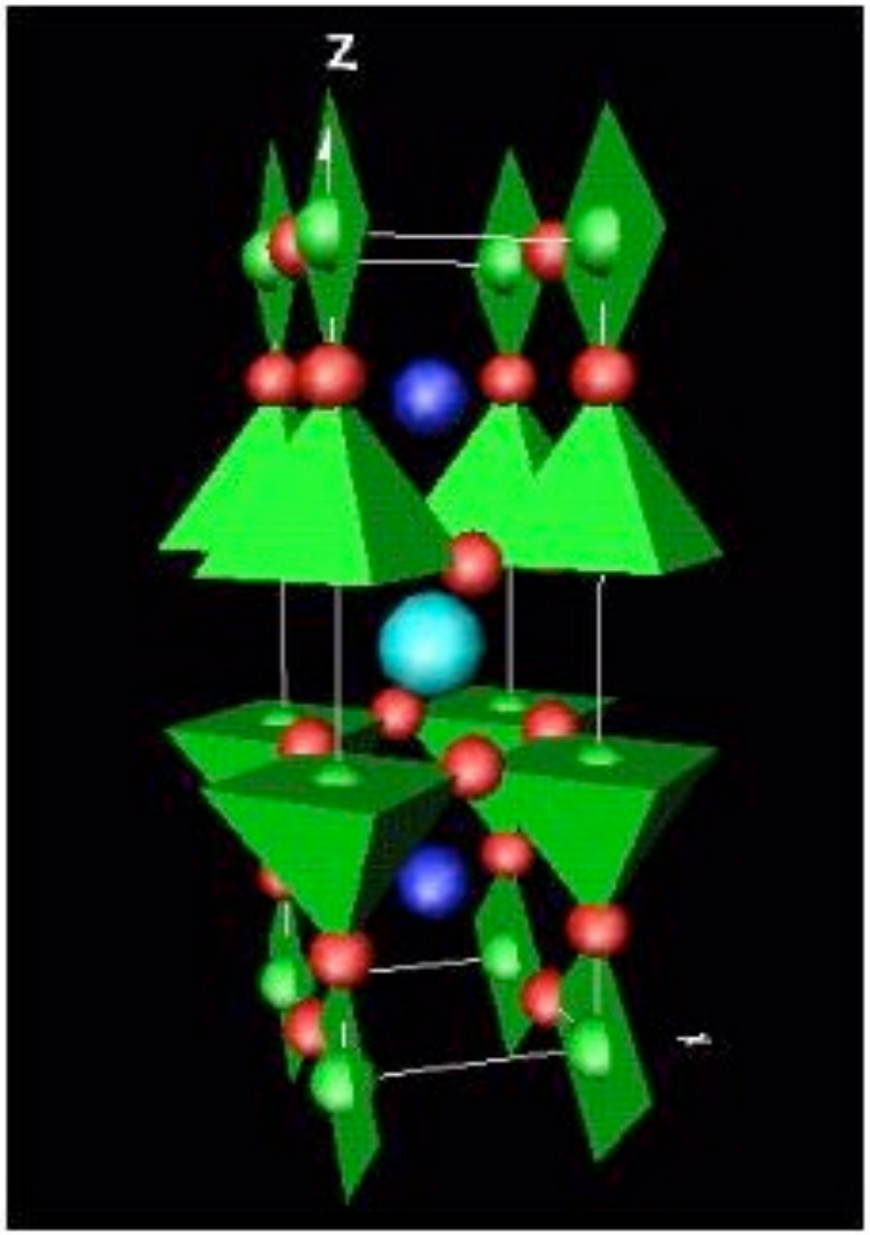
$$B = B_0 e^{-r/\lambda}$$



strange metal

$YBa_2Cu_3O_7$

Cuprate Superconductors

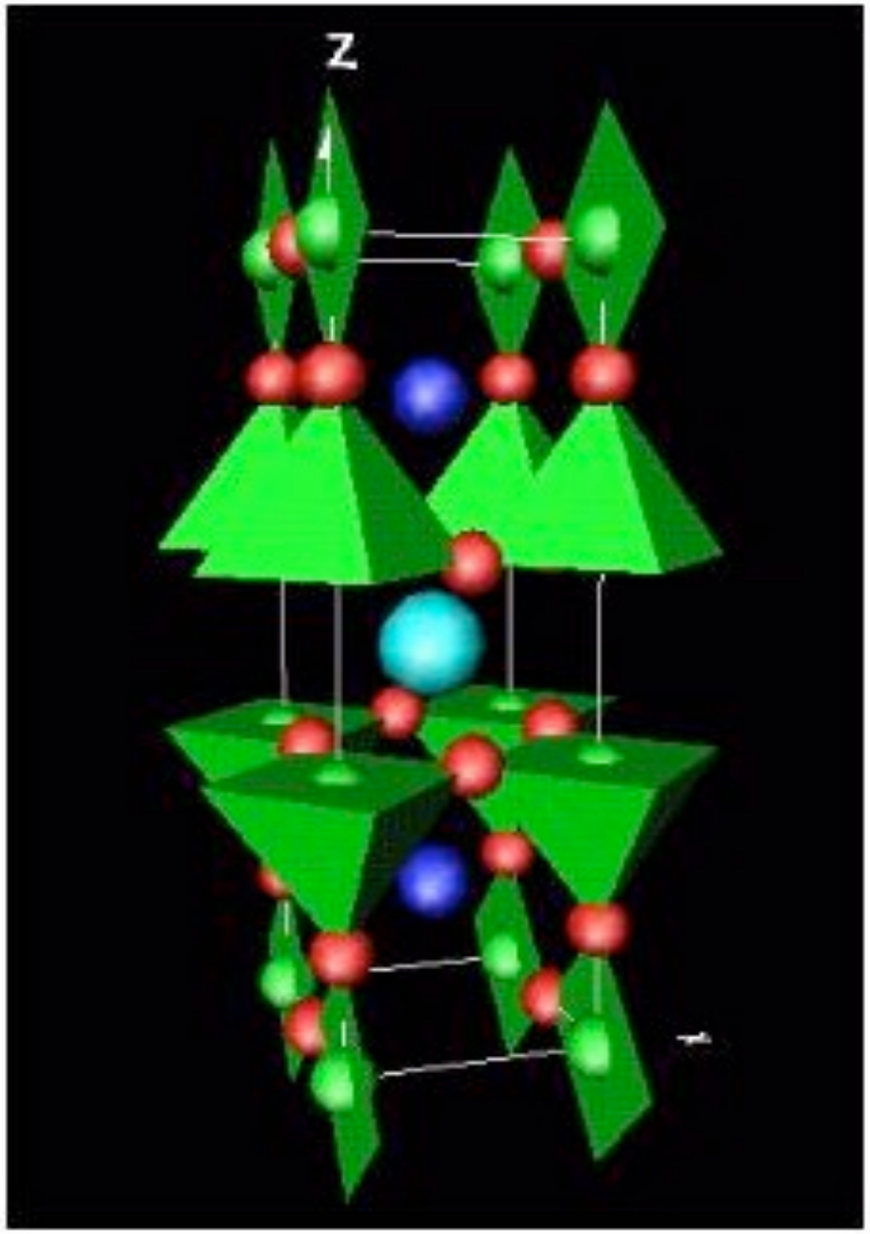


$Y\,Ba_2Cu_3O_7$

Cuprate Superconductors

strange metal

$$[J] = d_U$$



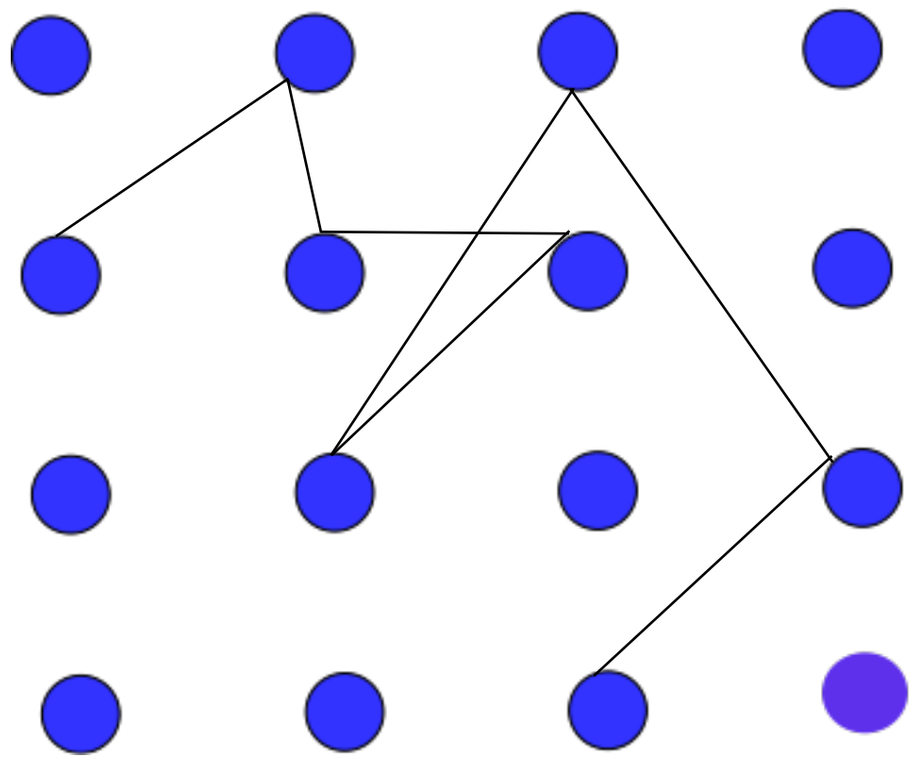
$YBa_2Cu_3O_7$
Cuprate Superconductors

strange metal

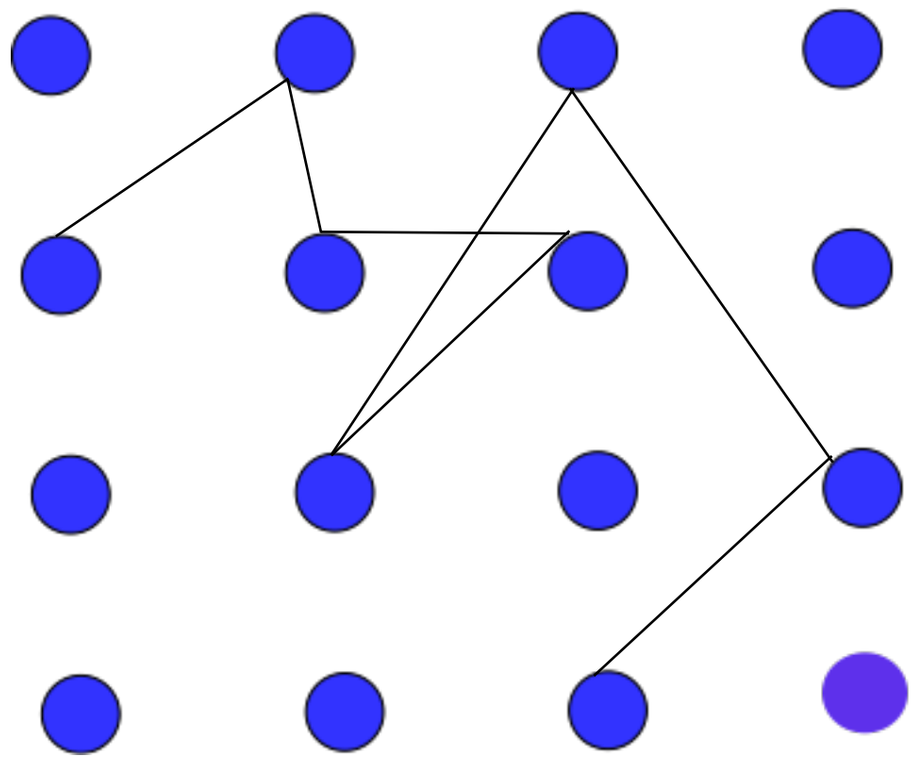
$[J] = d_U$ fractional

what is the strange metal?

Drude metal



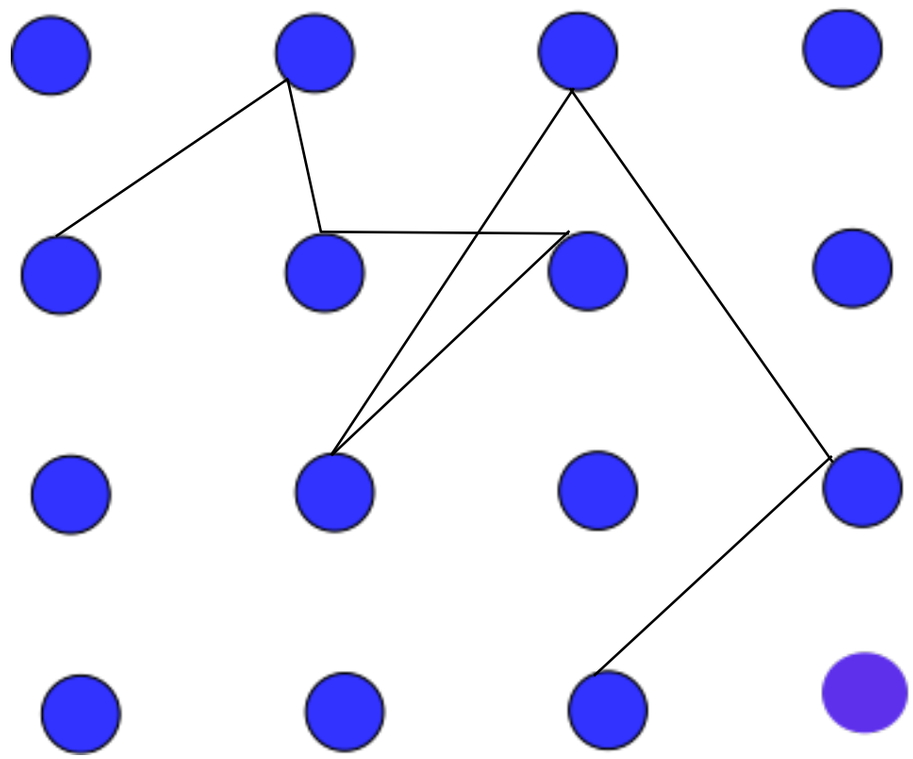
Drude metal



$$\dot{p} = e(E + \frac{p \times B}{m}) - \frac{p}{\tau}$$

momentum relaxation

Drude metal

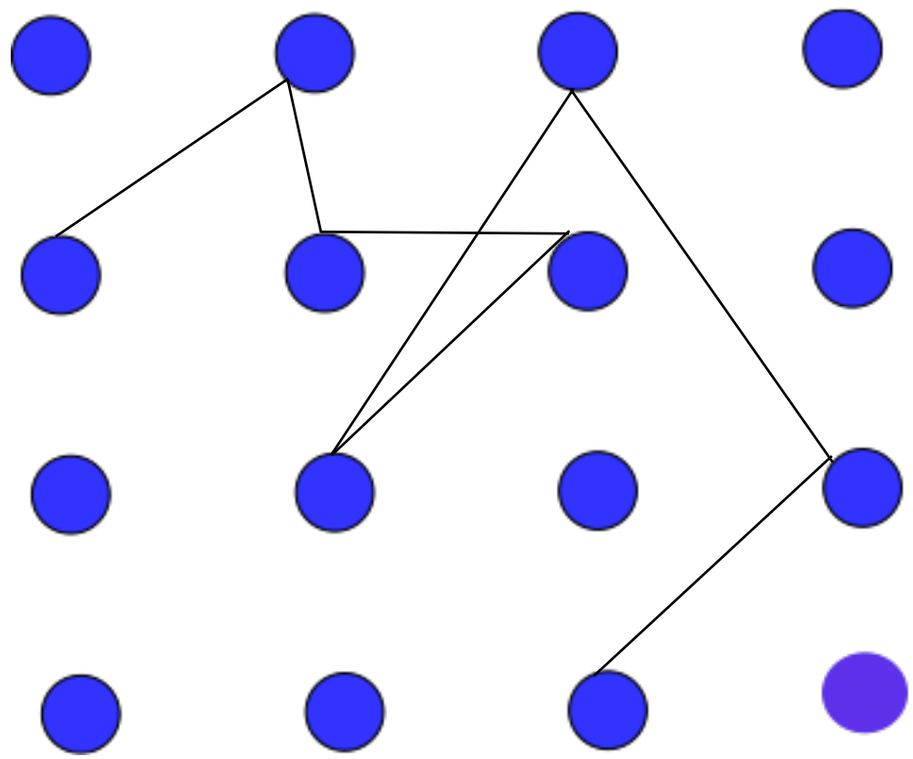


$$\dot{p} = e(E + \frac{p \times B}{m}) - \frac{p}{\tau}$$

momentum relaxation

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}$$

Drude metal



$$\dot{p} = e(E + \frac{p \times B}{m}) - \frac{p}{\tau}$$

momentum relaxation

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau} \Rightarrow \lim_{\tau \to 0} \Re\sigma \to \infty$$

f-sum rule

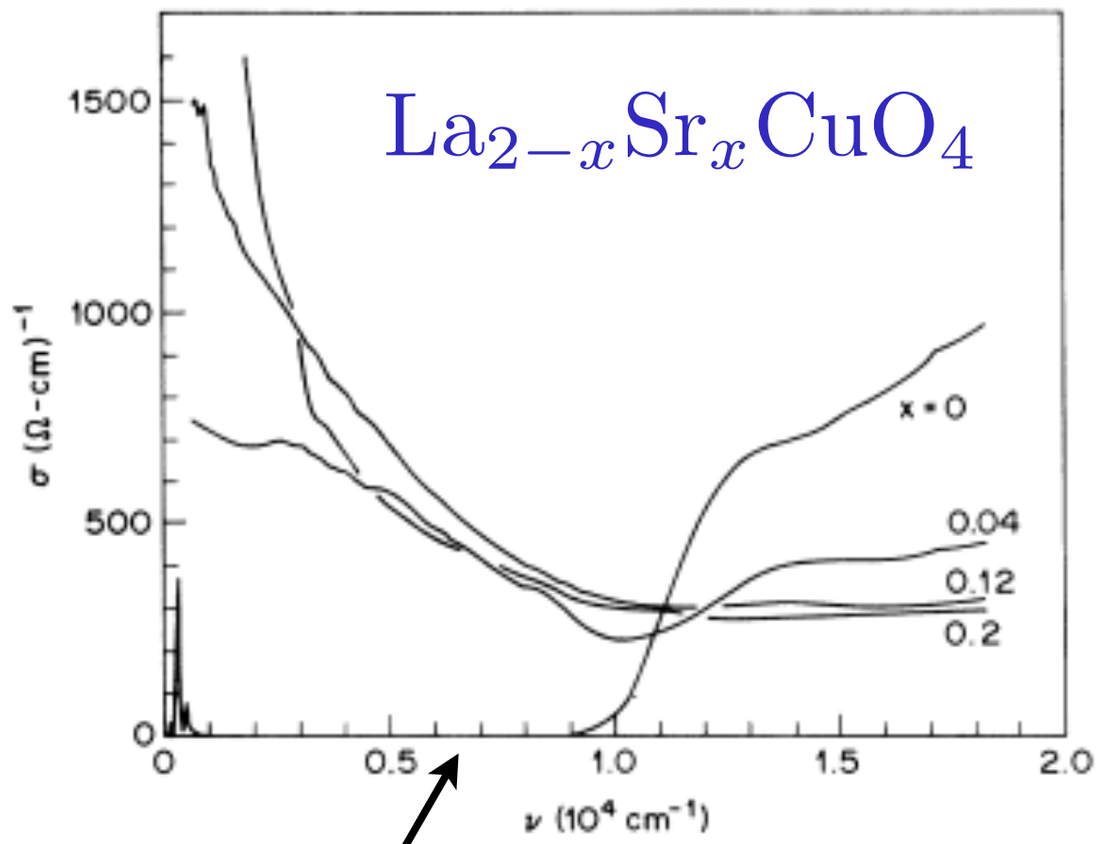
$$N_{\text{eff}} = \frac{2mV_{\text{cell}}}{\pi e^2} \int_0^\infty \sigma(\omega) d\omega$$

strange metal

Growth of the optical conductivity in the Cu-O planes

S. L. Cooper, G. A. Thomas, J. Orenstein, D. H. Rapkine, A. J. Millis, S-W. Cheong, and A. S. Cooper
AT&T Bell Laboratories, Murray Hill, New Jersey 07974

Z. Fisk
Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 7 March 1990)



optical gap
(Mott gap)

What is a Mott Insulator?

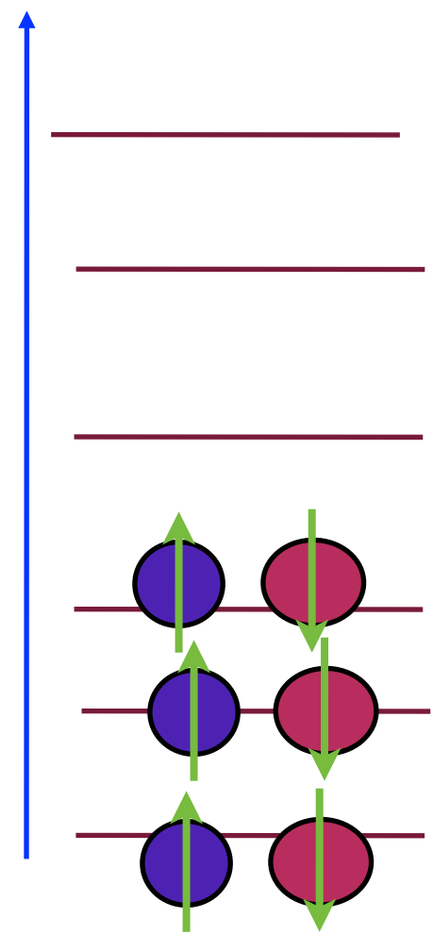
NiO insulates

d^8 ?

What is a Mott Insulator?

NiO insulates

d^8 ?

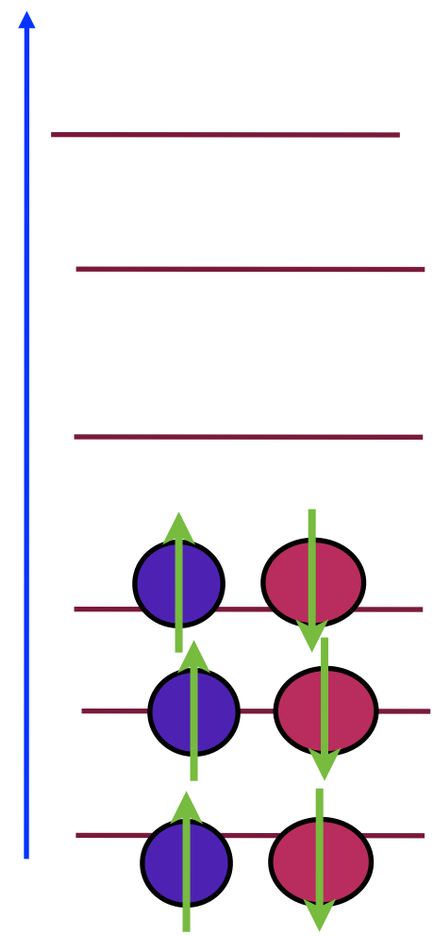


EMPTY STATES=
METAL

What is a Mott Insulator?

NiO insulates

d^8 ?



EMPTY STATES=
METAL

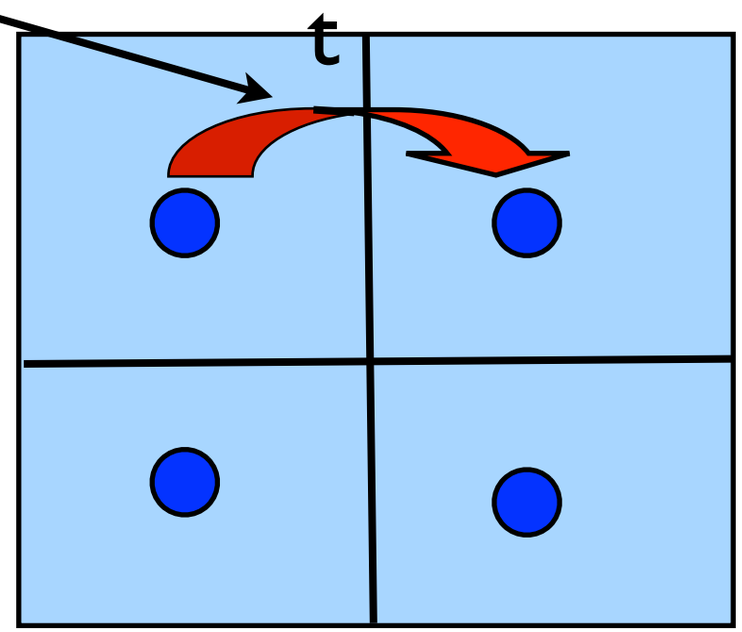
band theory fails!

Mott mechanism

NiO insulates

d^8 ?

perhaps this costs energy



Mott mechanism

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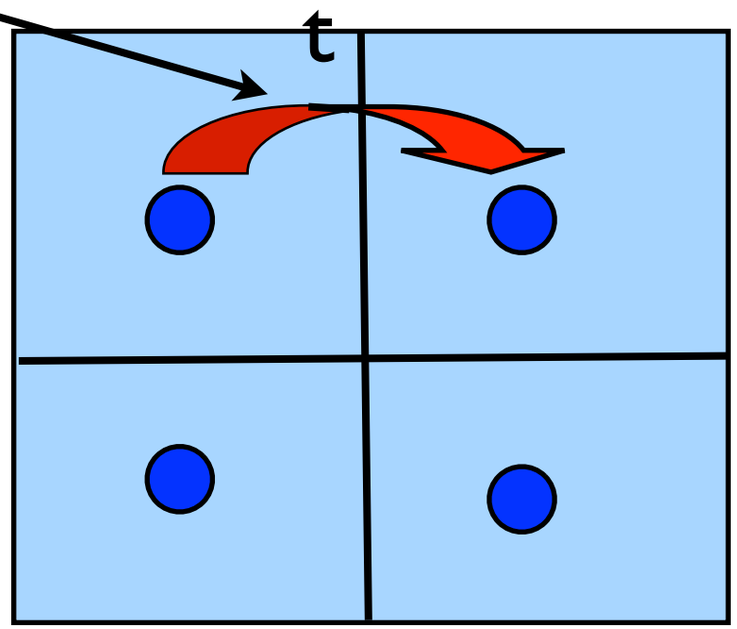
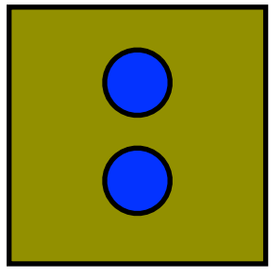




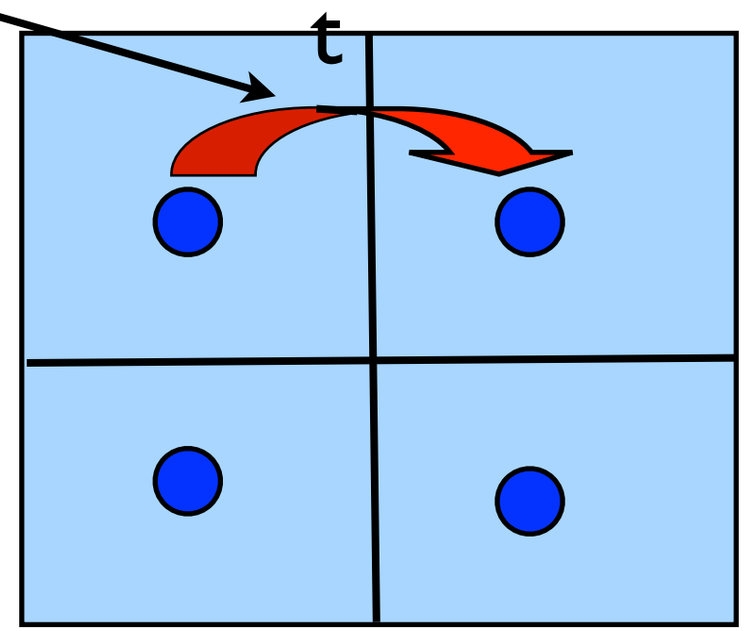
$$U \gg t$$

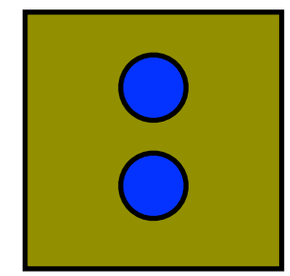
Mott mechanism

NiO insulates

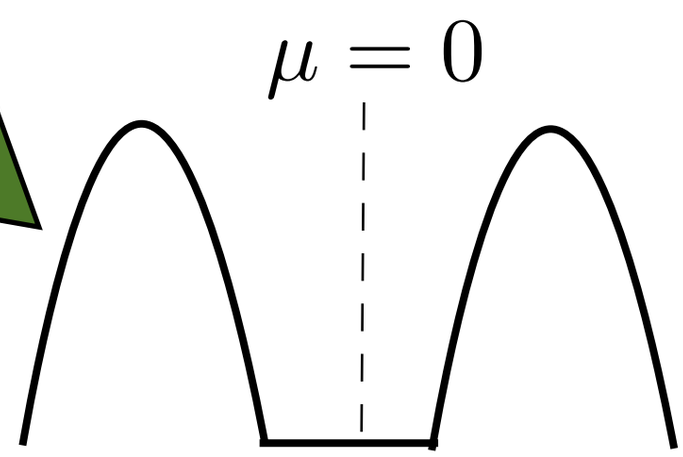
d^8 ?

perhaps this costs energy





$U \gg t$



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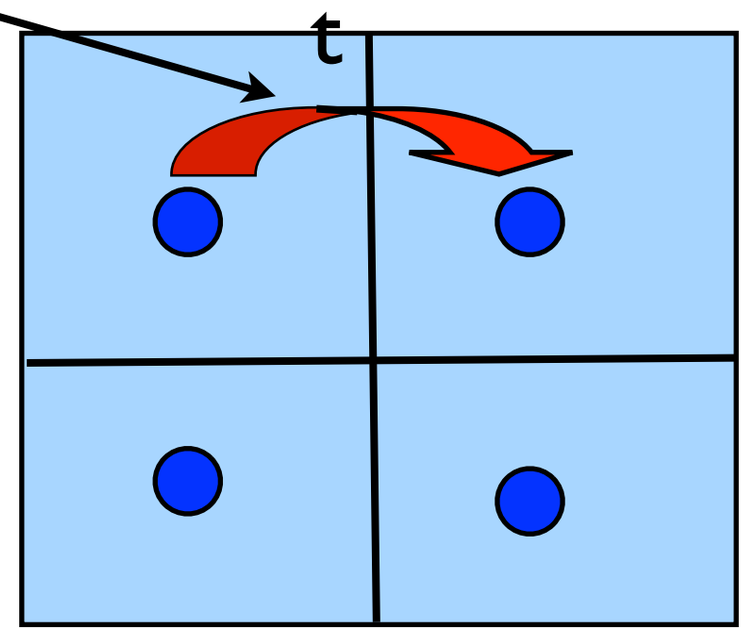

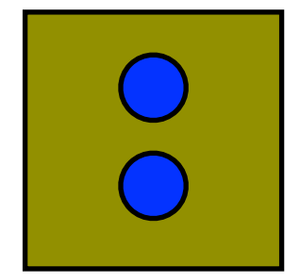

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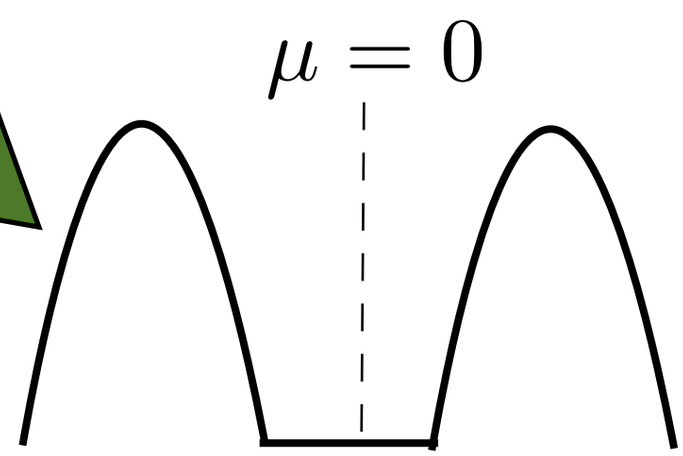


no change in size of Brillouin zone

doping dependence

modified f-sum rule

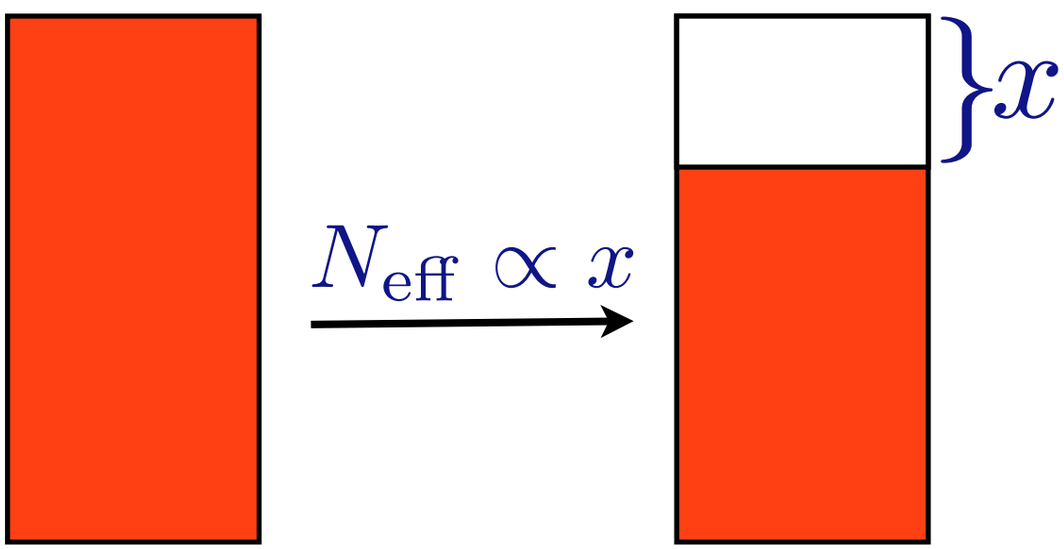
optical gap

$$N_{\text{eff}}(\Omega) = \frac{2mV_{\text{cell}}}{\pi e^2} \int_0^{\Omega} \sigma(\omega) d\omega$$

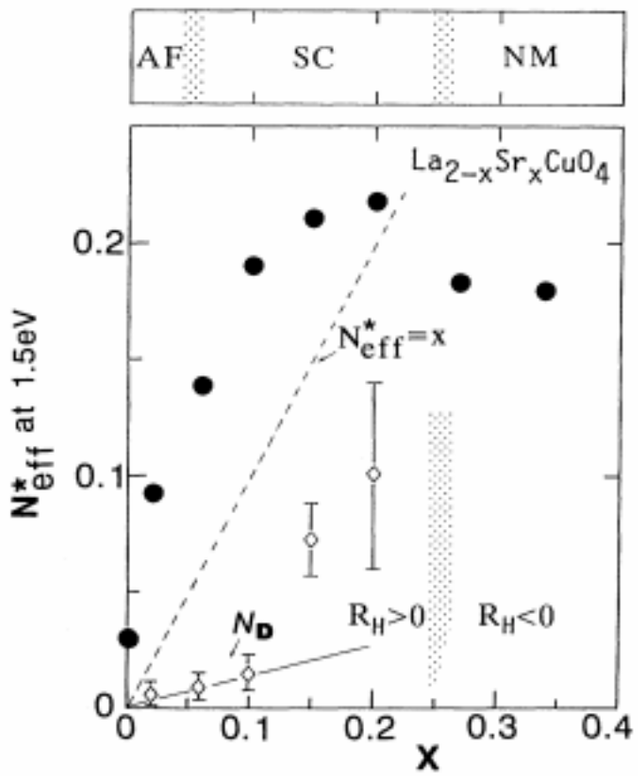
modified f-sum rule

optical gap

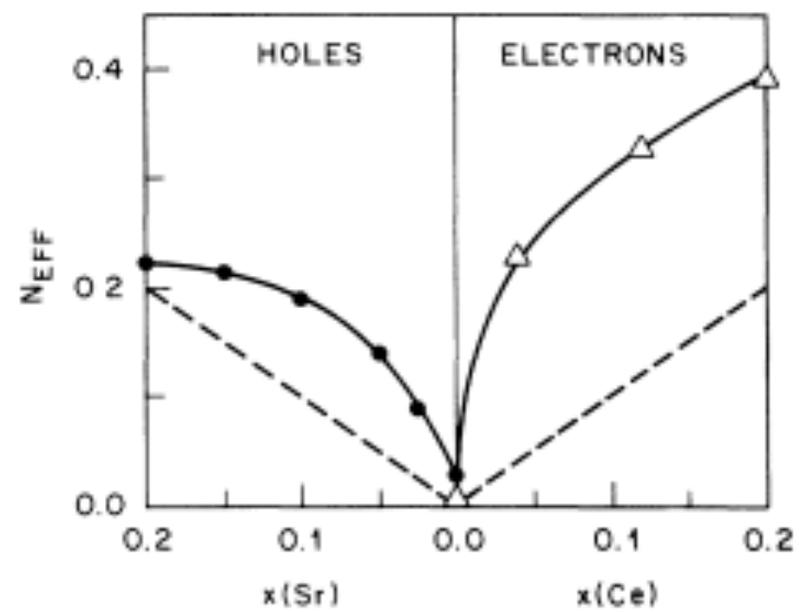
$$N_{\text{eff}}(\Omega) = \frac{2mV_{\text{cell}}}{\pi e^2} \int_0^{\Omega} \sigma(\omega) d\omega$$



Uchida, et al.



Cooper, et al.



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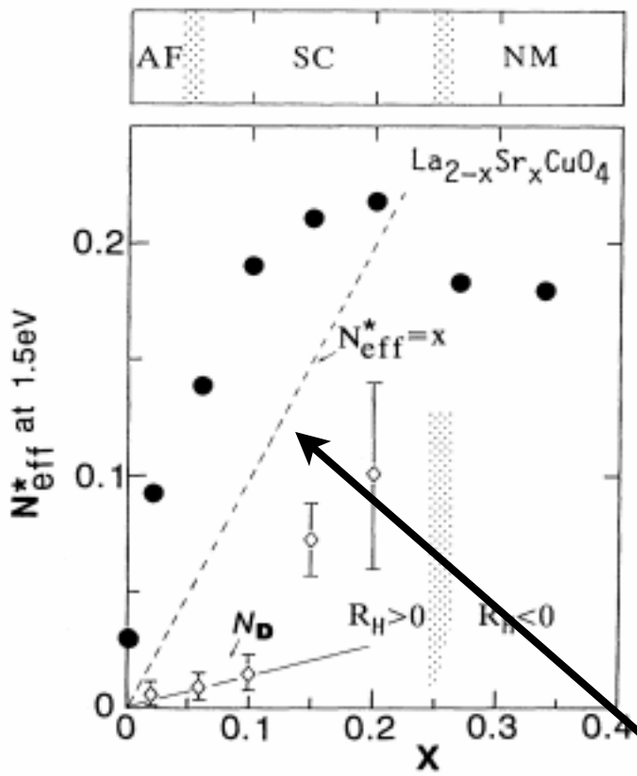
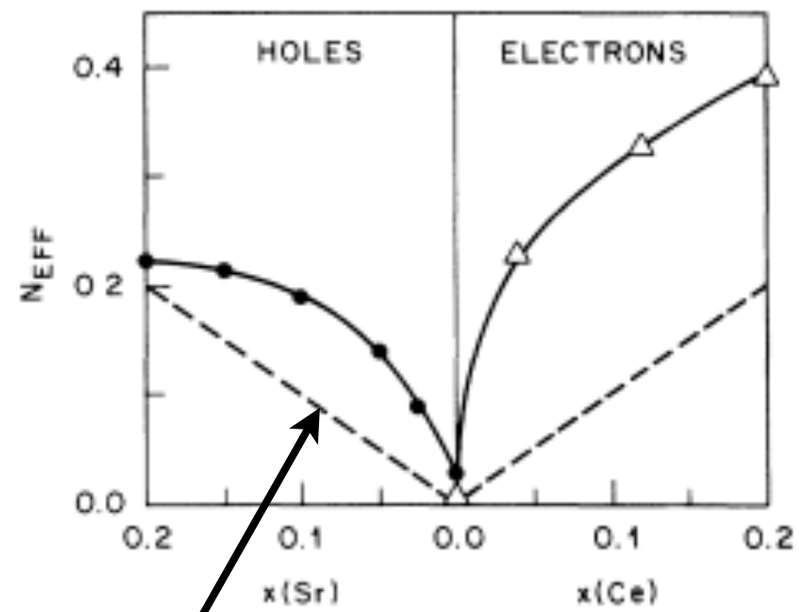


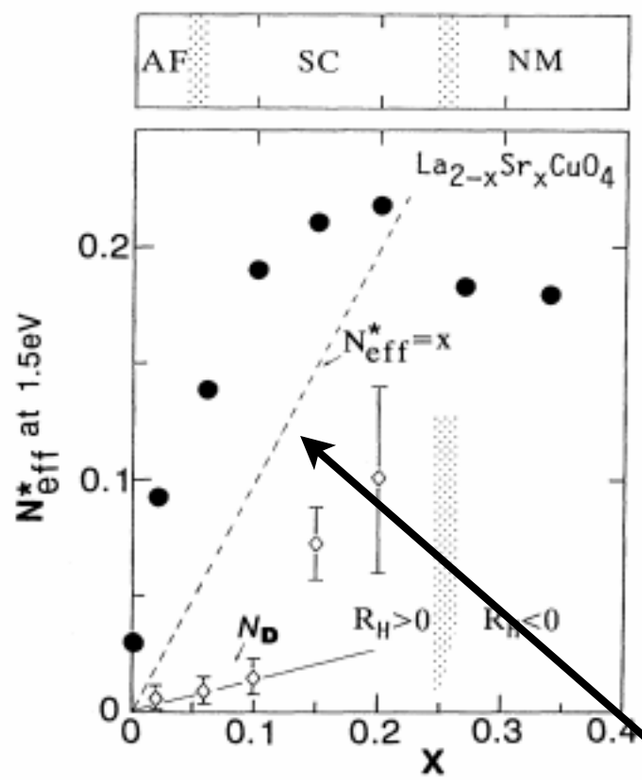
Cooper, et al.

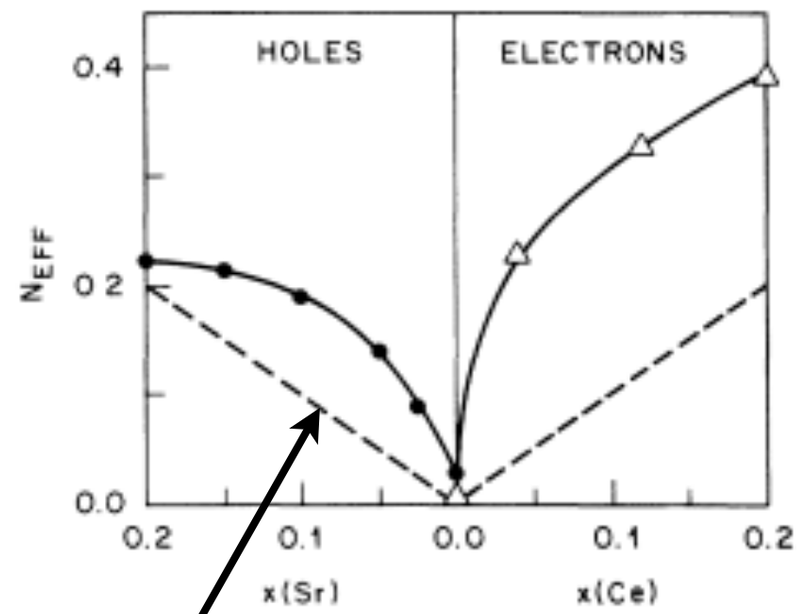


x

Uchida, et al.

Cooper, et al.





x

low-energy model for $N_{\text{eff}} > x$??

excess carriers?

excess carriers?

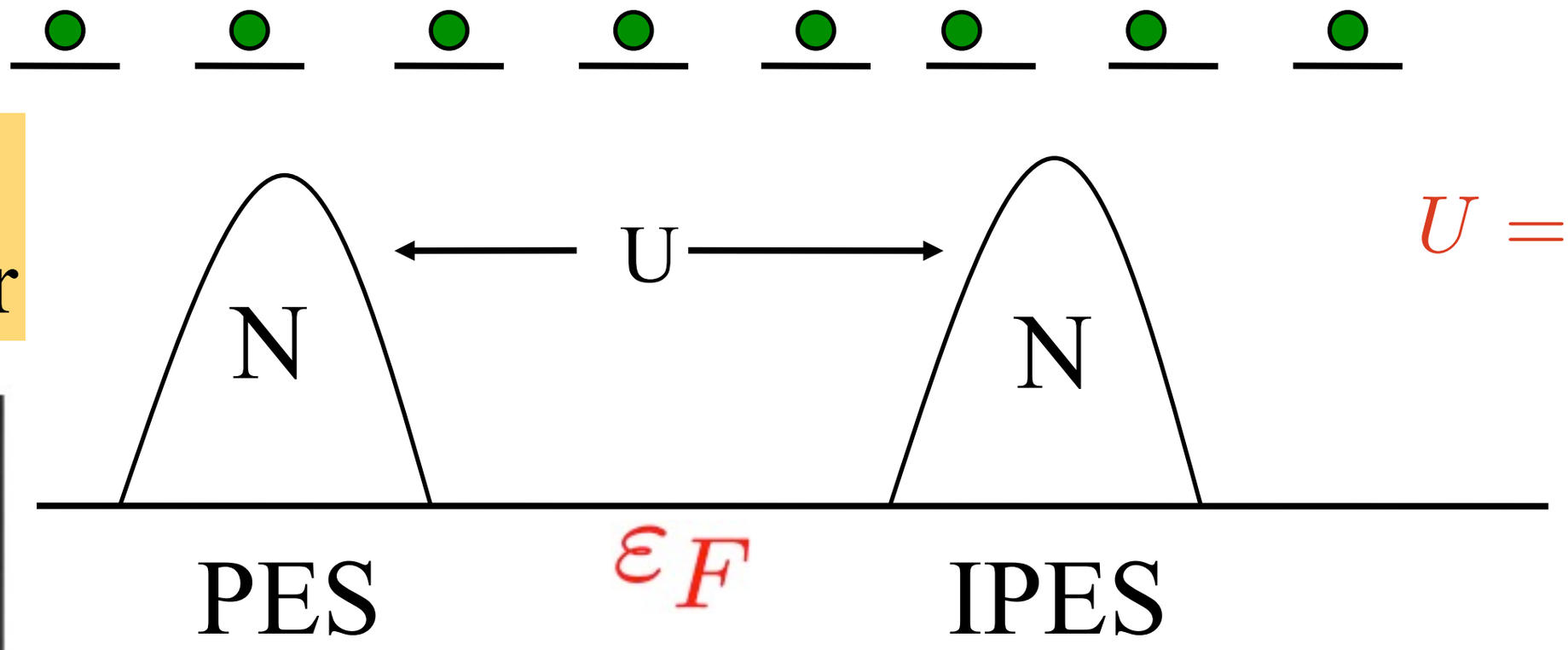
charge carriers have no particle content?

Mott insulator

Mott insulator

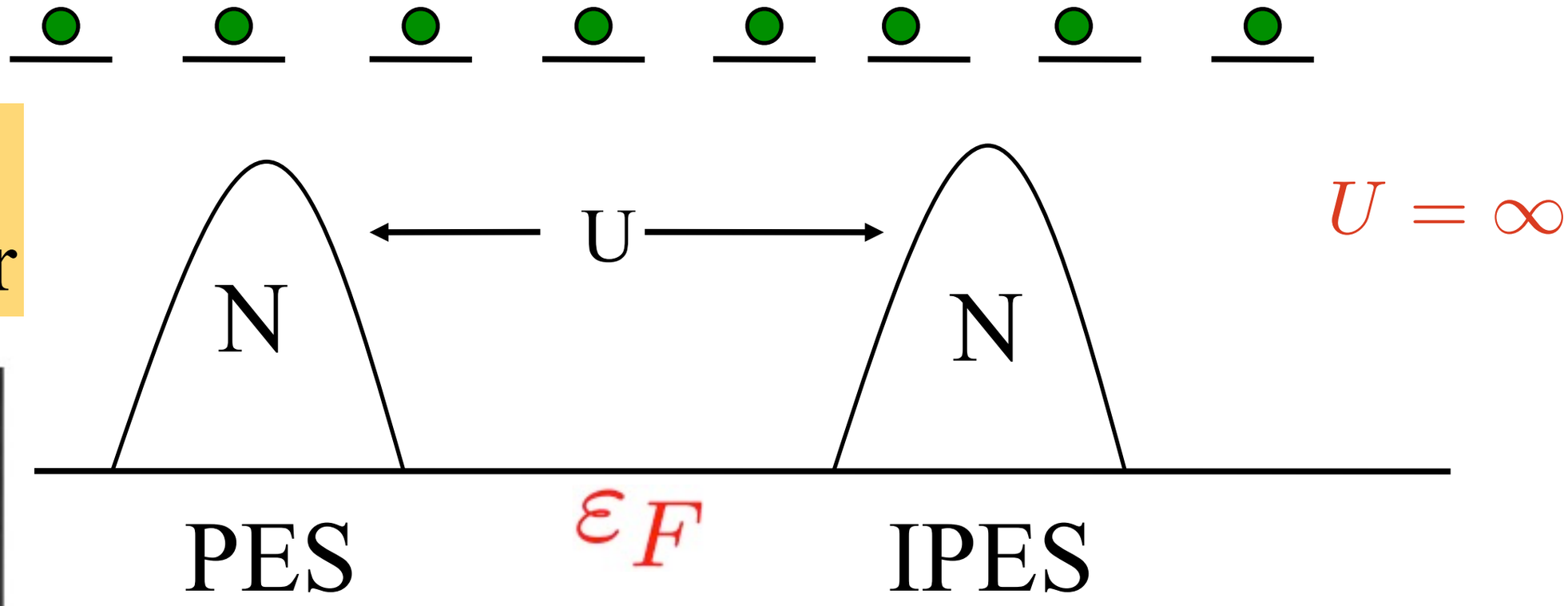
$U = \infty$

Mott insulator



$U = \infty$

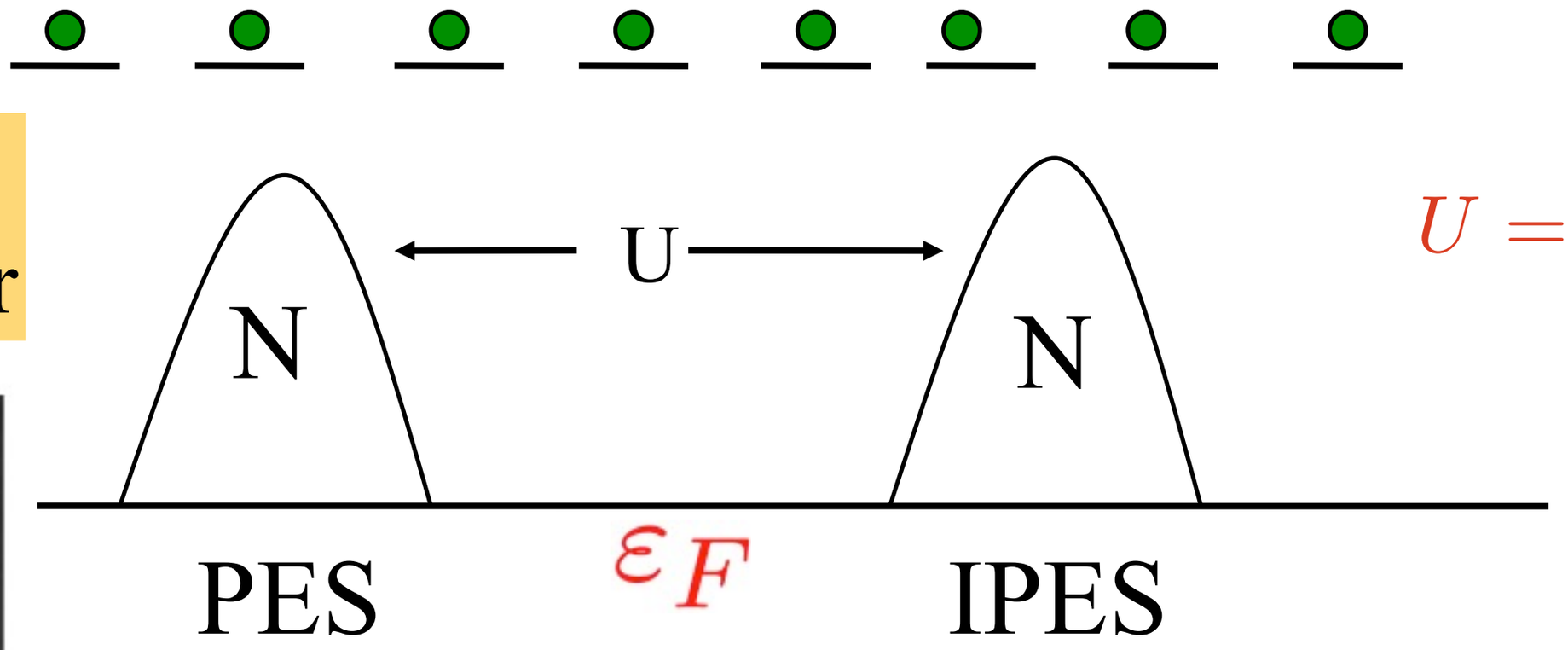
Mott insulator



$U = \infty$

no hopping:

Mott insulator



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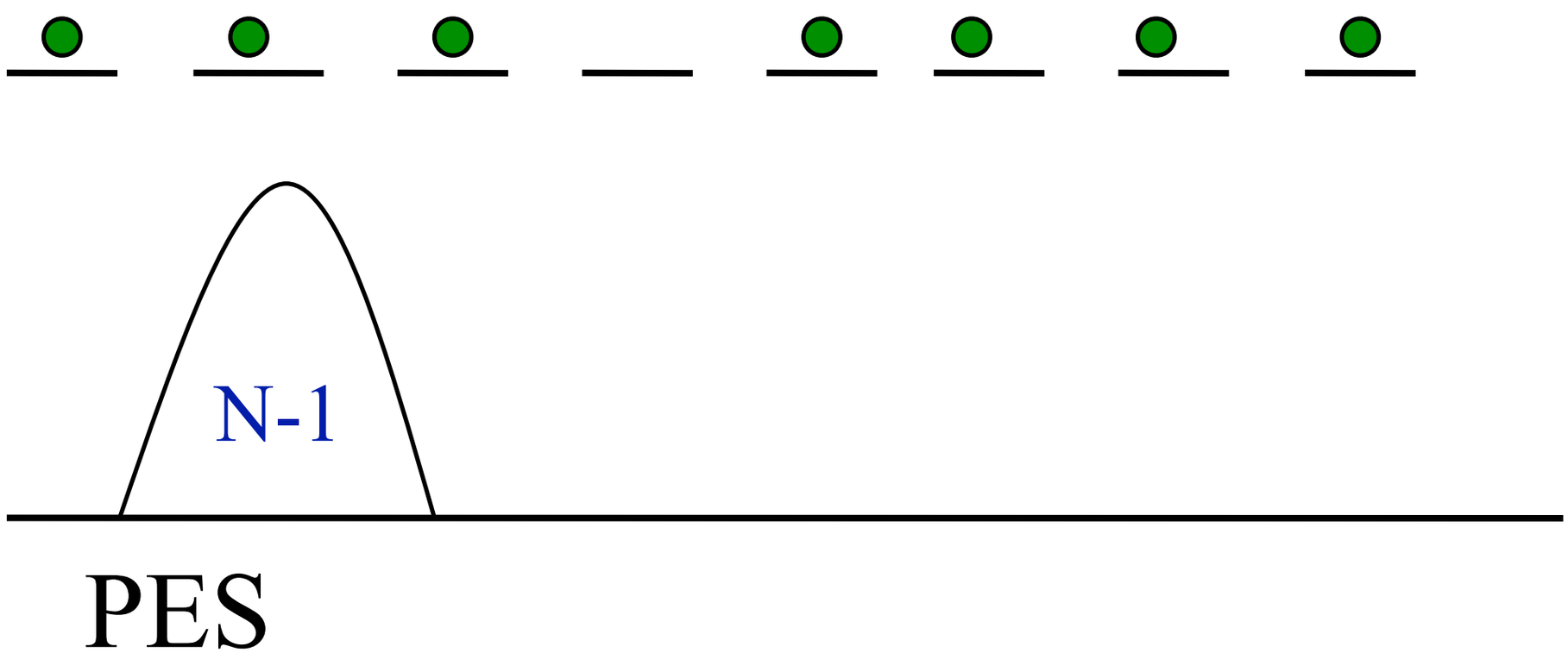
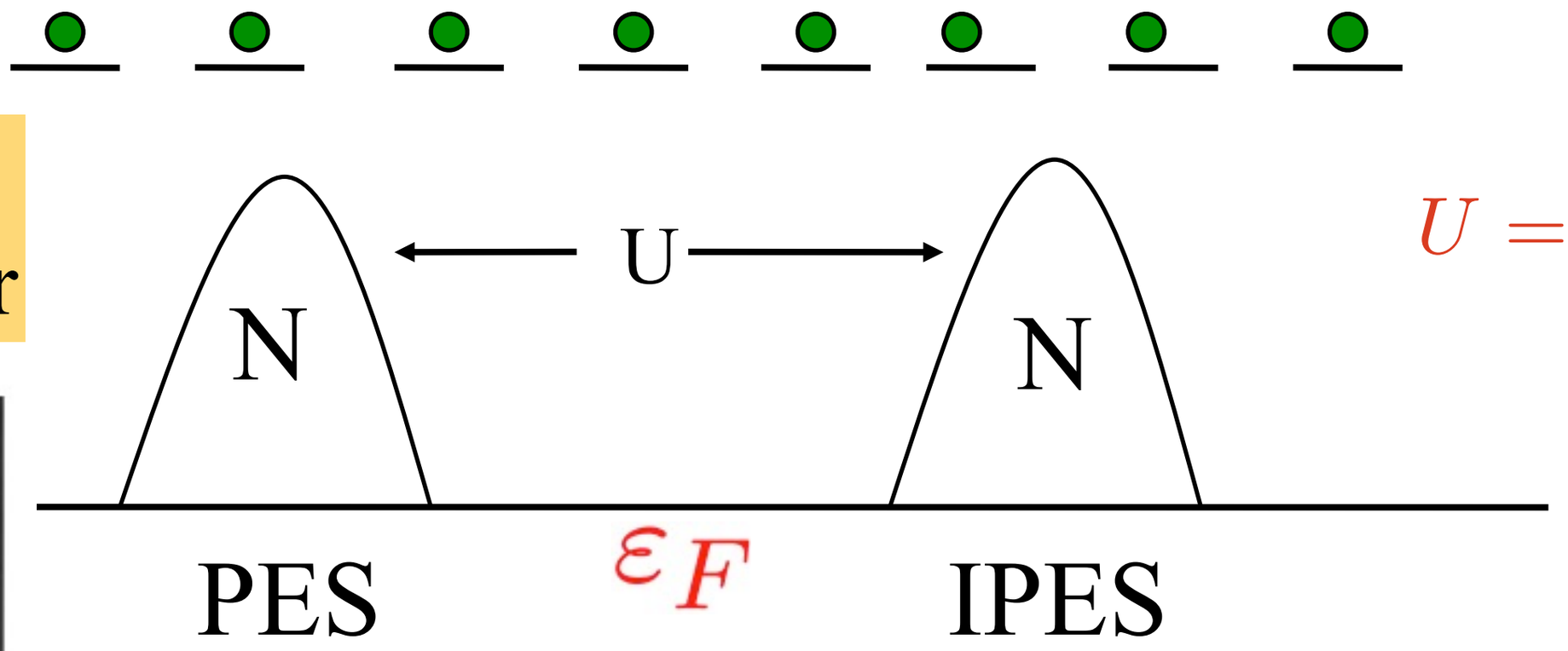


Mott insulator



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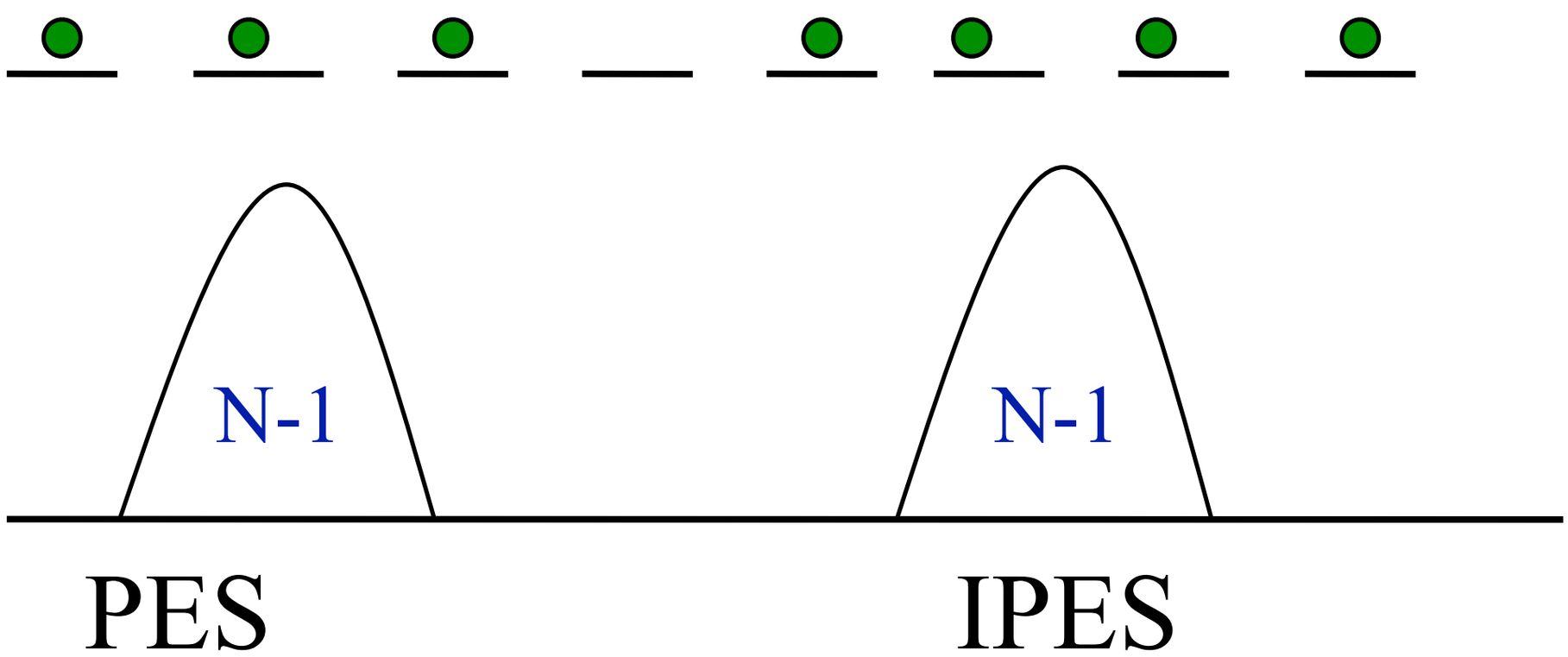
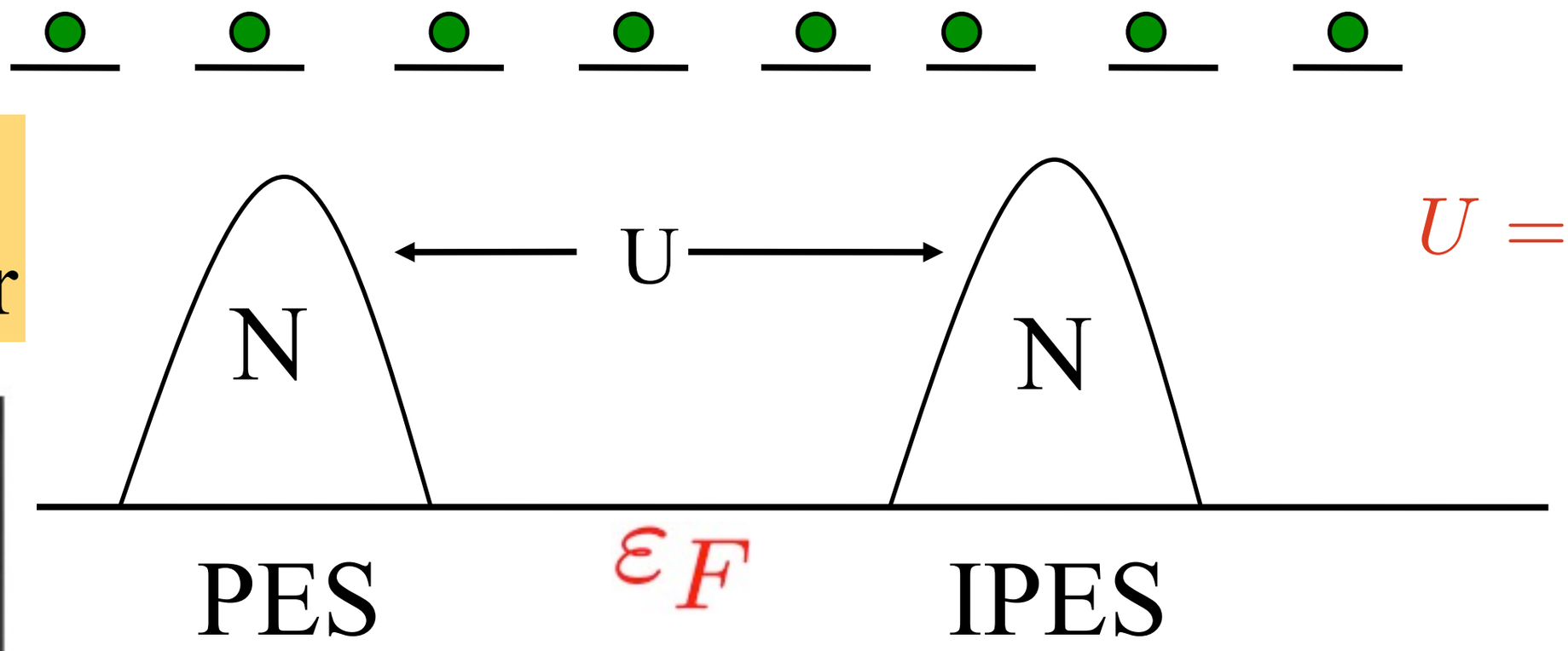
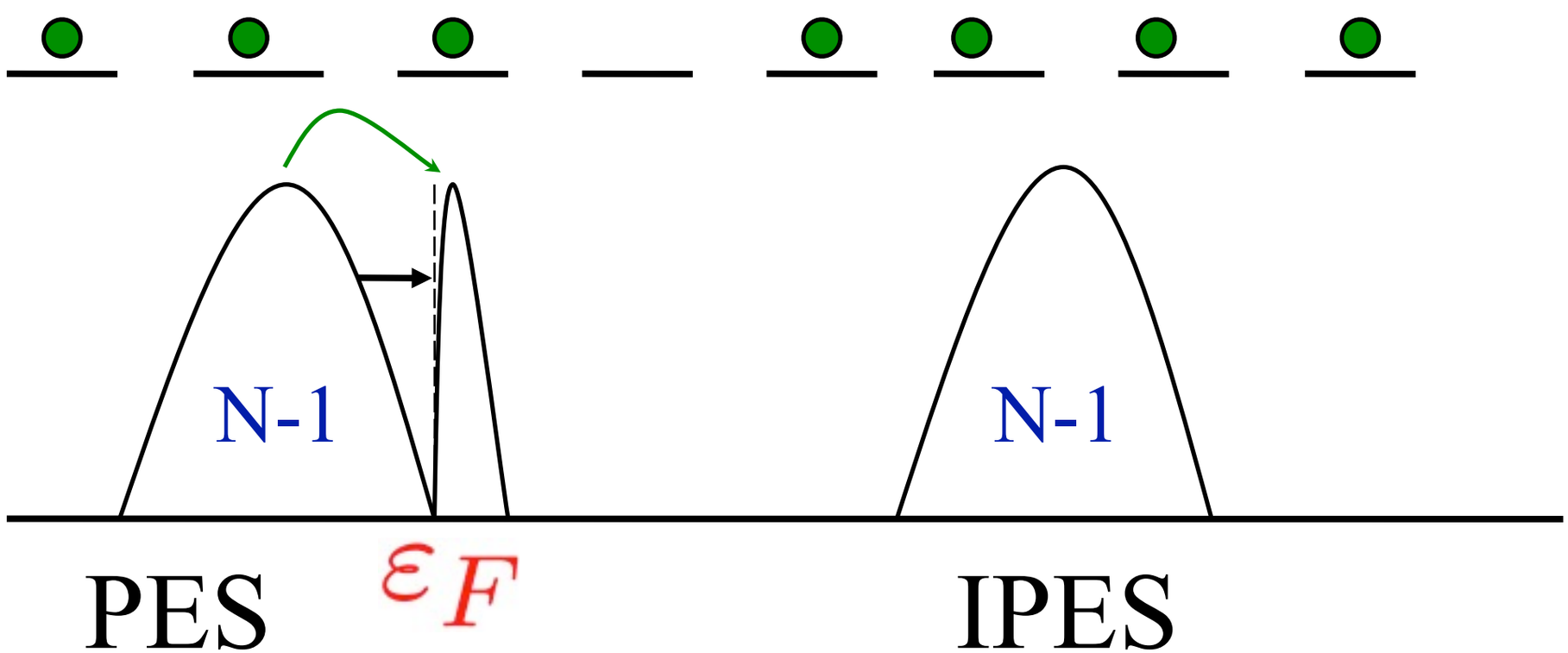


Mott insulator

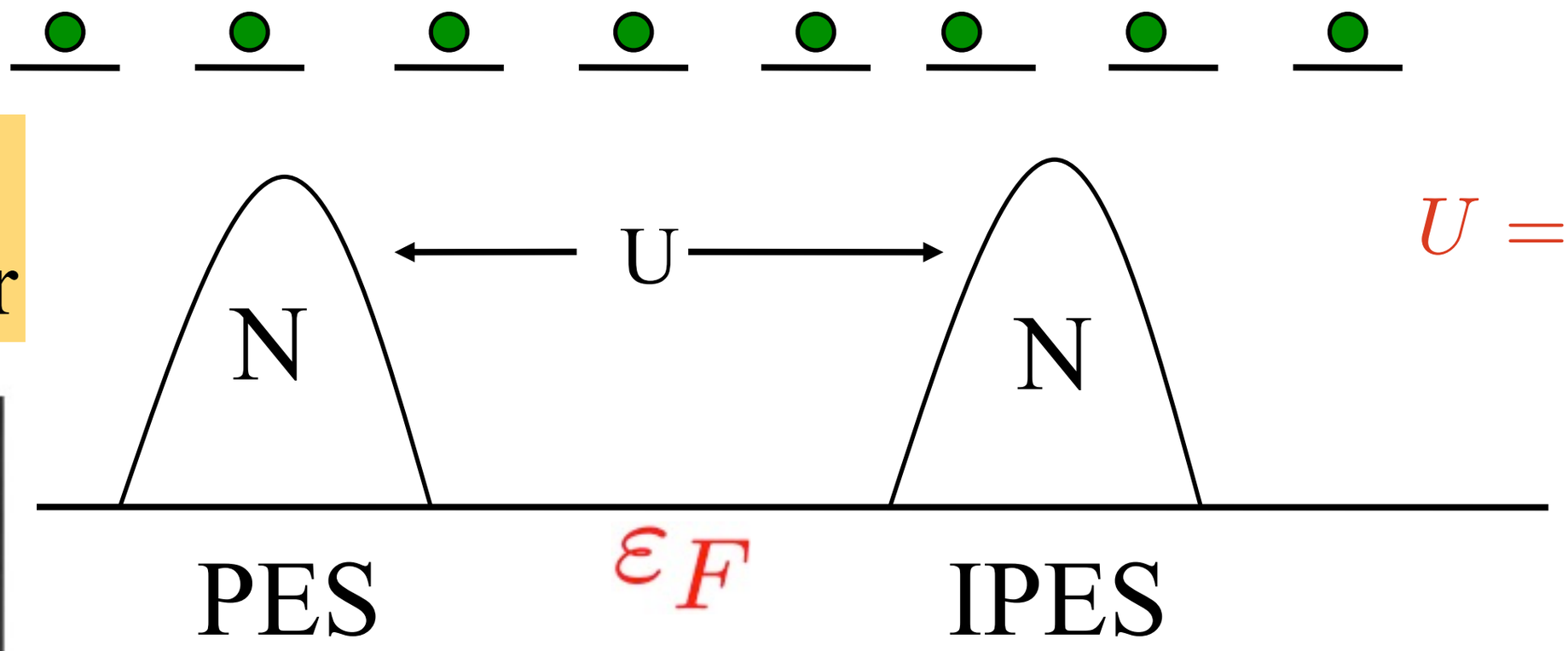


$U = \infty$

no hopping:



Mott insulator



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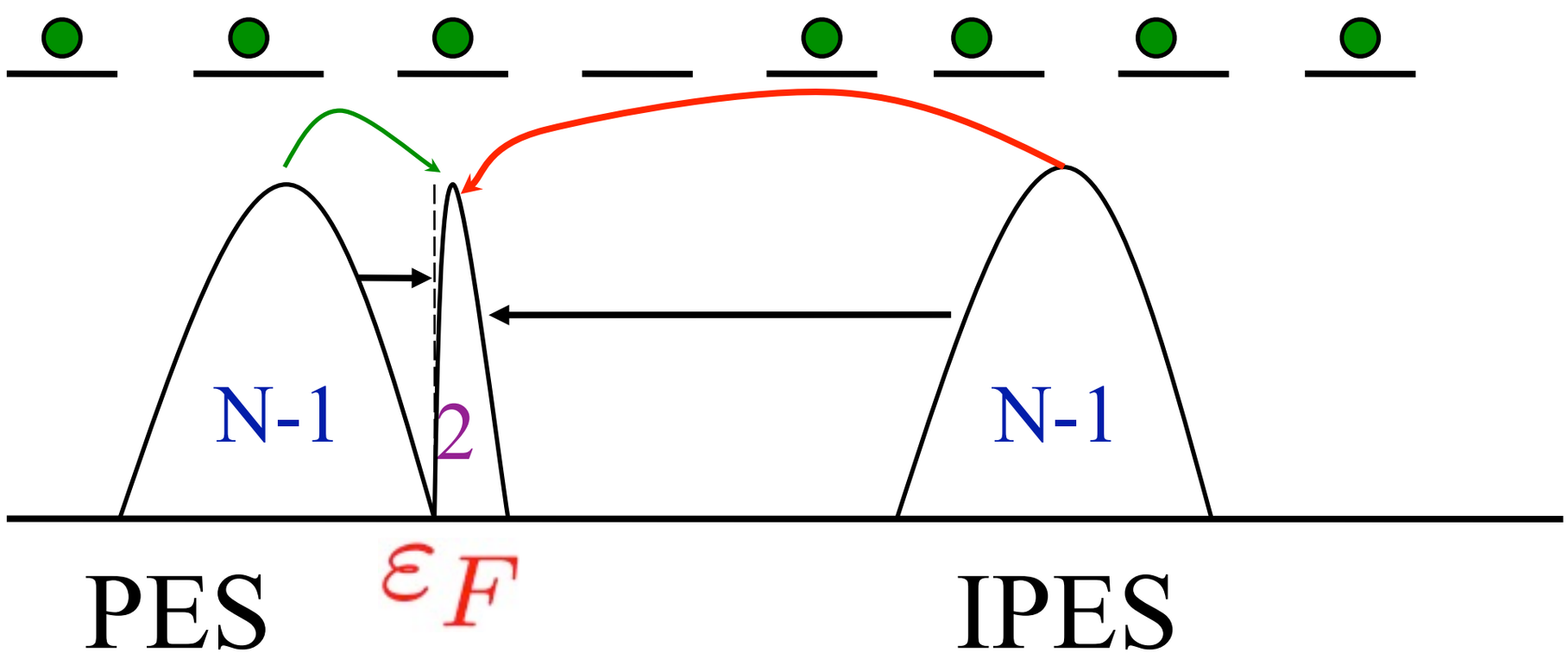
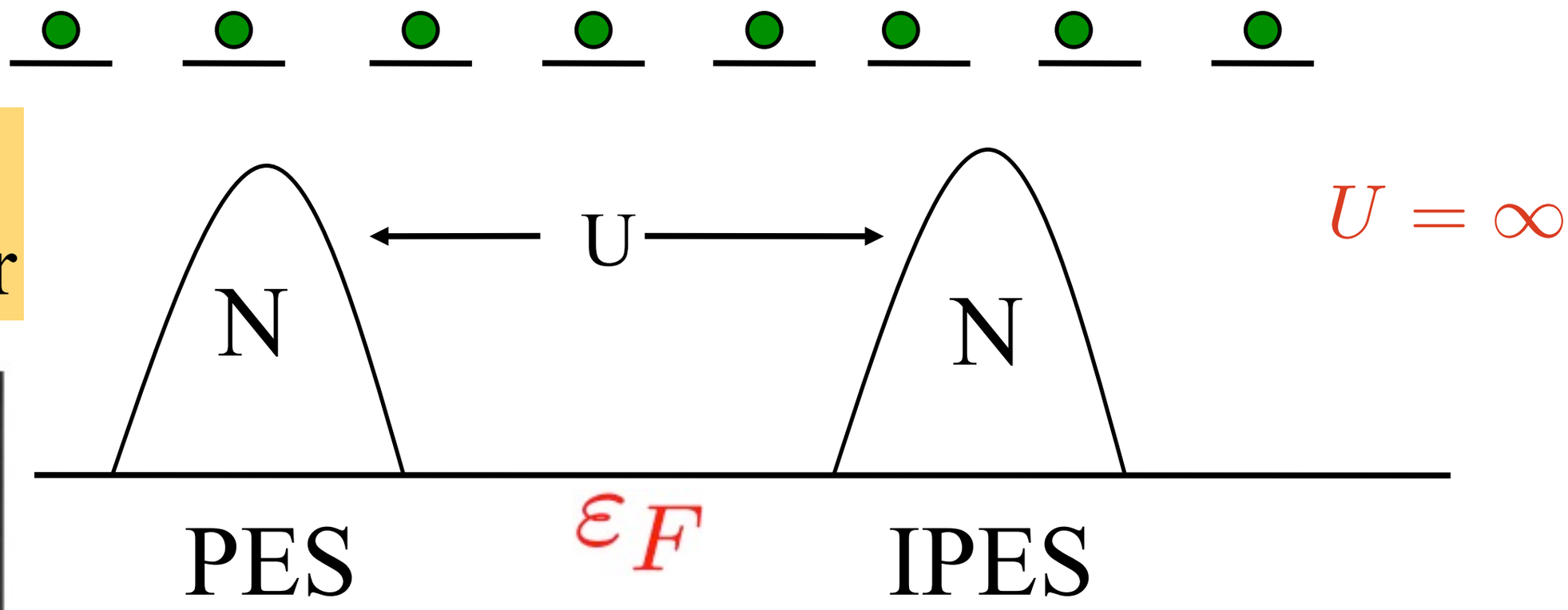
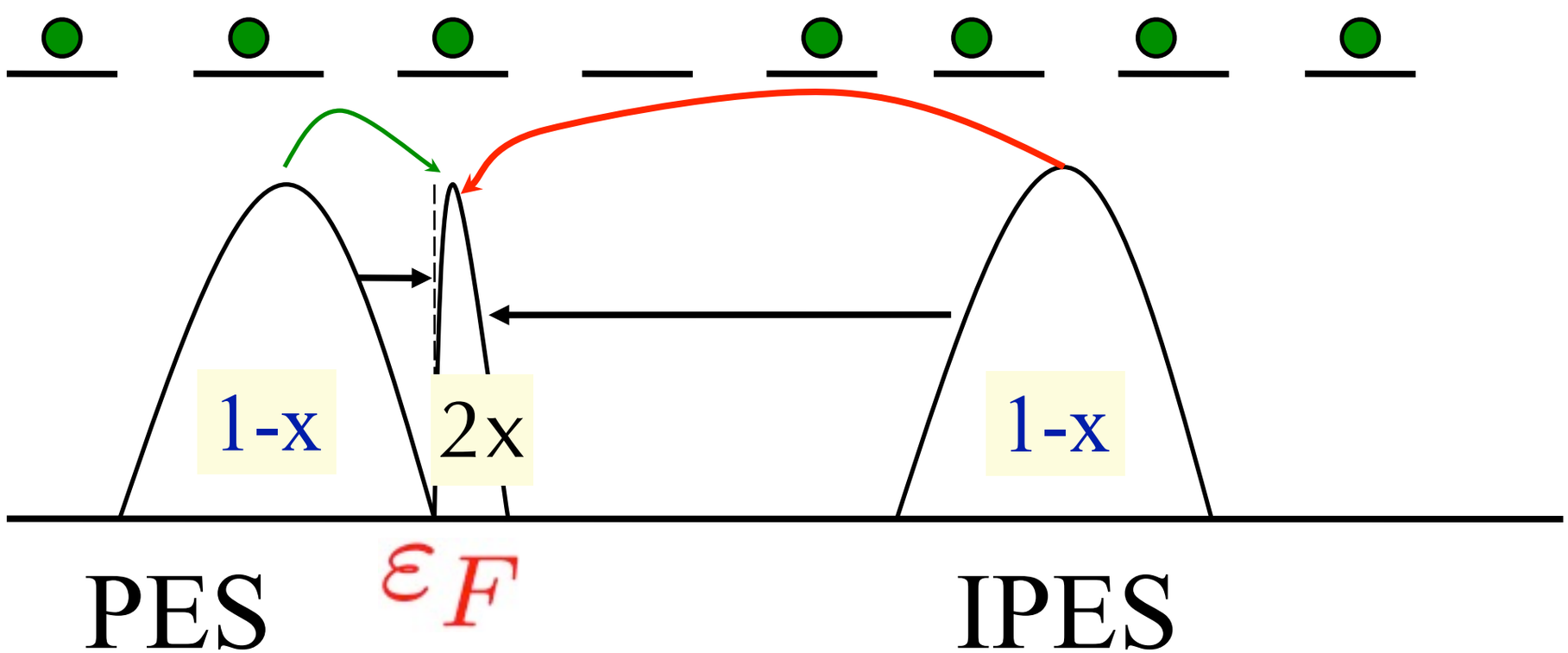


Sawatzky

Mott insulator



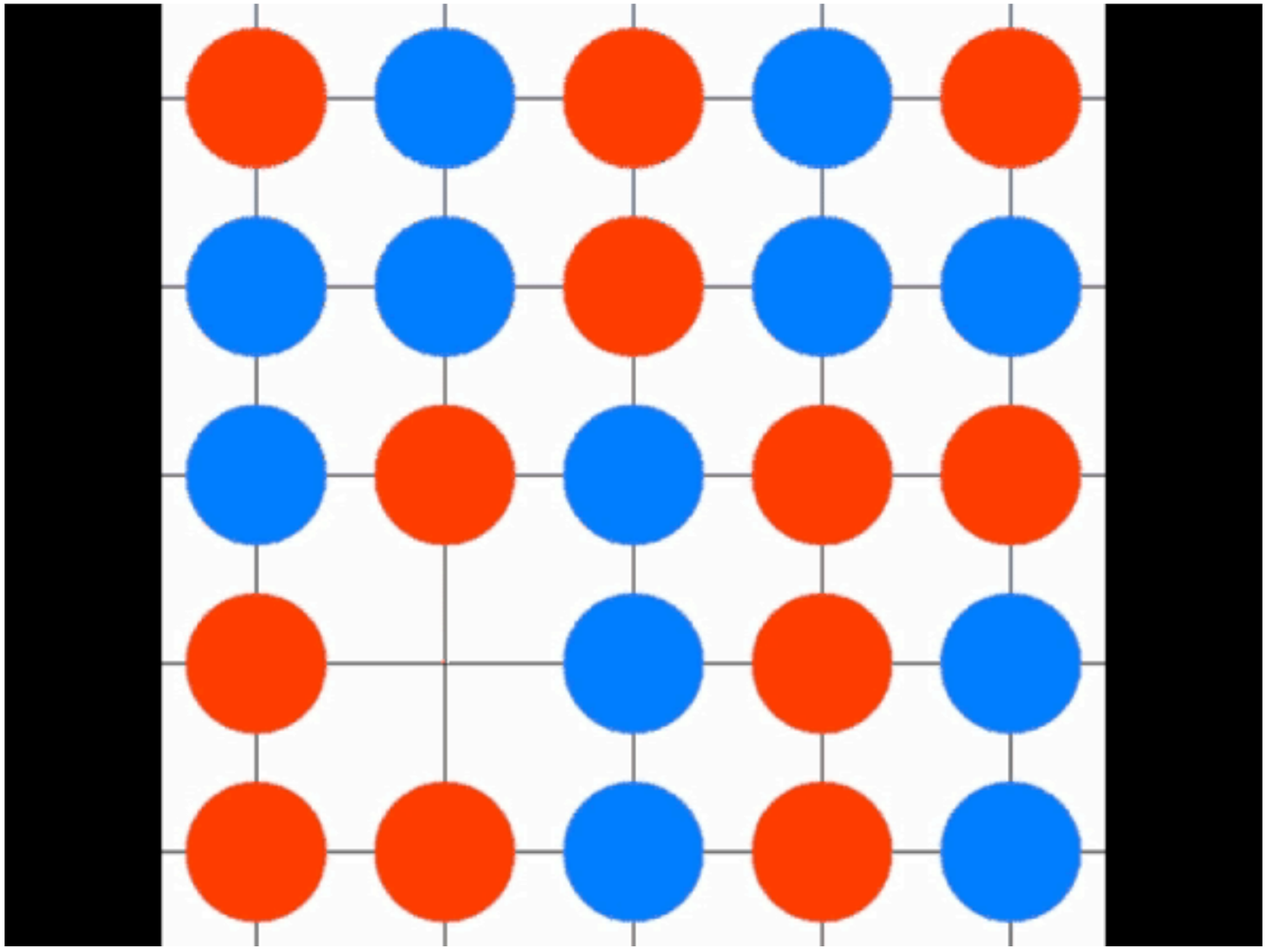
no hopping:



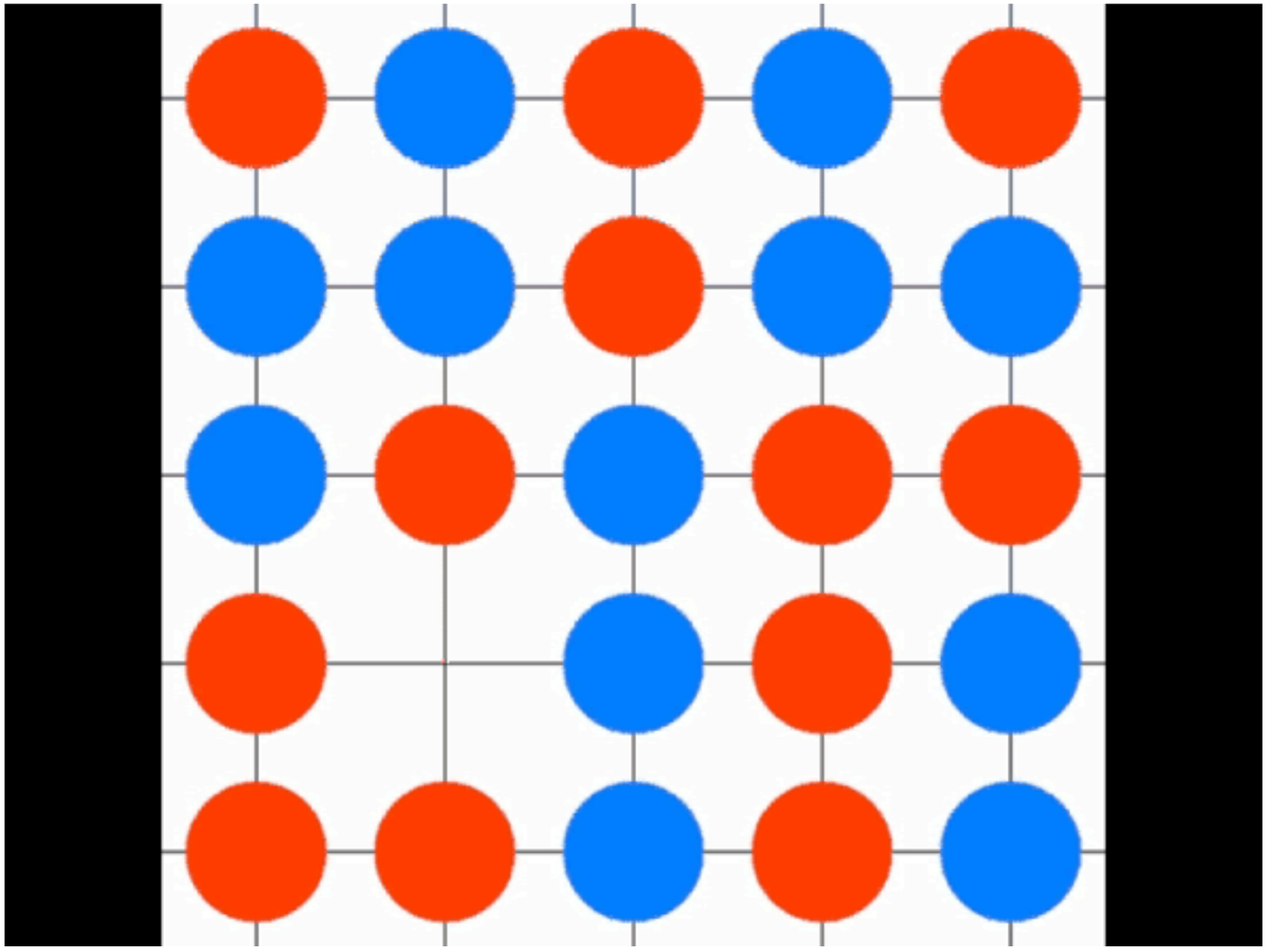
Sawatzky

spectral function (dynamics)

spectral function (dynamics)

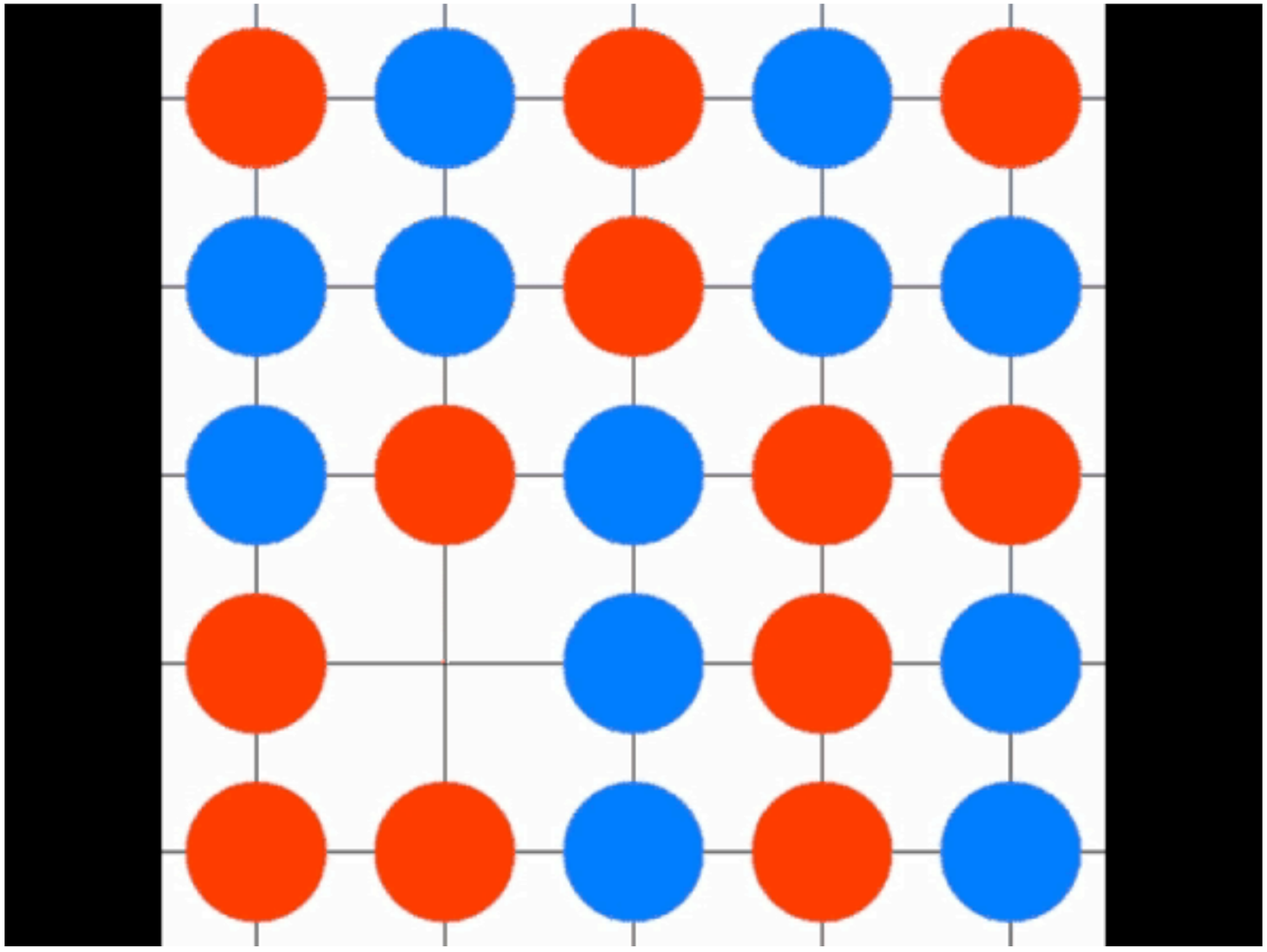


spectral function (dynamics)



low-energy states $> 1 + x$

spectral function (dynamics)



low-energy states $> 1 + x$ Harris/Lange 1967

why is this a problem?

counting electron states

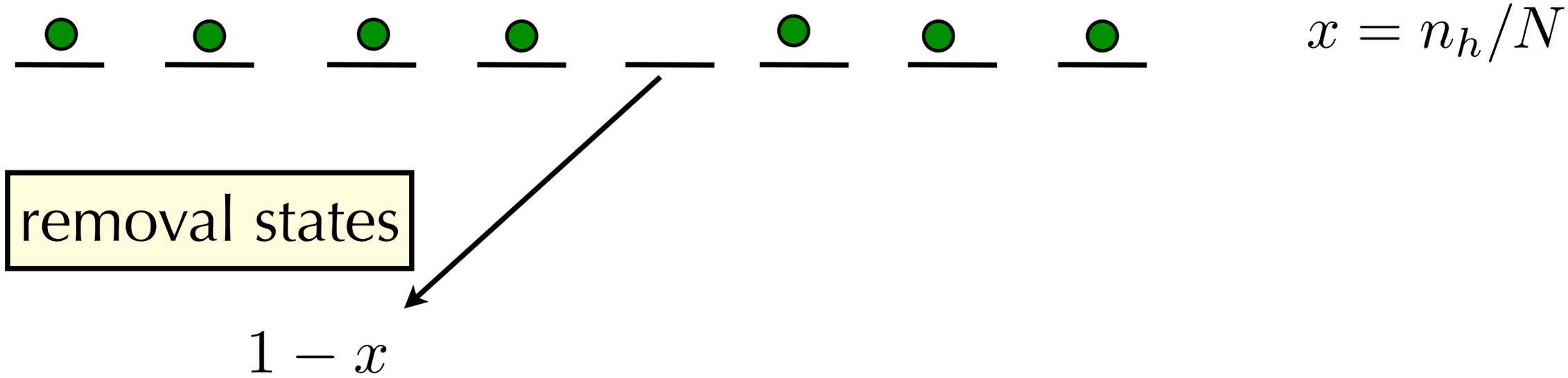
need to know: N (number of sites)

counting electron states

$x = n_h/N$

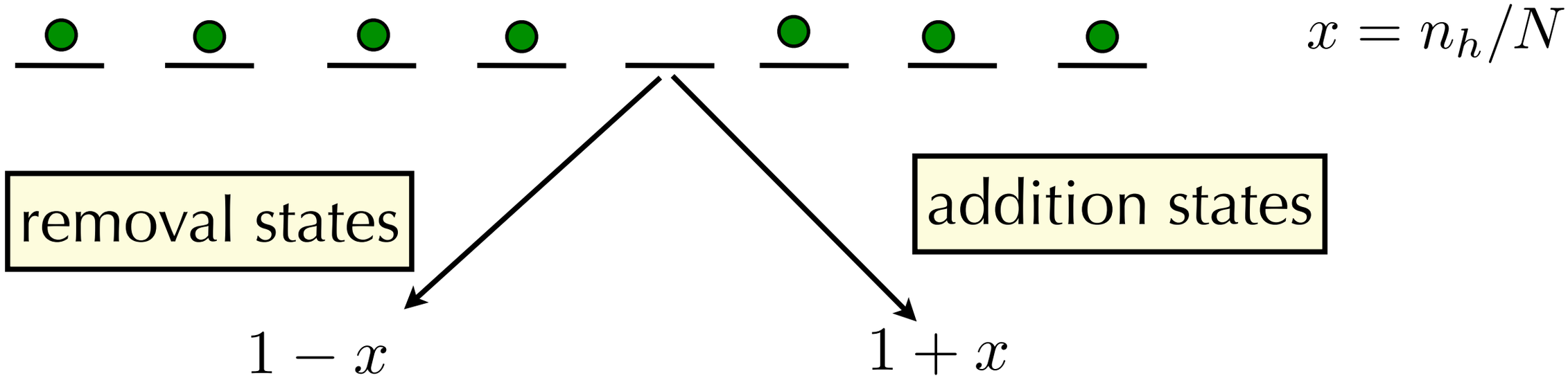
need to know: N (number of sites)

counting electron states



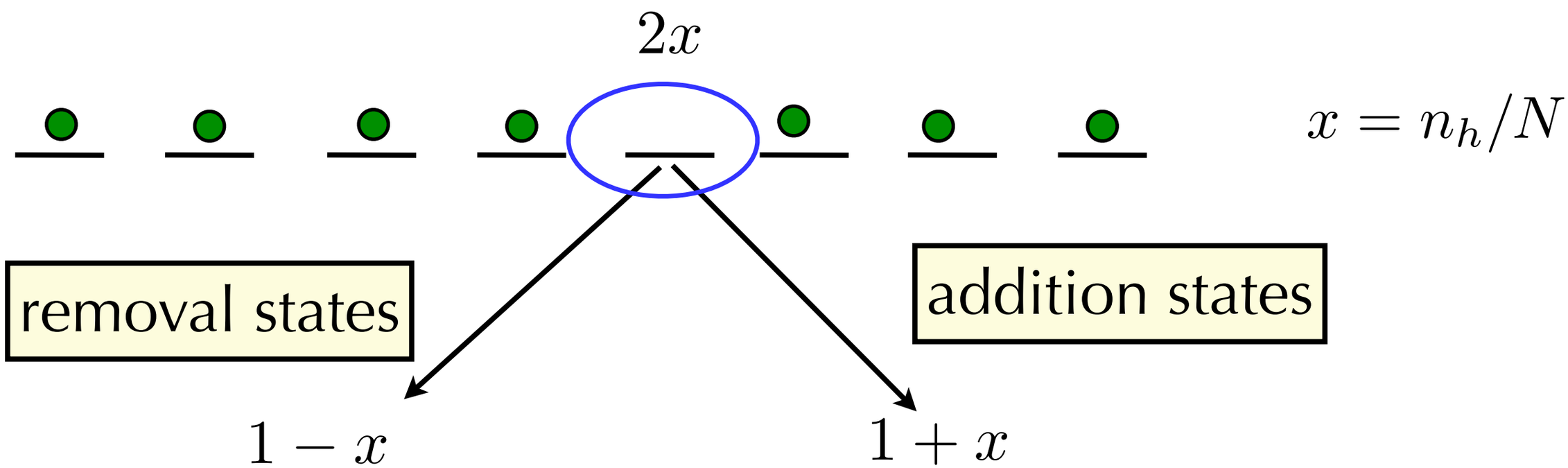
need to know: N (number of sites)

counting electron states



need to know: N (number of sites)

counting electron states



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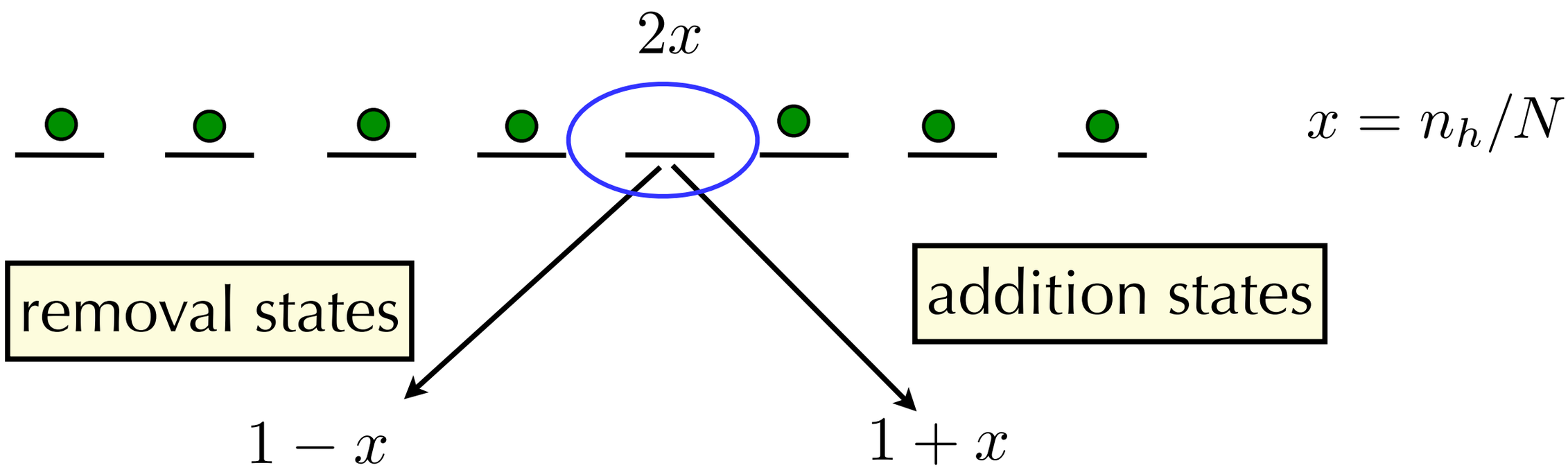
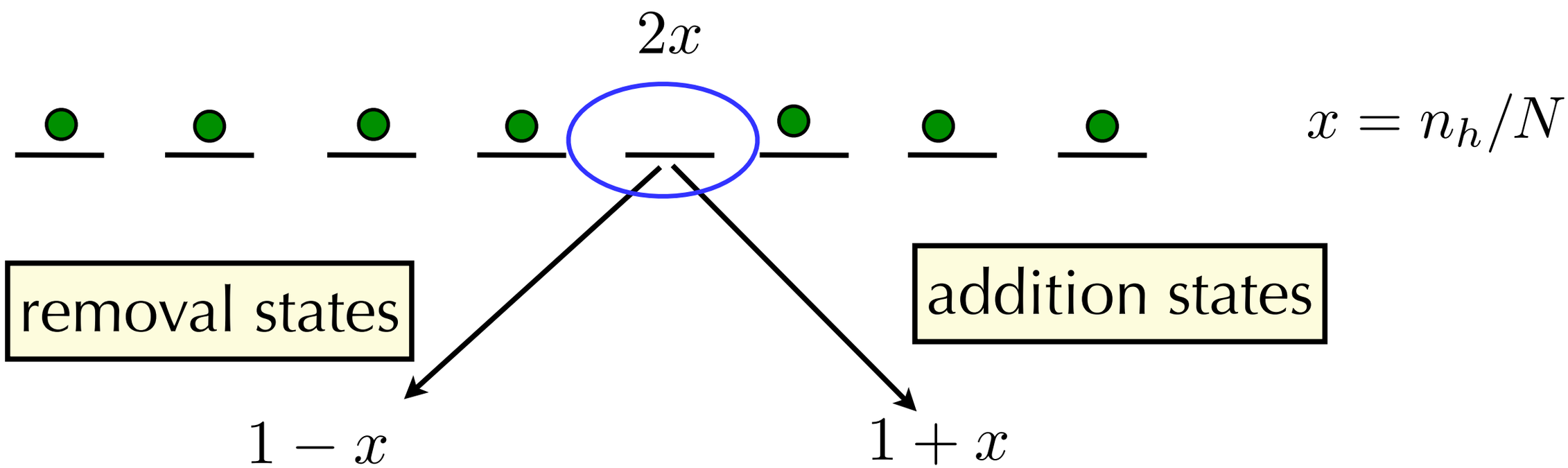
counting electron states



low-energy electron states

need to know: N (number of sites)

counting electron states

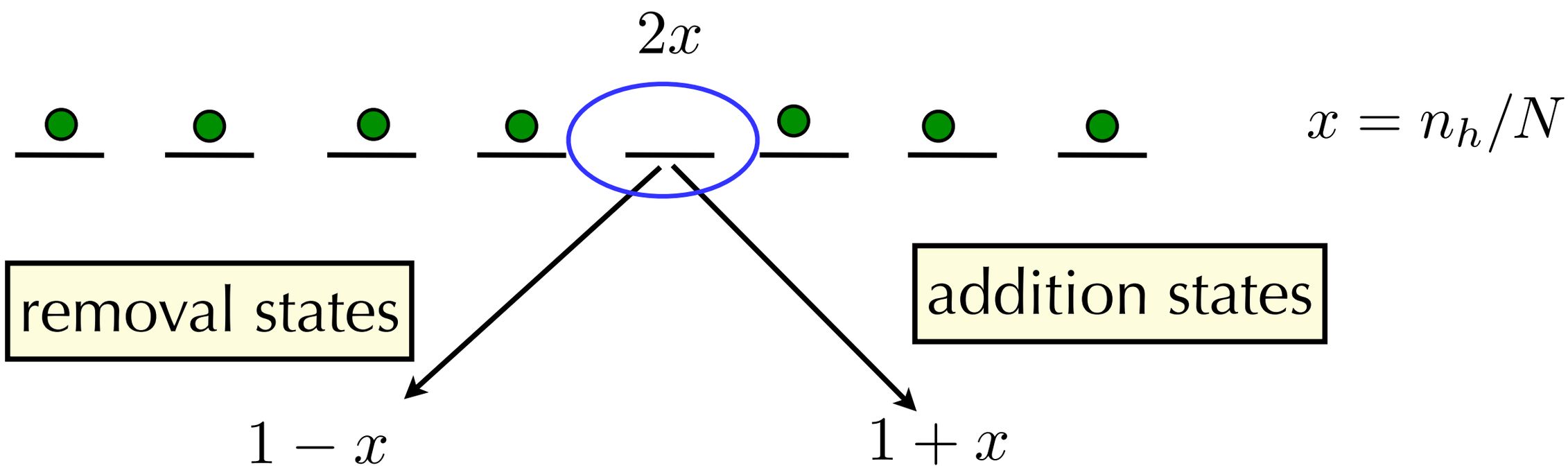


low-energy electron states

$$1 - x + 2x = 1 + x$$

need to know: N (number of sites)

counting electron states



low-energy electron states

$$1 - x + 2x = 1 + x$$

high energy

$$1 - x$$

need to know: N (number of sites)

dynamics are
not exhausted by counting electrons alone

dynamics are
not exhausted by counting electrons alone

what's the extra stuff?

what else is weird?

Quantum critical behaviour in a high-T_c superconductor

D. van der Marel[1*], H. J. A. Molegraaf[1*], J. Zaanen[2], Z. Nussinov[2*], F. Carbone[1*], A. Damascelli[3*], H. Eisaki[3*], M. Greven[3], P. H. Kes[2] & M. Li[2]

[1]*Materials Science Centre, University of Groningen, 9747 AG Groningen, The Netherlands*
[2]*Leiden Institute of Physics, Leiden University, 2300 RA Leiden, The Netherlands*
[3]*Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA*

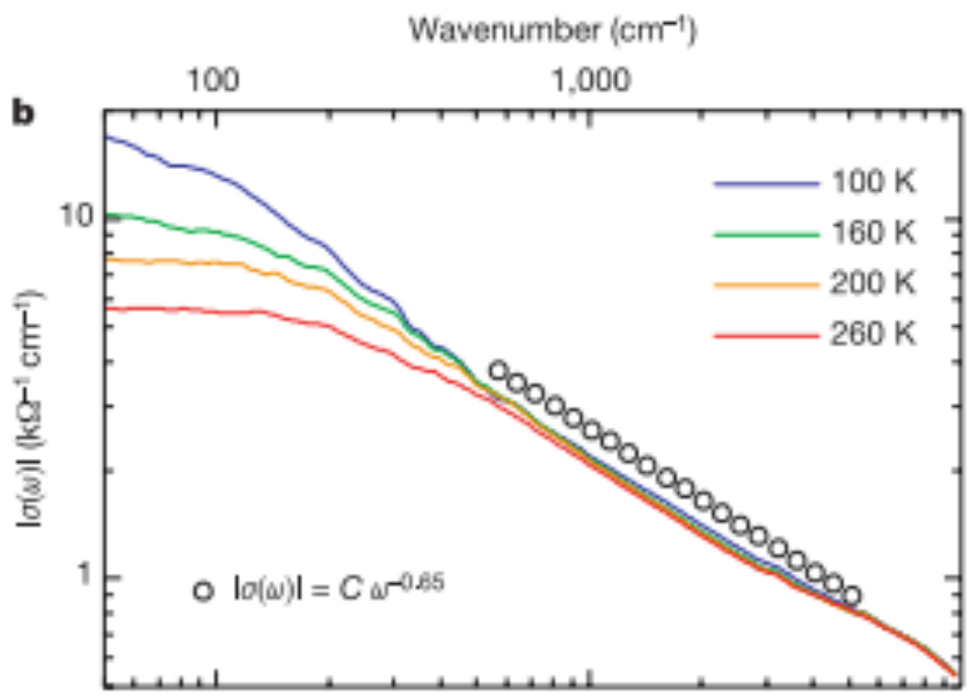
Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

Quantum critical behaviour in a high-T_c superconductor

D. van der Marel[1*], H. J. A. Molegraaf[1*], J. Zaanen[2], Z. Nussinov[2*], F. Carbone[1*], A. Damascelli[3*], H. Eisaki[3*], M. Greven[3], P. H. Kes[2] & M. Li[2]

[1]*Materials Science Centre, University of Groningen, 9747 AG Groningen, The Netherlands*
[2]*Leiden Institute of Physics, Leiden University, 2300 RA Leiden, The Netherlands*
[3]*Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA*



Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

$$\sigma(\omega) = C\omega^{-\frac{2}{3}}$$

YBCO thin films

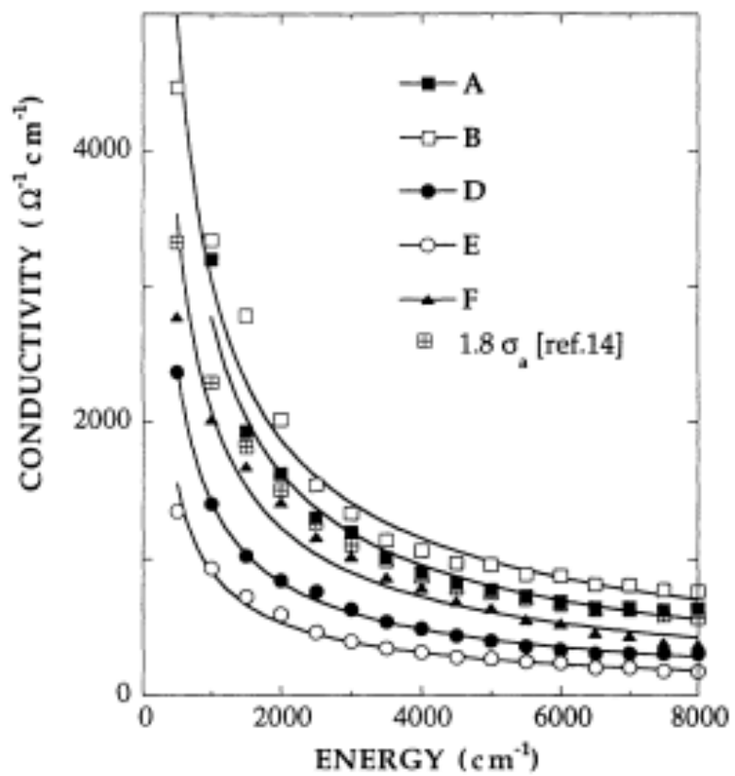


TABLE II. Values of the exponent α describing the best fit $\sigma \sim \omega^{-\alpha}$ for the various films labeled *A*, *B*, *D*, *E*, and *F* films. σ_a and σ_b stand for the conductivities measured along the *a* and *b* directions in an untwinned single crystal (Ref. 14). σ_{MFL} stands for the computed conductivity at 100 and 300 K for the marginal-Fermi-liquid model (MFL) (Ref. 16). The exponent is given within ±0.05.

σ_A (300 K)	σ_B (300 K)	σ_D (300 K)	σ_E (300 K)	σ_F (300 K)	σ_a (100 K)	σ_b (100 K)	σ_{MFL} (300 K)	σ_{MFL} (100 K)
0.77	0.70	0.77	0.76	0.76	0.67	none	0.74	0.68

criticality

↓

scale invariance

↓

power law correlations

criticality

scale invariance

power law correlations

criticality

↑

scale invariance

↑

power law correlations

Anderson: use scale-invariant propagators

$$G^R \propto \frac{1}{(\omega - v_s k)^\eta}$$

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d=1+1 system Luttinger liquid

compute conductivity without vertex corrections (PWA)

is flawed. In fact, in the Luttinger liquid such direct calculations are not to be trusted very firmly, since it is the nature of the Luttinger liquid that vertex corrections, if they must be included, will be singular; conventional transport theory is not applicable, and special methods such as the above are necessary.

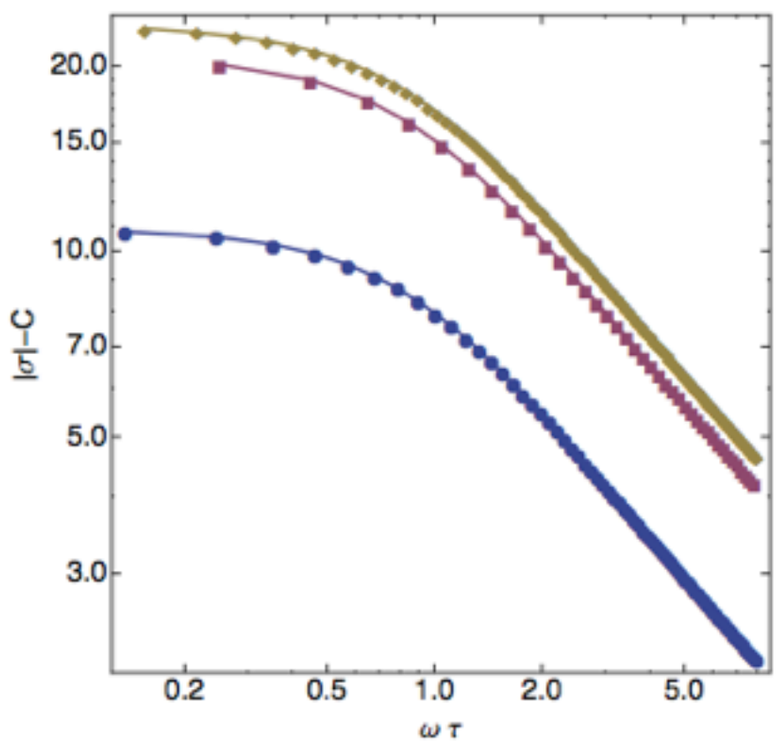
$$\sigma(\omega) \propto \frac{1}{\omega} \int dx \int dt G^e(x,t) G^h(x,t) e^{i\omega t} \propto (i\omega)^{-1+2\eta}$$

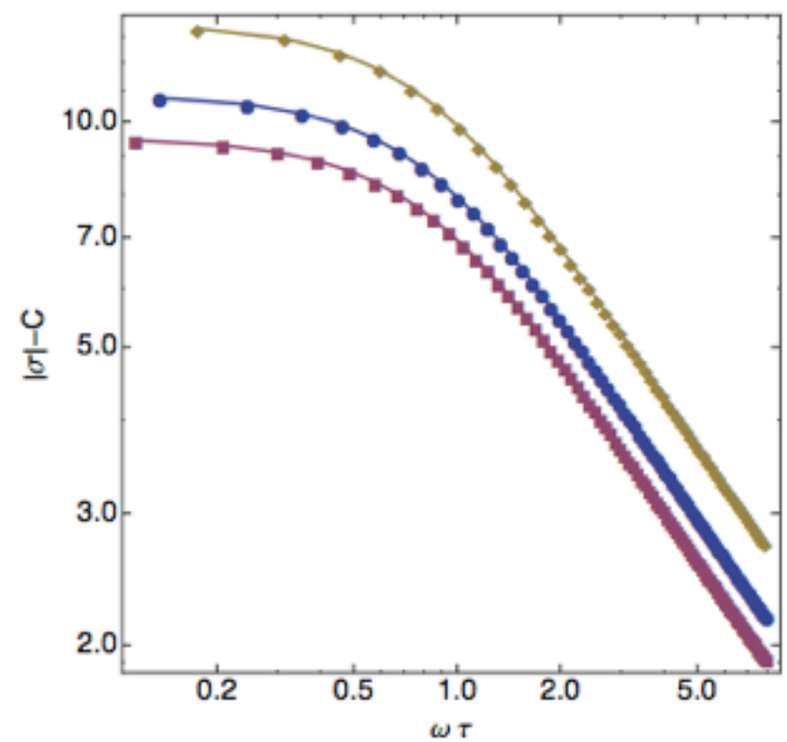
1997
Anderson

cuprates d=2+1,
vertex corrections
matter

cuprates d=2+1,
vertex corrections
matter

optical conductivity from a gravitational lattice





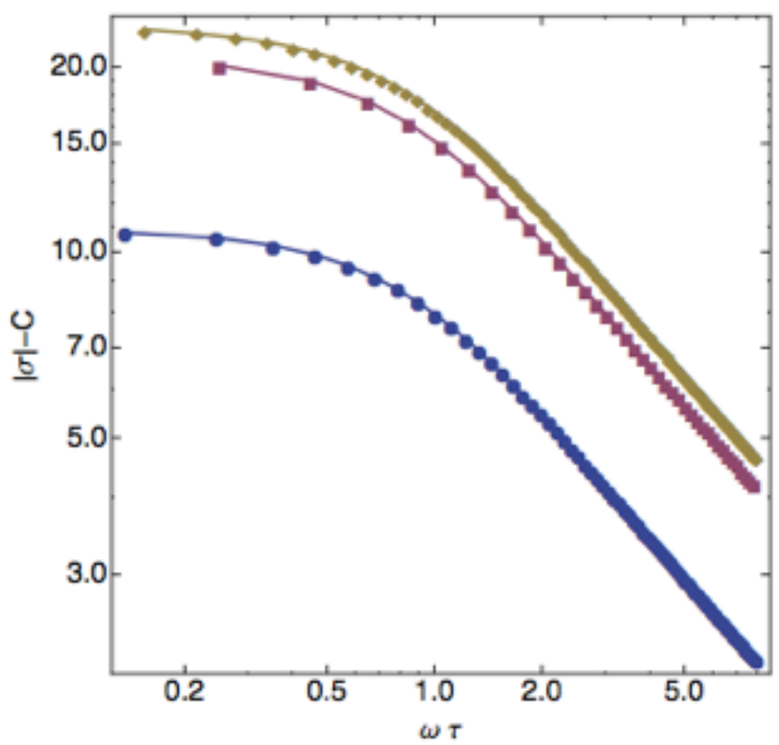
log-log plots for various parameters

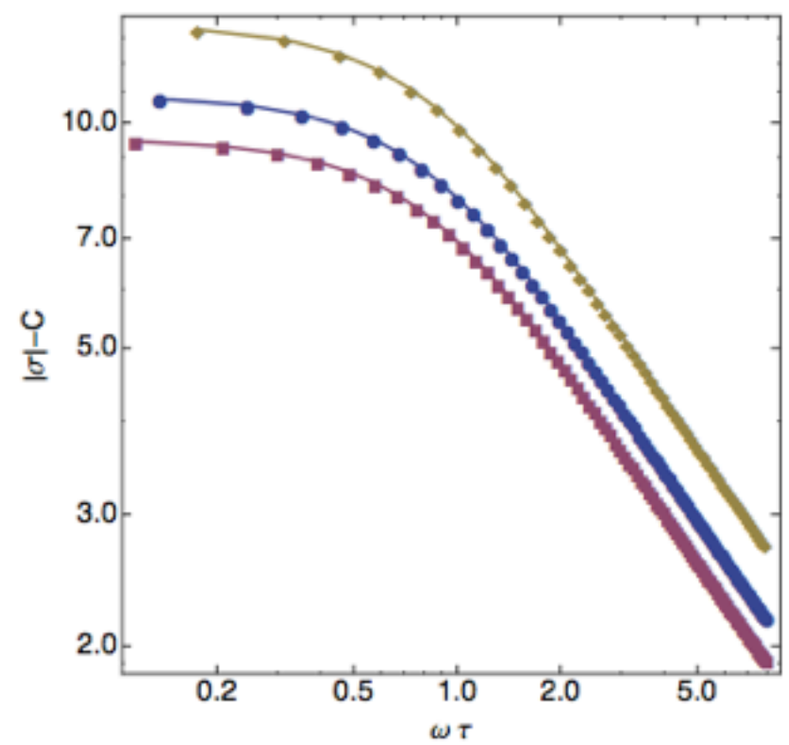
$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

for $0.2 \lesssim \omega\tau \lesssim 0.8$

G. Horowitz *et al*., Journal of High Energy Physics, 2012

optical conductivity from a gravitational lattice





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replicates features of the strange metal? how?

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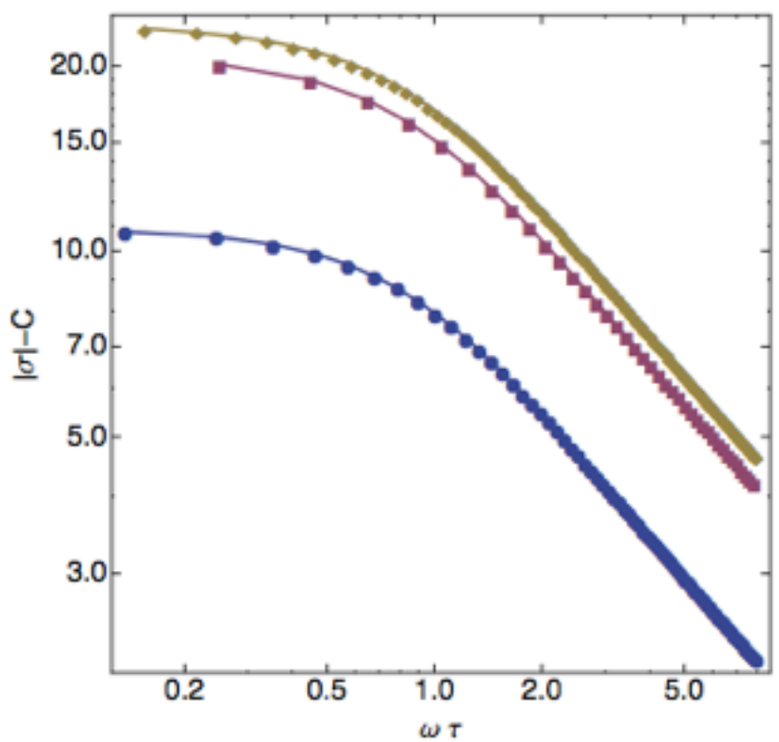

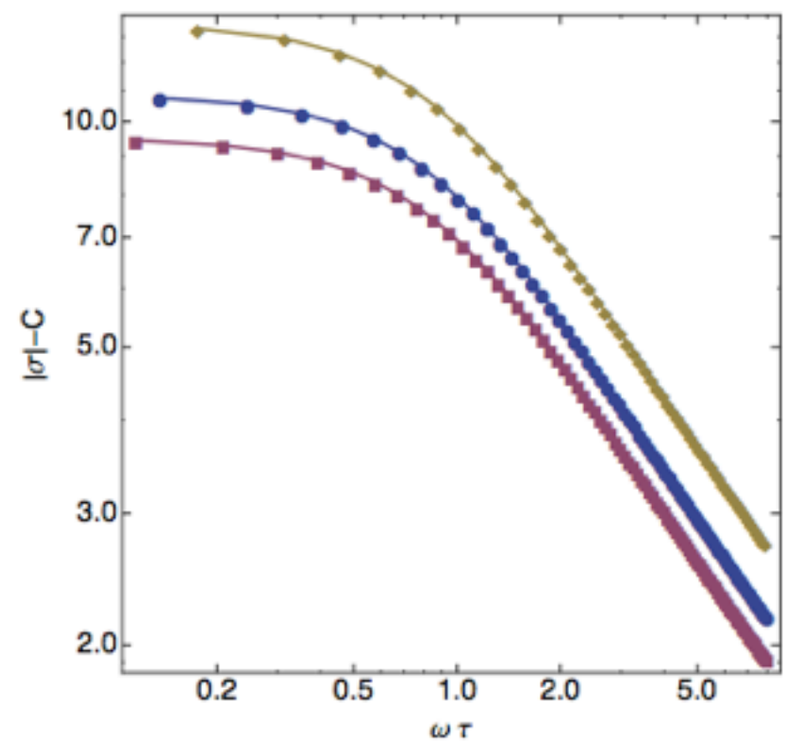

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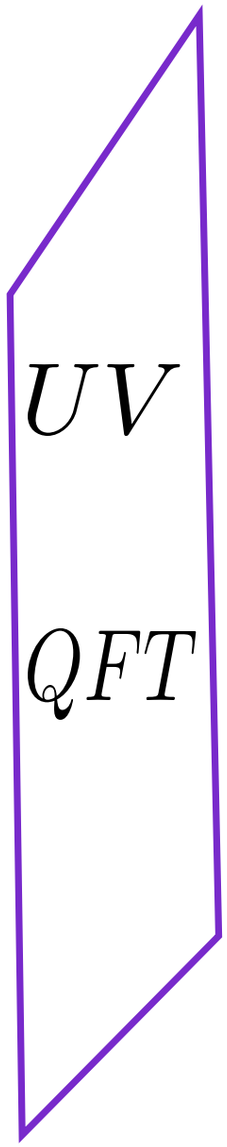
momentum relaxation??

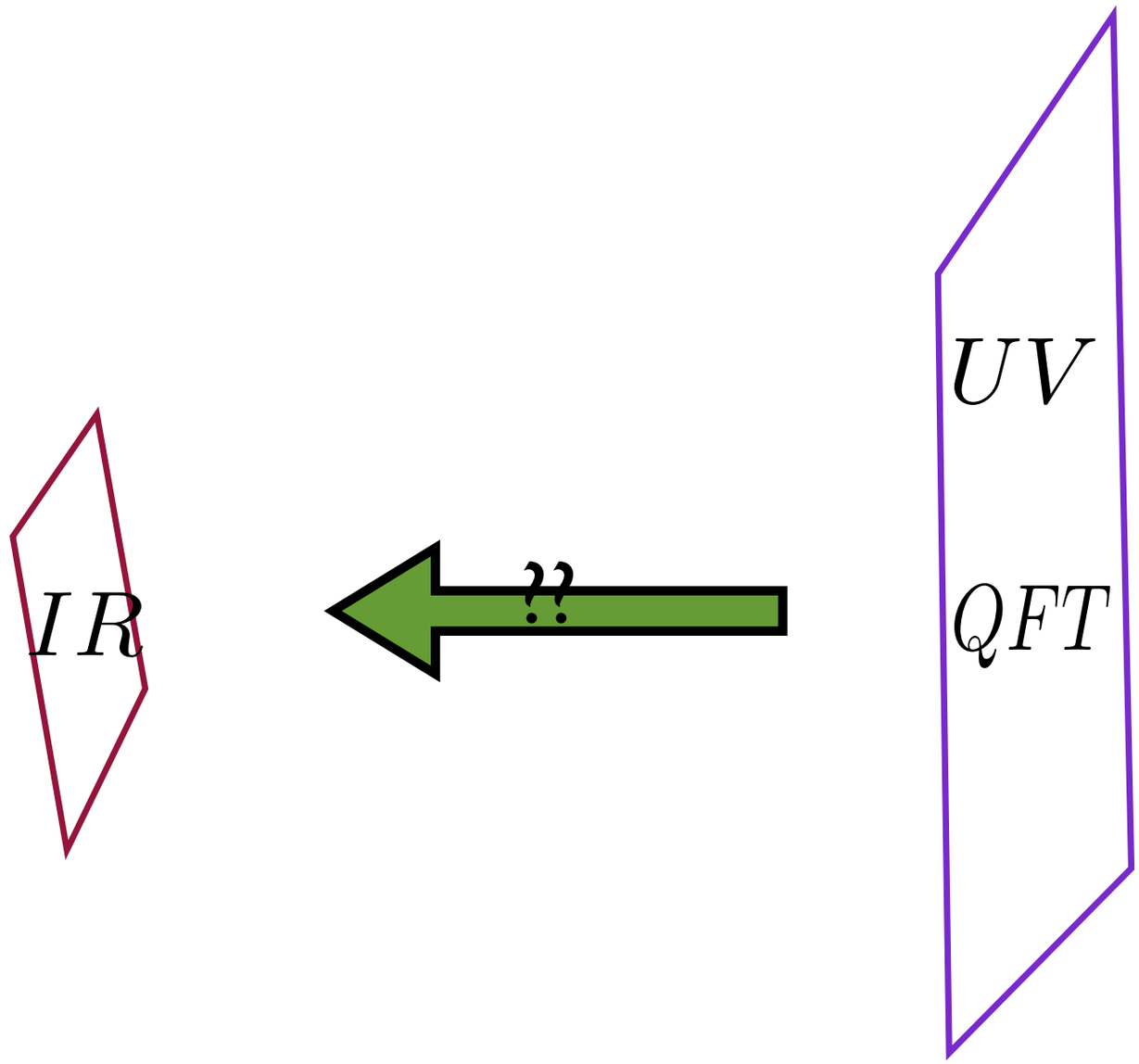
G. Horowitz *et al*., Journal of High Energy Physics, 2012

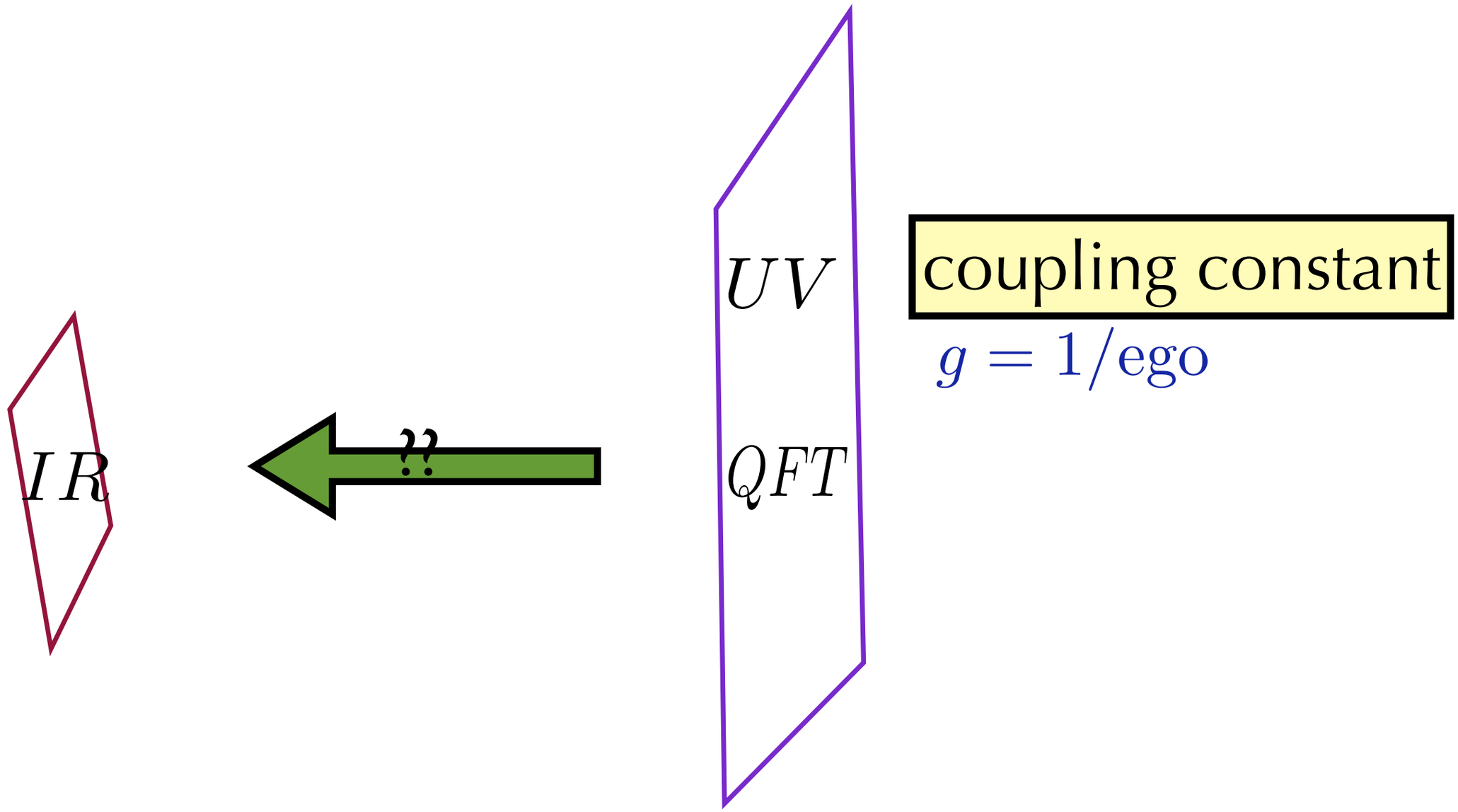
power law?

Could string theory be the answer?

momentum relaxation?


UV
QFT


UV
QFT
??
IR


UV
coupling constant
$g = 1/\text{ego}$
IR
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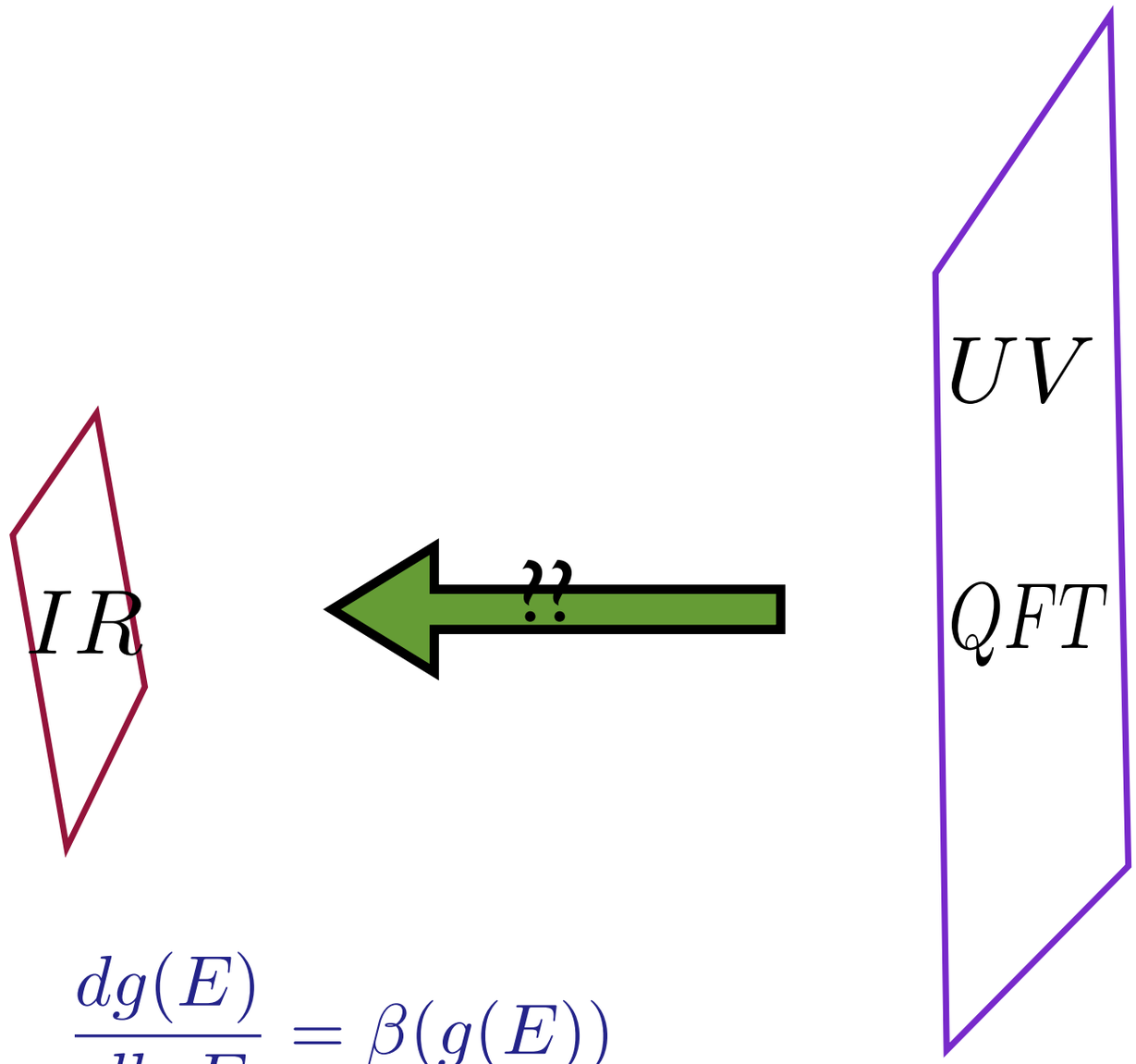


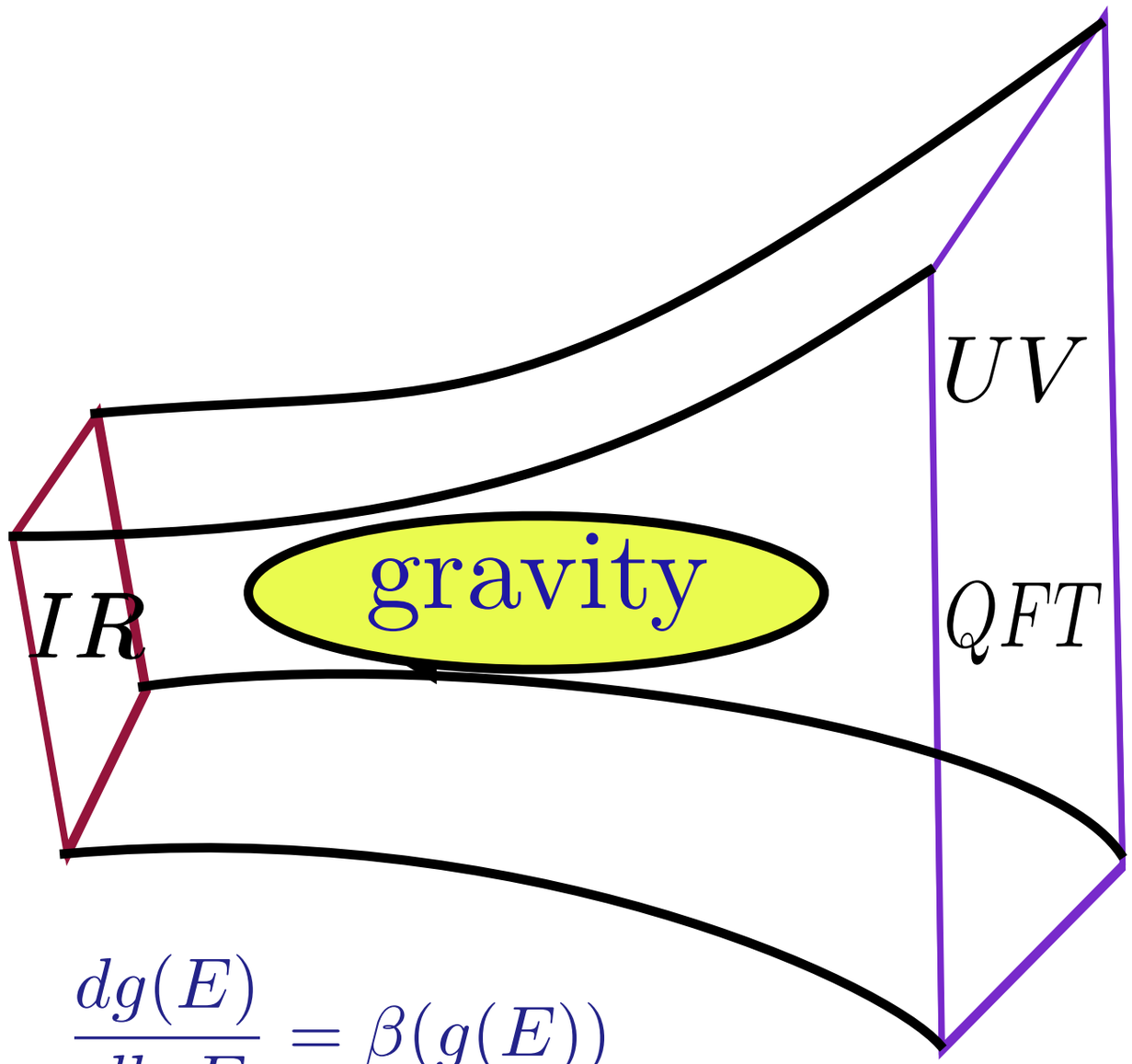
coupling constant

$g = 1/\text{ego}$

$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

locality in energy

gauge-gravity duality
(Maldacena, 1997)



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scale invariance
(continuous)

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$$E \to \lambda E$$

$$x^\mu \to x^\mu / \lambda$$

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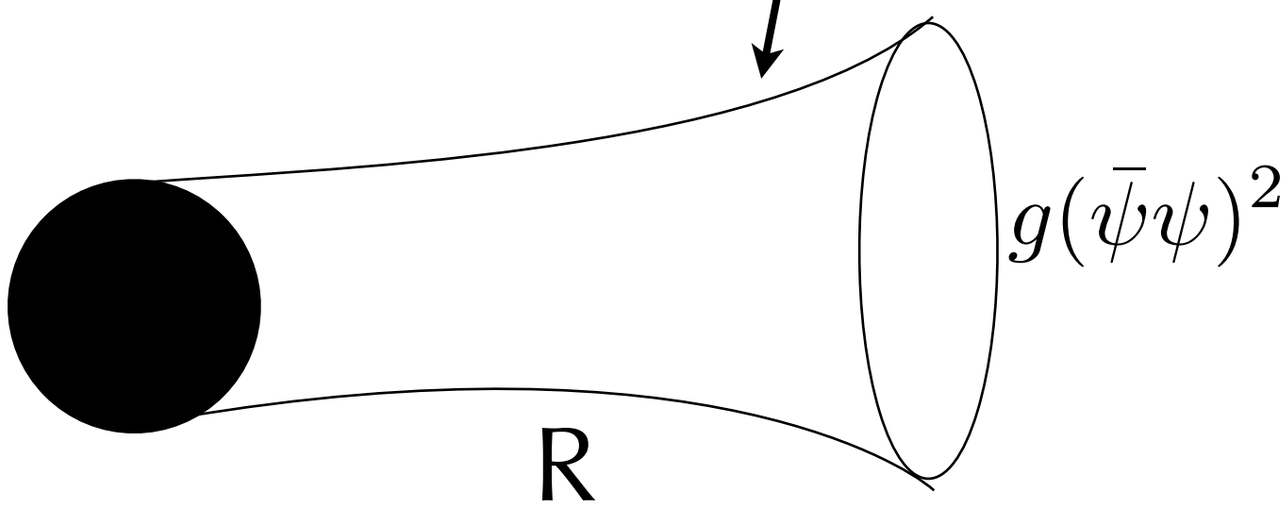
solve Einstein equations

$$ds^2 = \left(\frac{u}{L}\right)^2 \eta_{\mu\nu} dx^\mu dx^\nu + \left(\frac{L}{E}\right)^2 dE^2$$

anti de-Sitter space

replace coupled theory with geometry

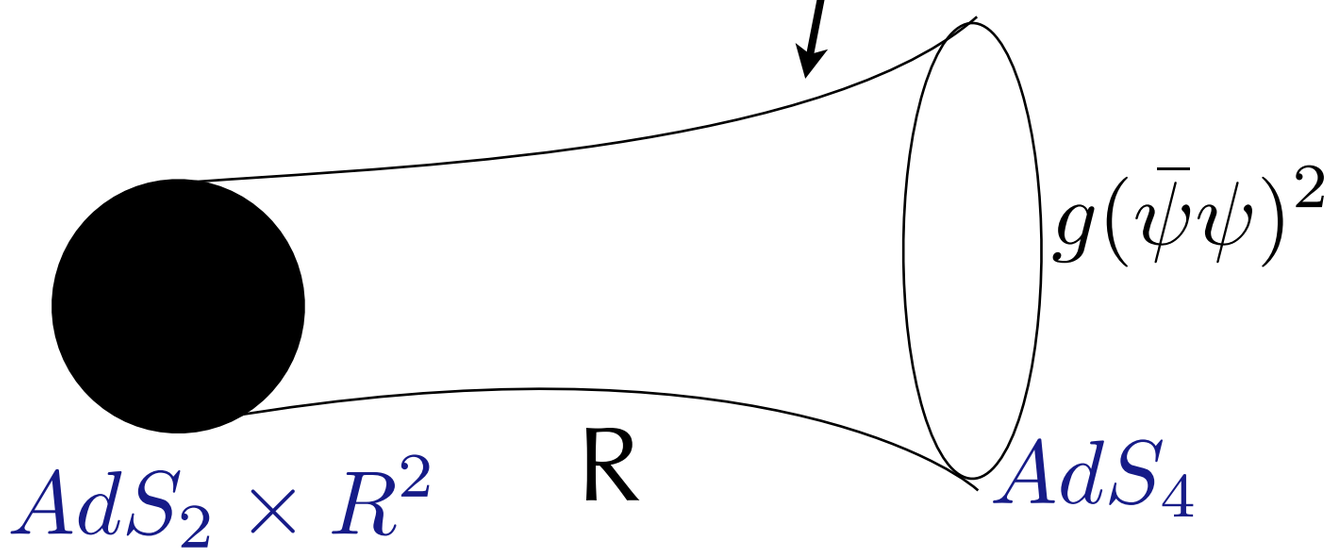
scale-invariant anti de-Sitter



$$R \propto \sqrt{g}$$

replace coupled theory with geometry

scale-invariant anti de-Sitter



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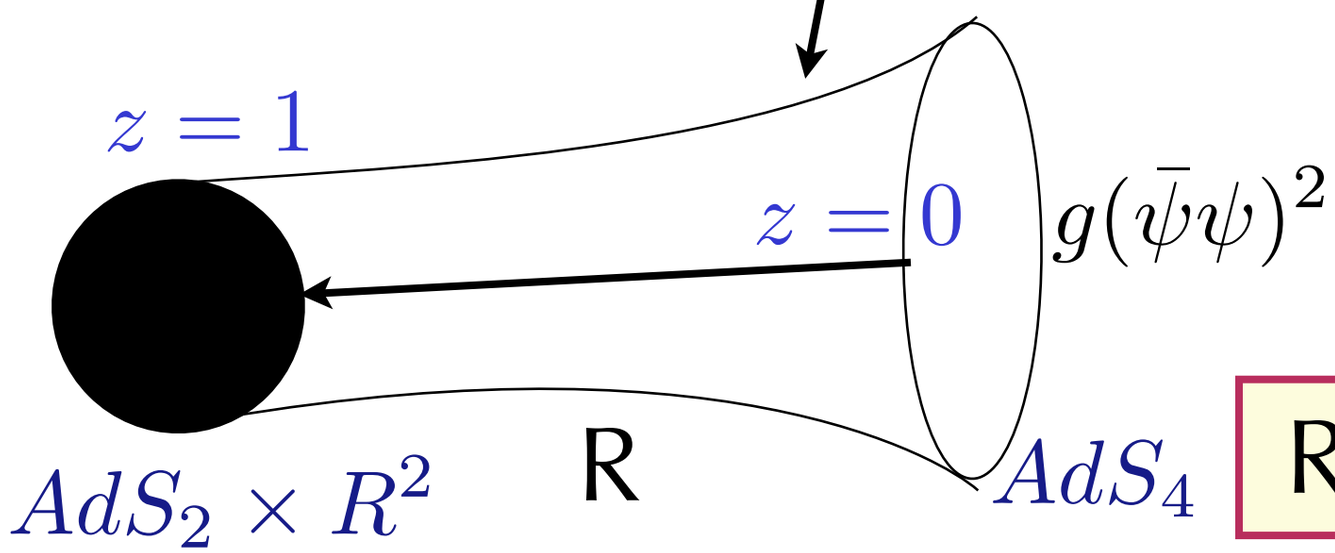

RG=GR

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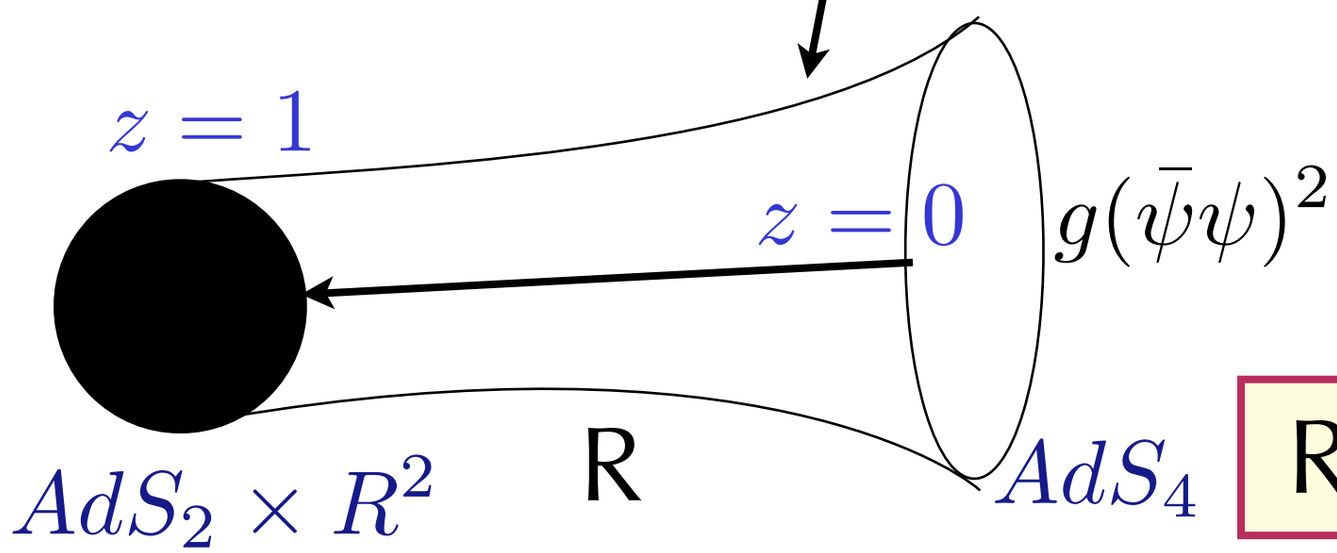
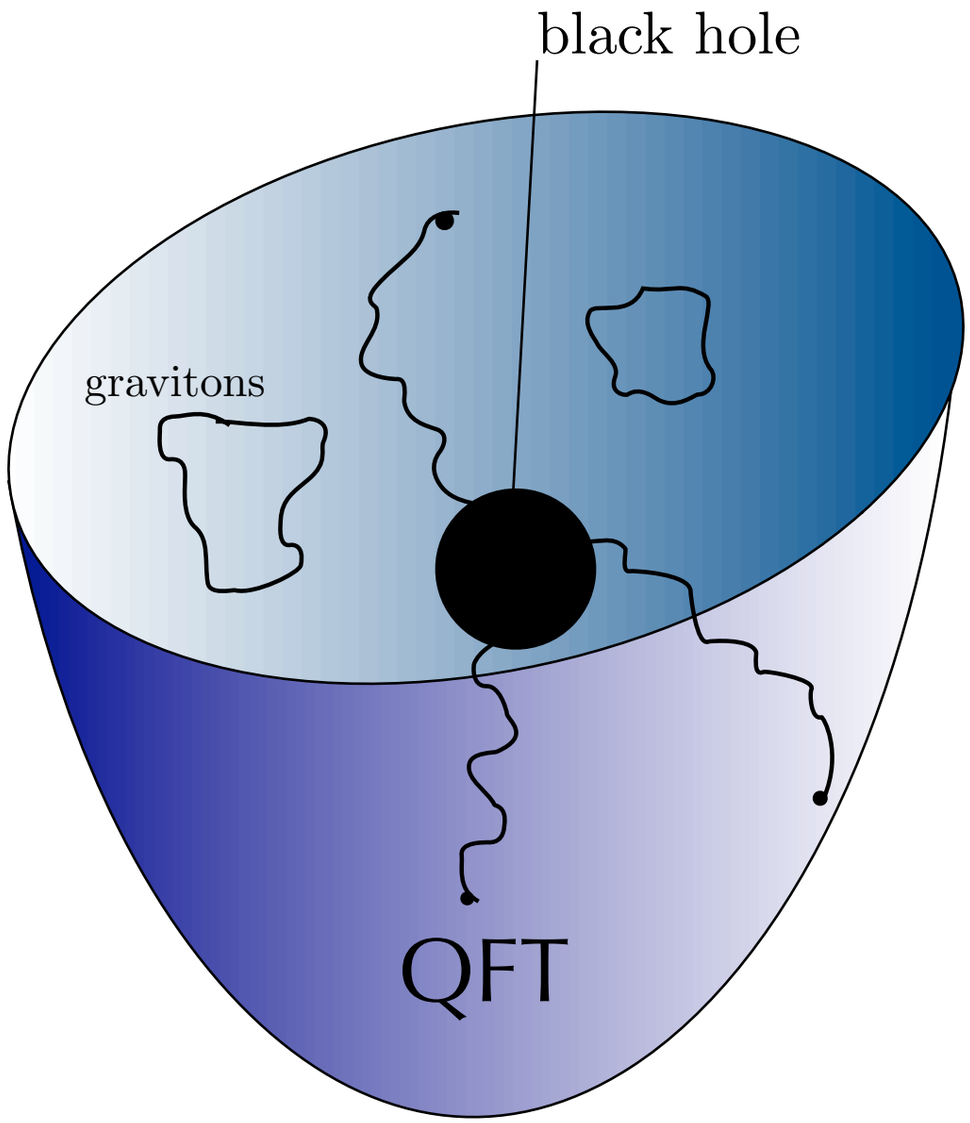


RG=GR

$$R \propto \sqrt{g}$$

cannot describe systems at g=0!

certain strongly coupled theories have gravity duals



absence of locality

new equation!

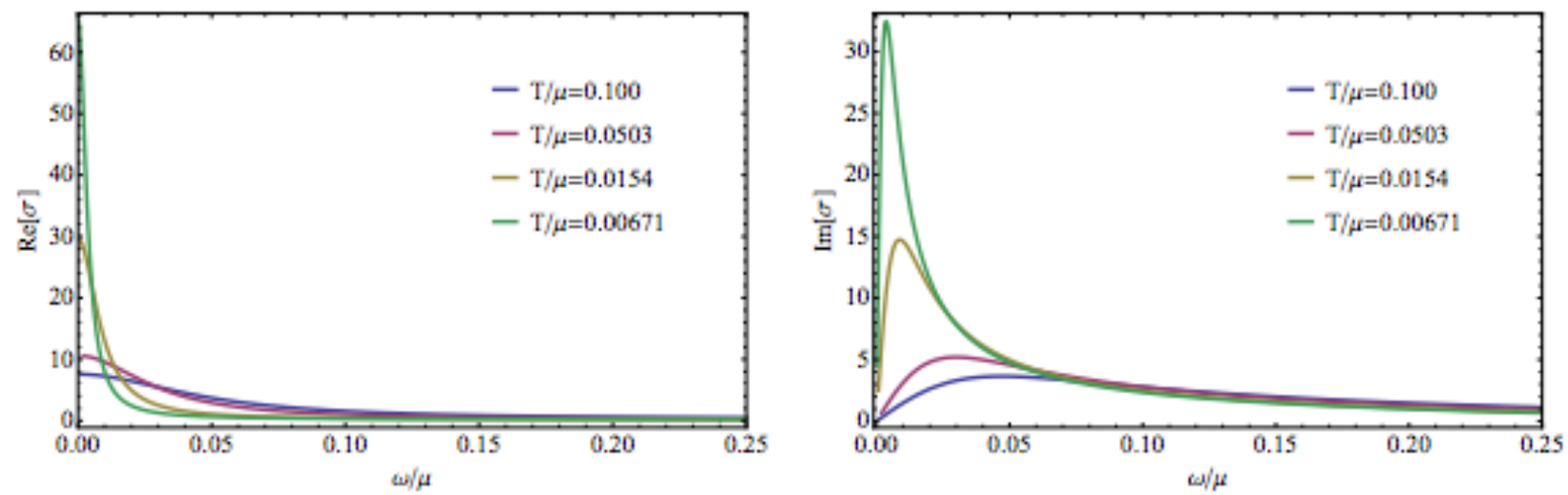
Einstein-Maxwell equations + non-uniform charge density = $B\omega^{-2/3}$

not so fast!

Donos and Gauntlett
(gravitational crystal)

Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$



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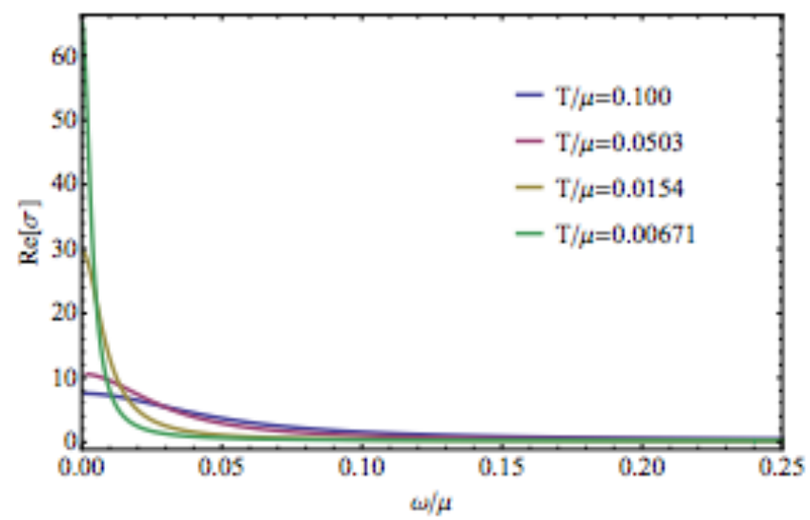

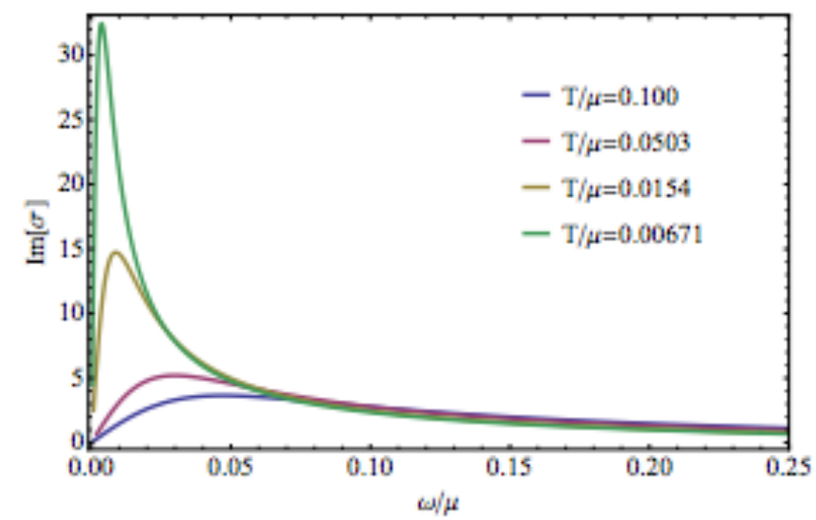

$$-\frac{2}{3} = 1 + \omega\frac{|\sigma|''}{|\sigma|'}$$

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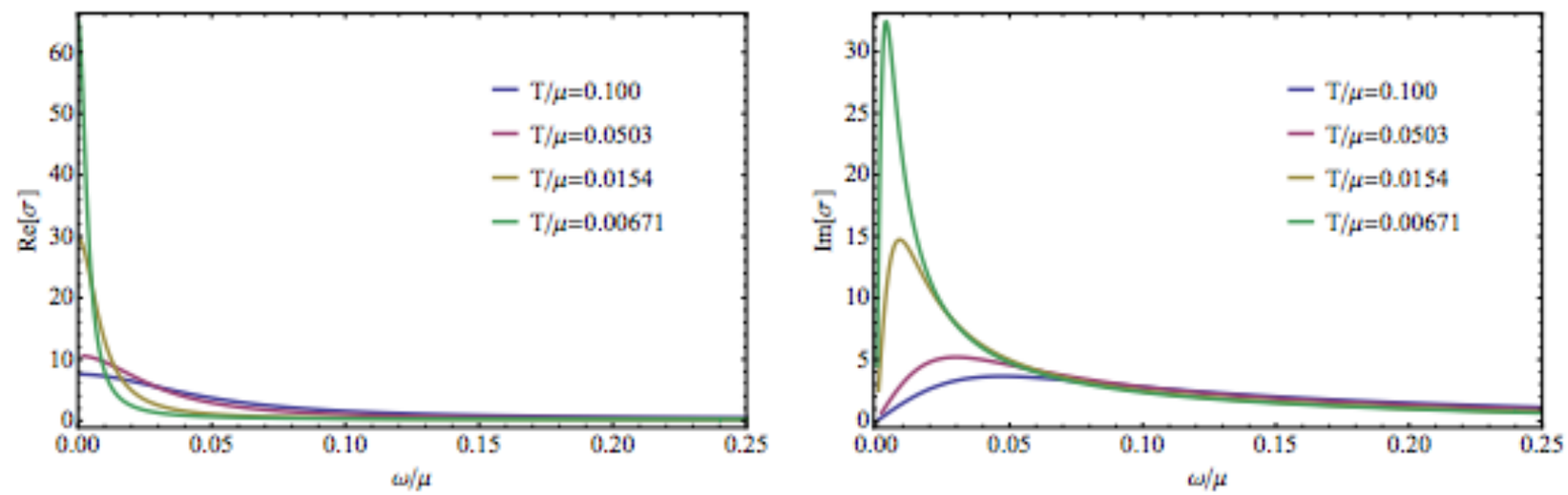

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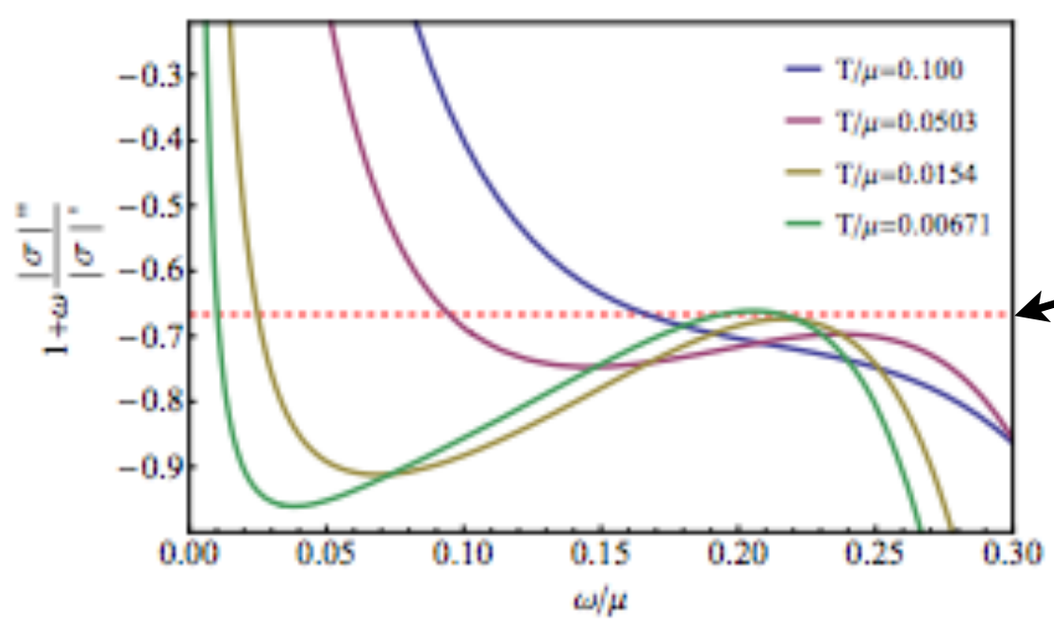

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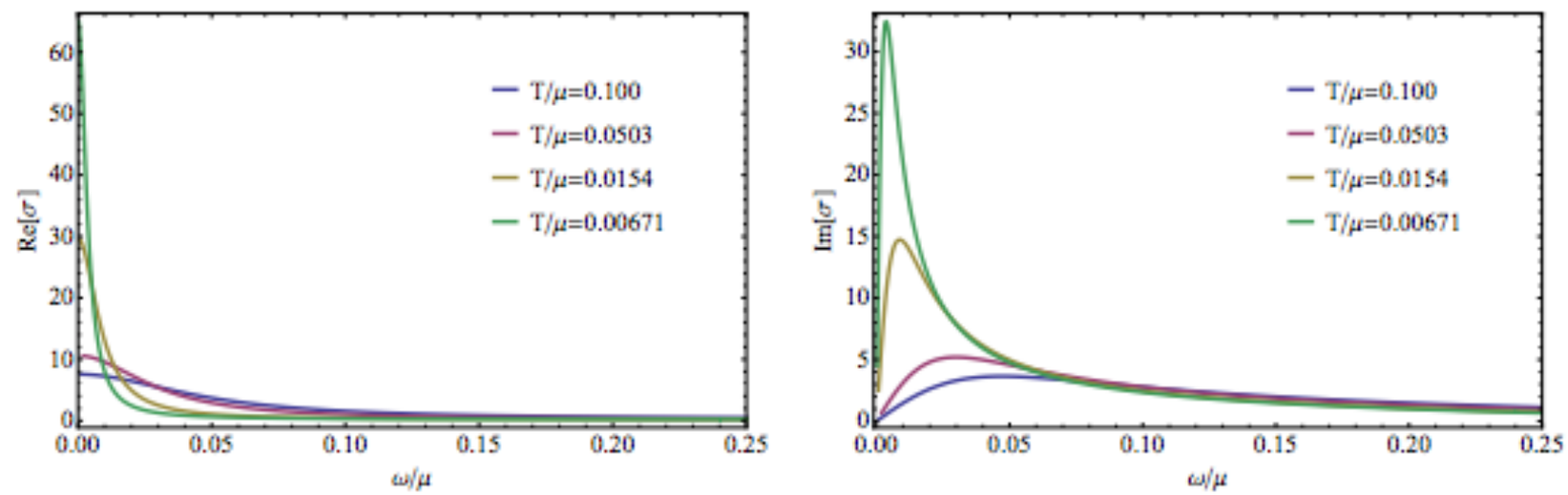

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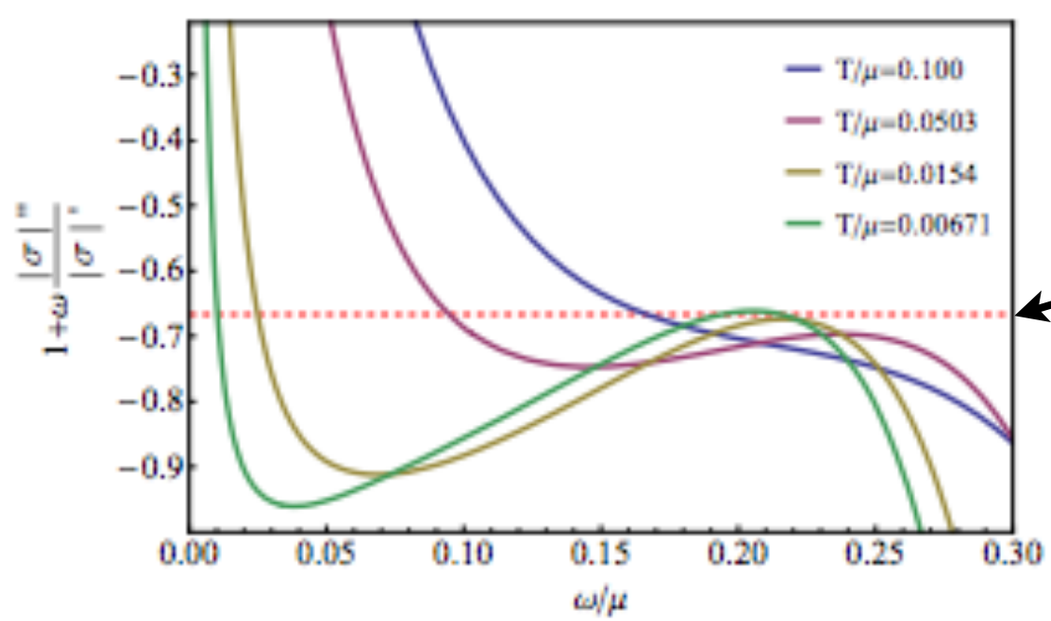

$$B\omega^{-2/3}$$

no power law!!

who is correct?

who is correct?

let's redo the calculation

model

model

action = gravity + EM + lattice

model

action = gravity + EM + lattice

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left(R - 2\Lambda - \frac{1}{2} F^2 \right),$$

model

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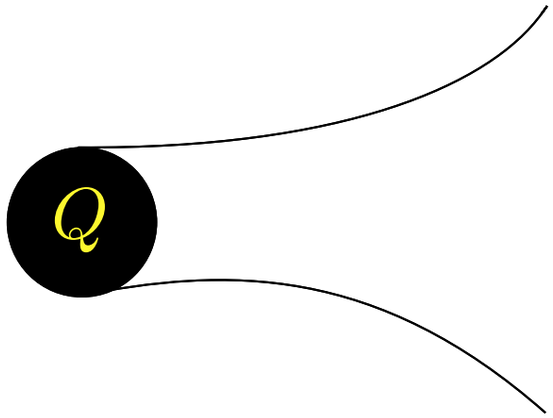
$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left(R - 2\Lambda - \frac{1}{2} F^2 \right),$$

$$\mathcal{L}(\phi) = \sqrt{-g}[-|\partial\phi|^2 - V(|\phi|)]$$

conductivity within AdS

$$(g_{\mathrm{ab}}, V(\Phi), A_t)$$

$$(\text{metric}\,, \text{potential}\,, \text{gaugefield})$$



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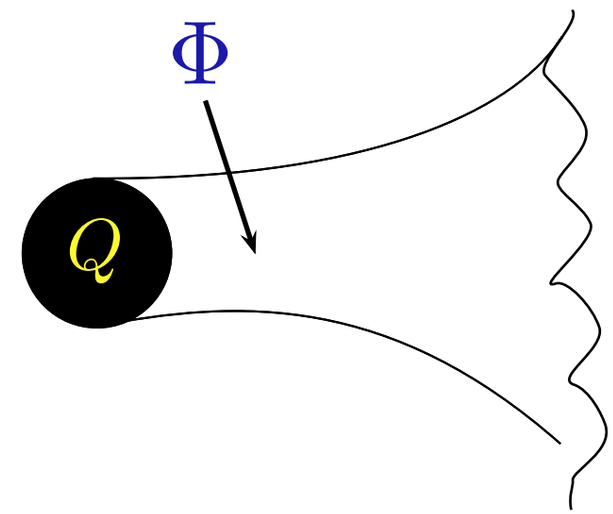

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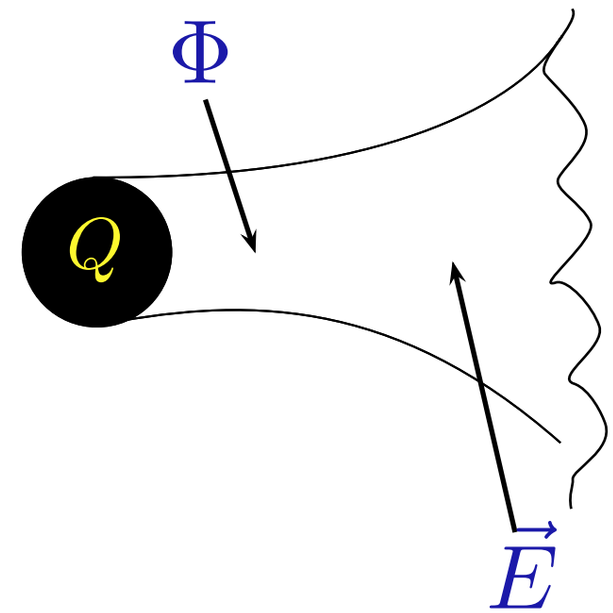
perturb with electric field



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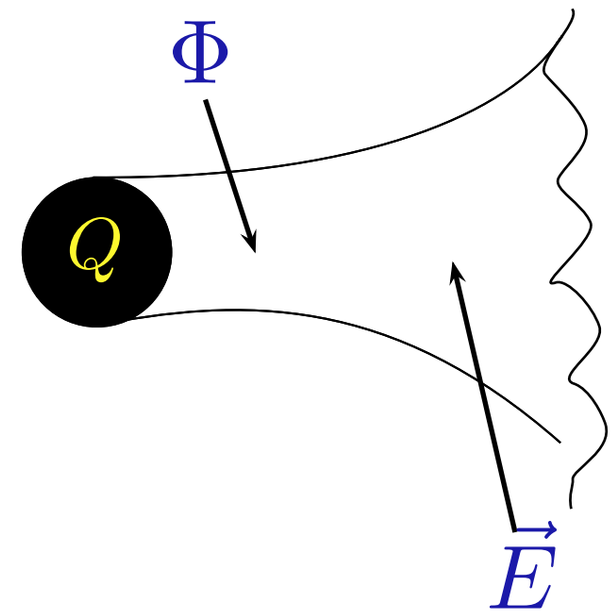

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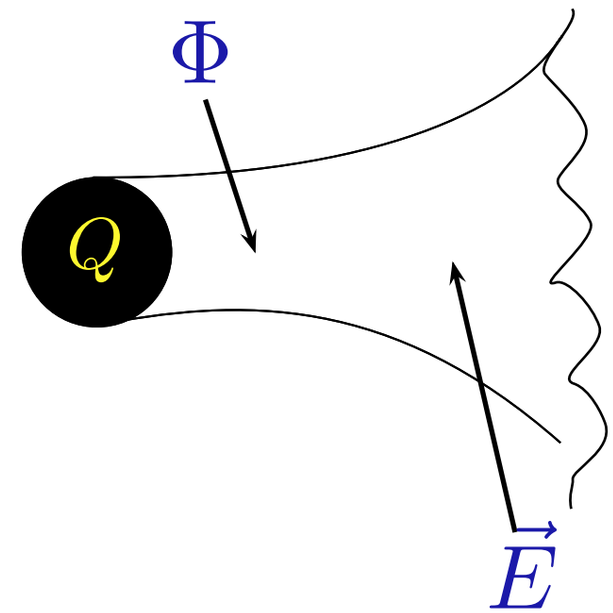

perturb with electric field

$$g_{\rm ab} = \bar{g}_{\rm ab} + h_{\rm ab}$$
$$A_a = \bar{A}_a + b_a$$
$$\Phi_i = \bar{\Phi}_i + \eta_i$$

$$\sigma = J_x(x,\omega)/E$$

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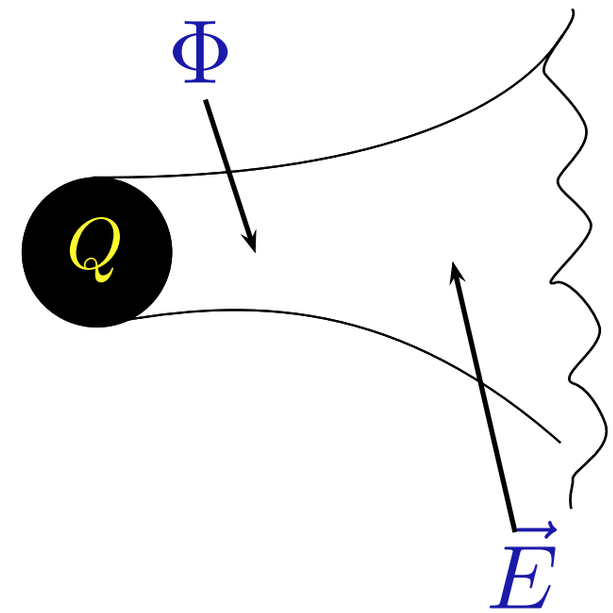

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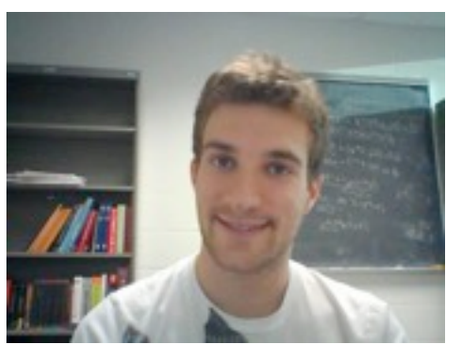
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solve equations of motion with gauge invariance (without mistakes)

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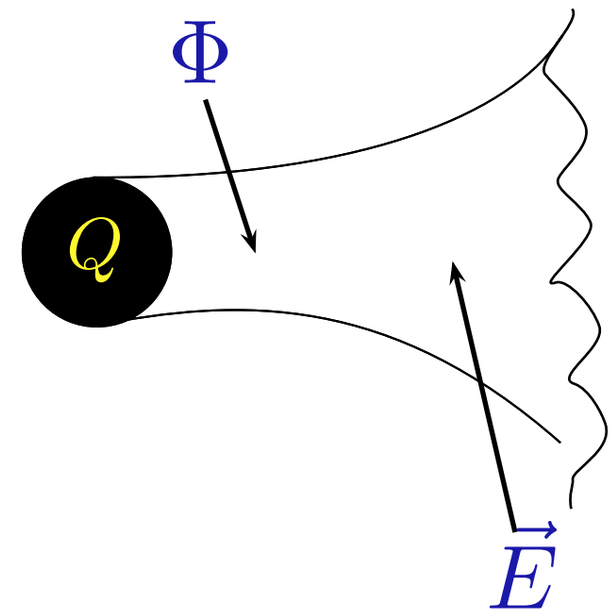

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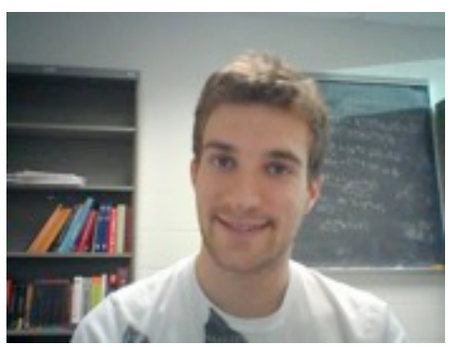
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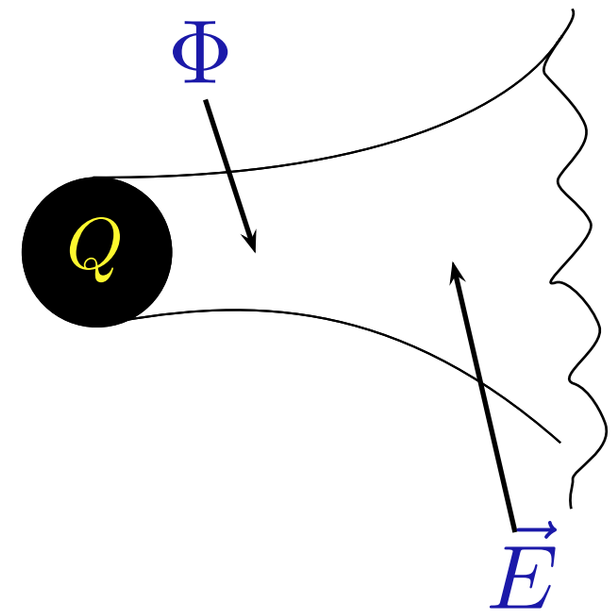

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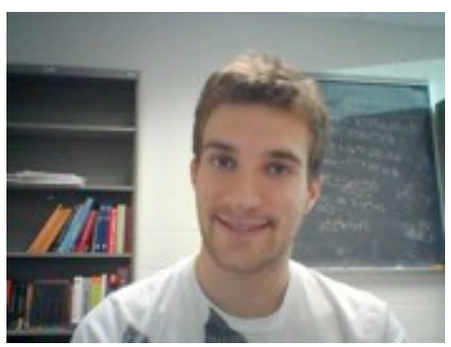
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$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

gravitational crystals

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

$$\frac{1}{2}\Box h_{\mu\nu} - 2m^2 h_{\mu\nu} + \cdots$$

$$m^2 \propto V(\Phi))$$

graviton has a mass!

gravitational crystals

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$$\partial_t T^{ti} \neq 0 = -\tau^{-1} T^{ti}$$

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in Drude formula

HST vs. DG

Horowitz, Santos, Tong (HST)

DG

HST vs. DG

Horowitz, Santos, Tong (HST)

DG

$$V(\Phi) = -\Phi^2/L^2$$

$$\Phi(x) = A_0 \cos(kx)$$

inhomogeneous in x

HST vs. DG

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DG

$$V\left(|\Phi|^2\right)$$

$$\Phi(z,x) = \phi(z)e^{ikx}$$

no inhomogeneity in x

Our Model

$$\mathcal{L}_\Phi = (\nabla\Phi_1)^2 + (\nabla\Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2)$$

Our Model

$$\mathcal{L}_\Phi = (\nabla\Phi_1)^2 + (\nabla\Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2)$$

$$\Phi_1(x) = A_0 \cos\left(kx - \frac{\theta}{2}\right),$$

Our Model

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$$\Phi_1(x) = A_0 \cos\left(kx - \frac{\theta}{2}\right), \qquad \Phi_2 = A_0 \cos\left(kx + \frac{\theta}{2}\right)$$

Our Model

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$$\theta = 0$$

HST

Our Model

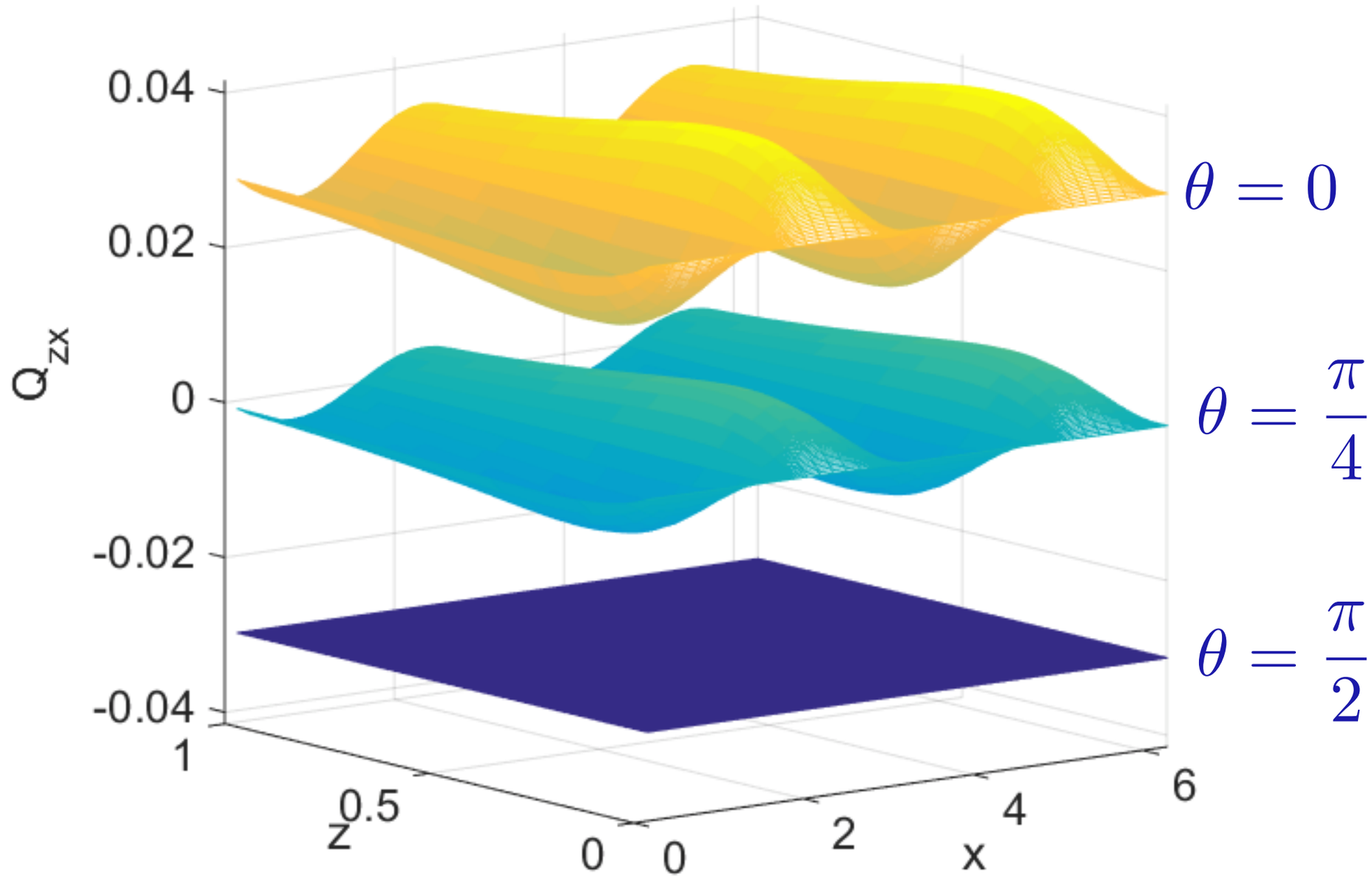
$$\mathcal{L}_\Phi = (\nabla\Phi_1)^2 + (\nabla\Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2)$$

$$\Phi_1(x) = A_0 \cos\left(kx - \frac{\theta}{2}\right), \qquad \Phi_2 = A_0 \cos\left(kx + \frac{\theta}{2}\right)$$

$\theta = 0$ → HST

$\theta = \frac{\pi}{2}$ → DG

$A_0 = 0.75,\ k = 1,\ \mu = 1.4,\ T/\mu = 0.115$



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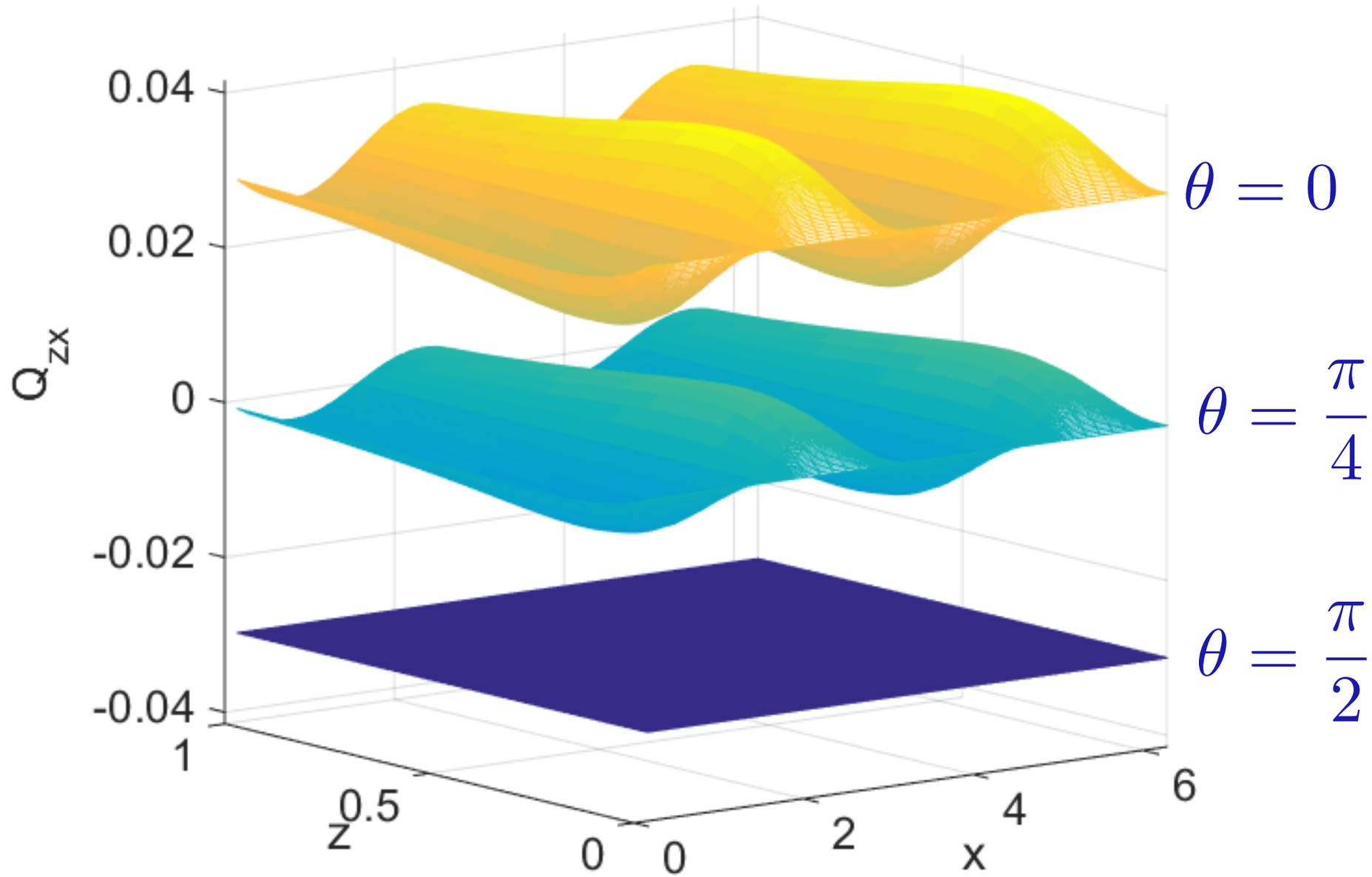
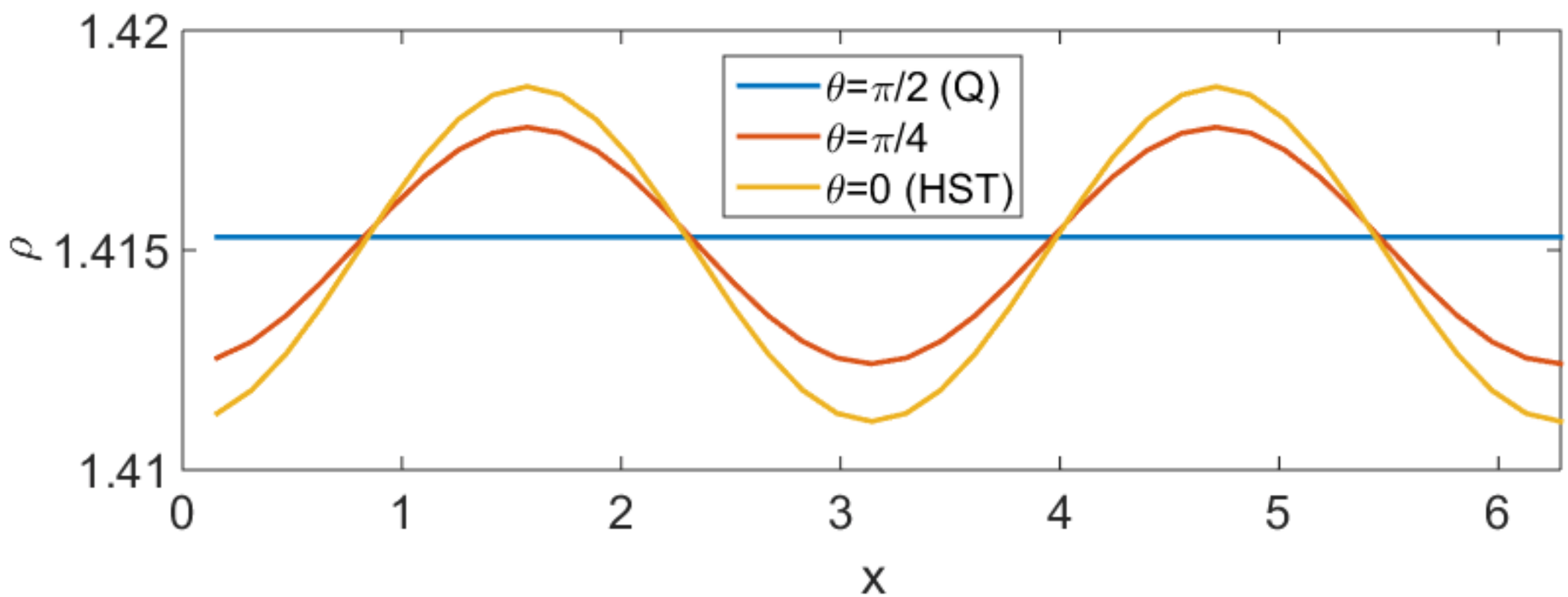


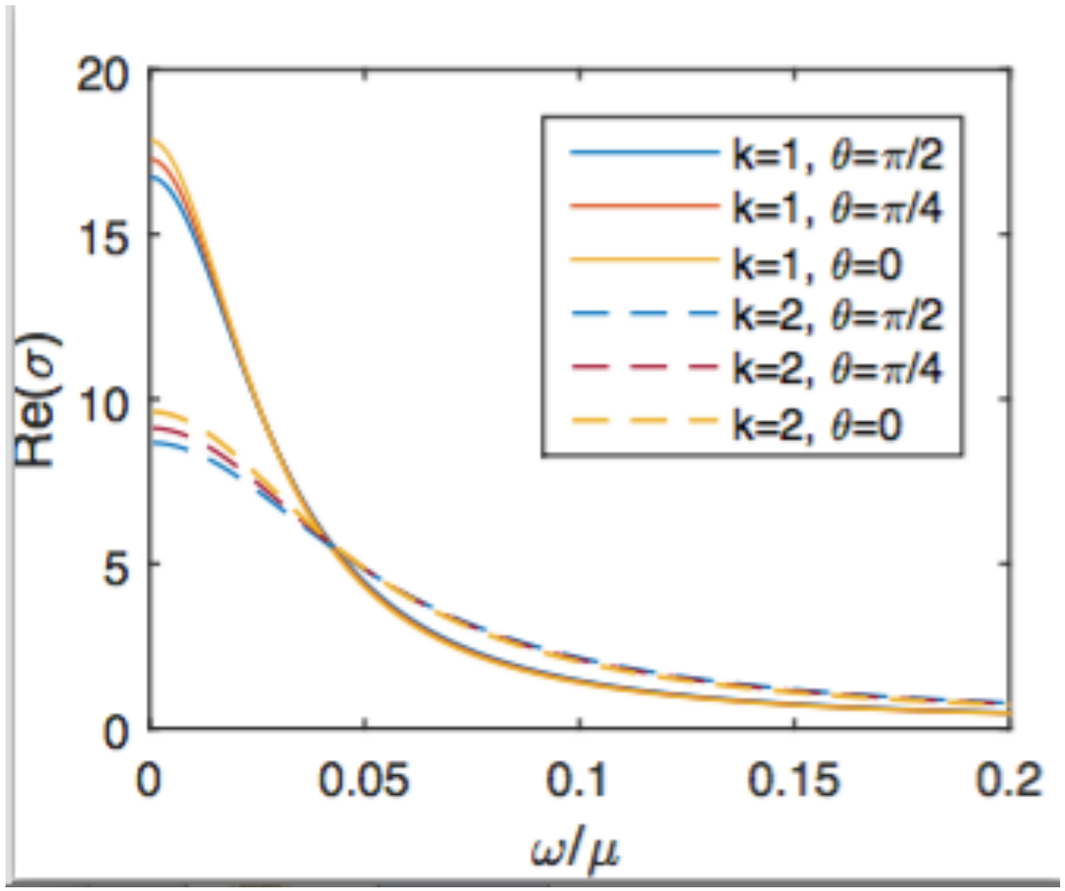
translational invariance is broken in metric in multiples of 2k

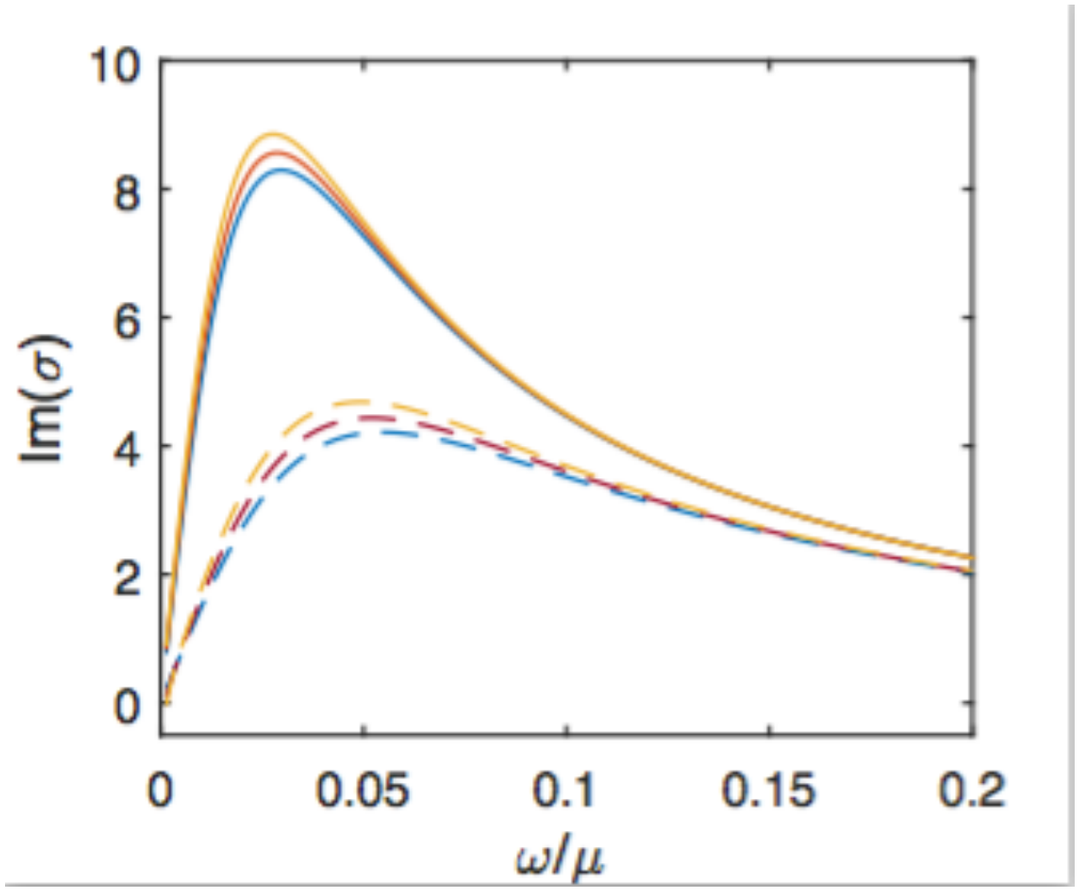
charge density

$$\rho = \lim_{z \to 0} \sqrt{-g} F^{tz}$$



conductivity



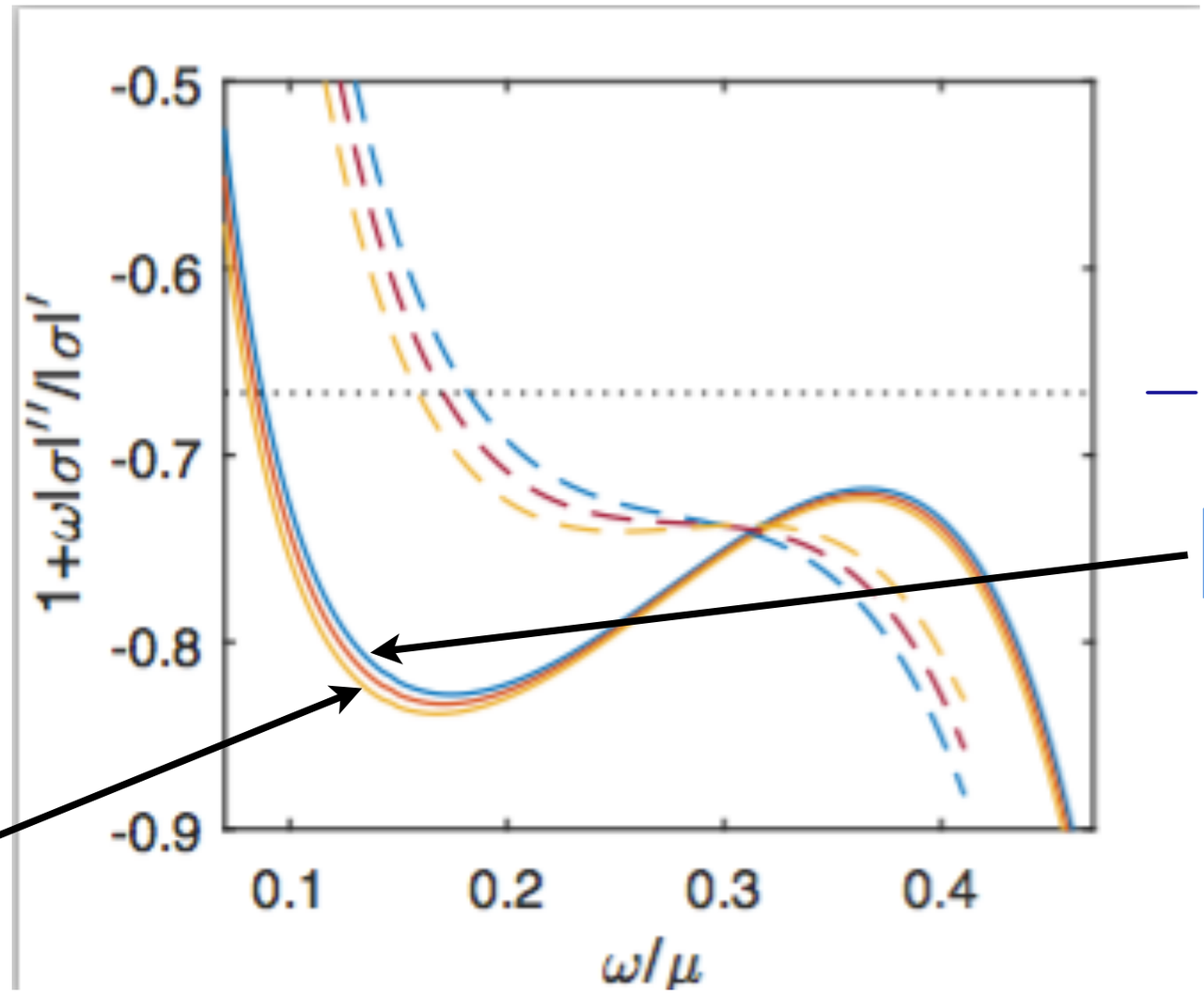


- high-frequency behavior is identical
- low-frequency RN has $\mathrm{Re}(\sigma) \sim \delta(\omega),\ \mathrm{Im}(\sigma) \sim 1/\omega$
- low-frequency lattice has Drude form

is there a power law?

is there a power law?

Results

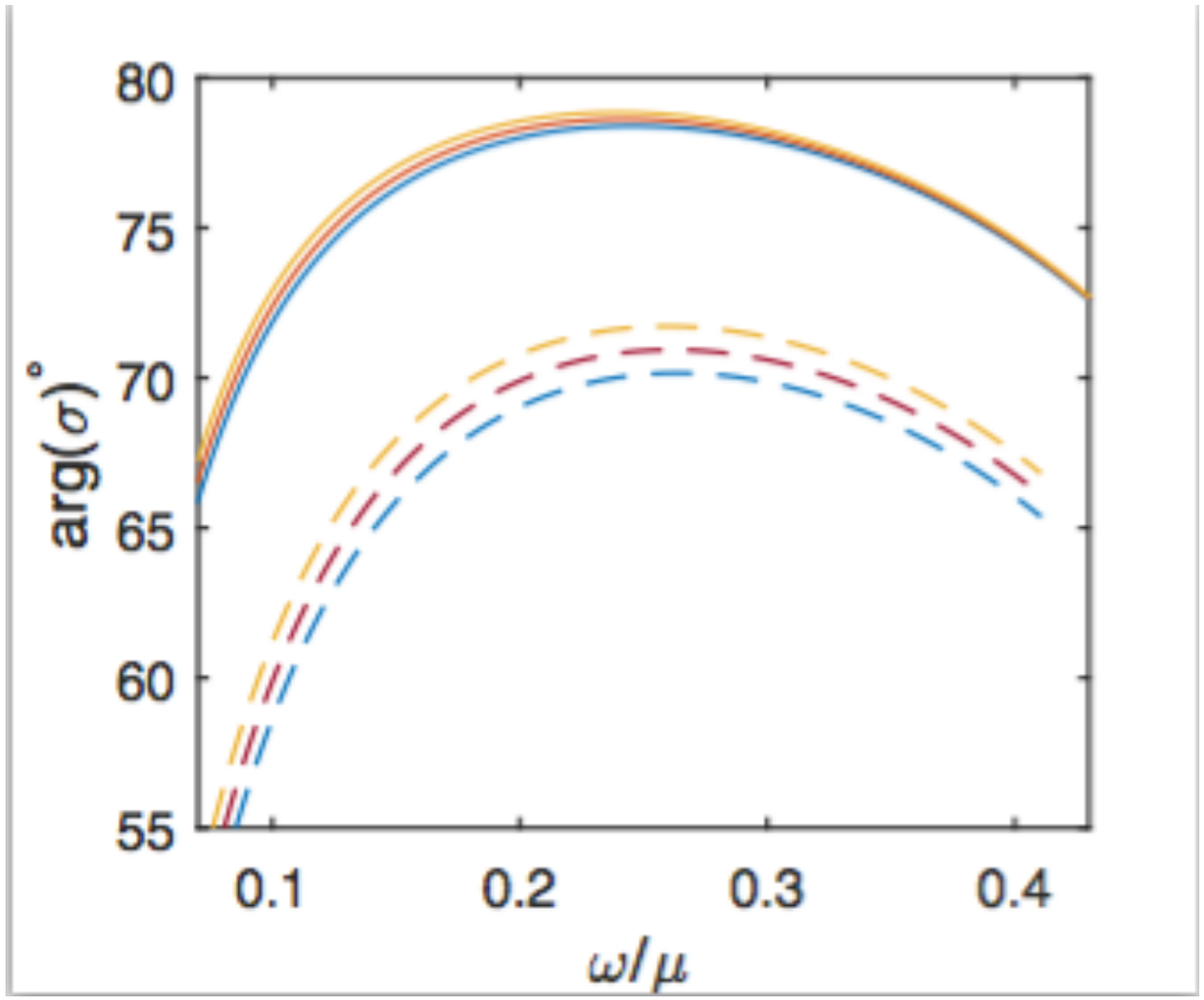


$-2/3$

DG

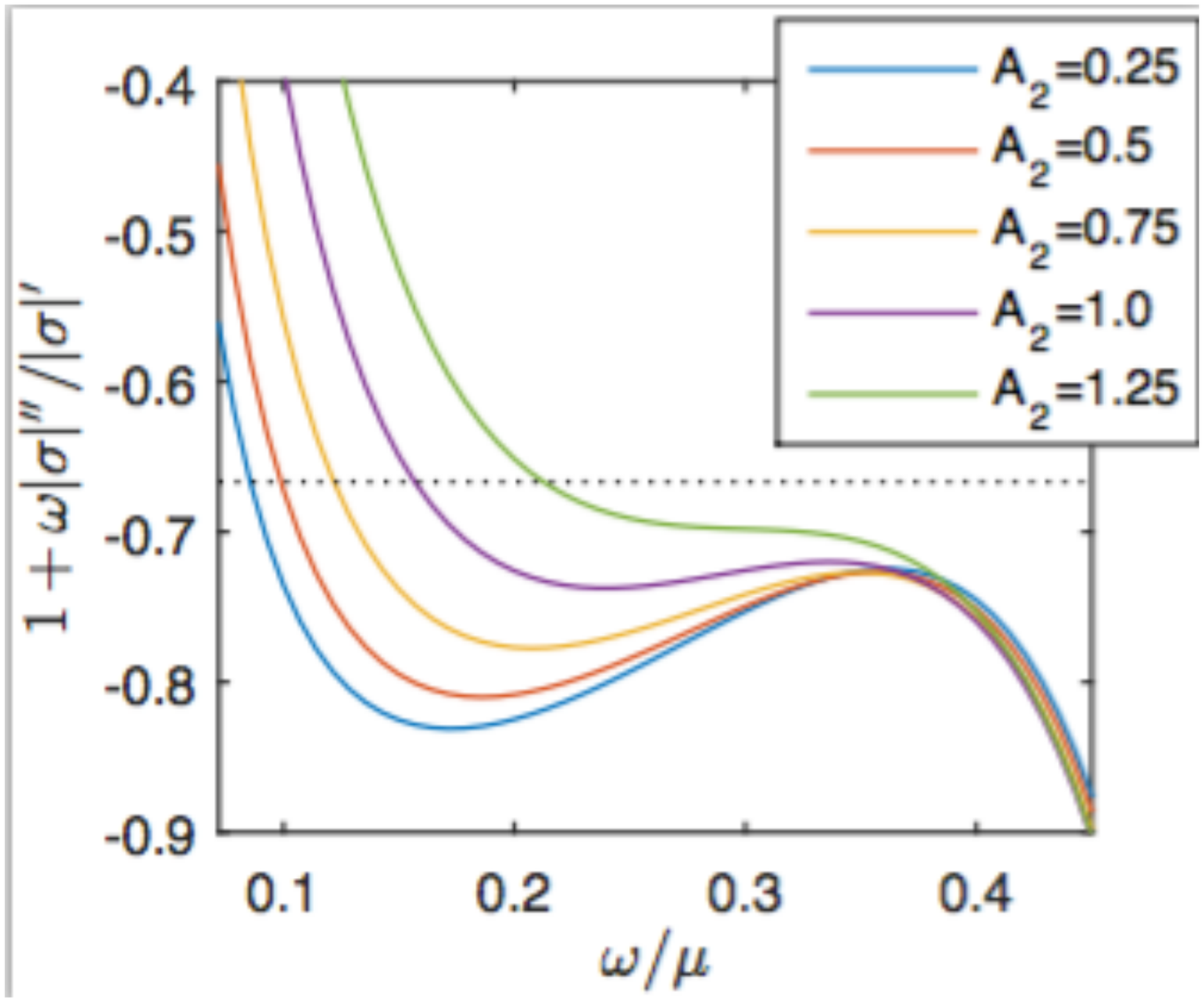
HST

Results

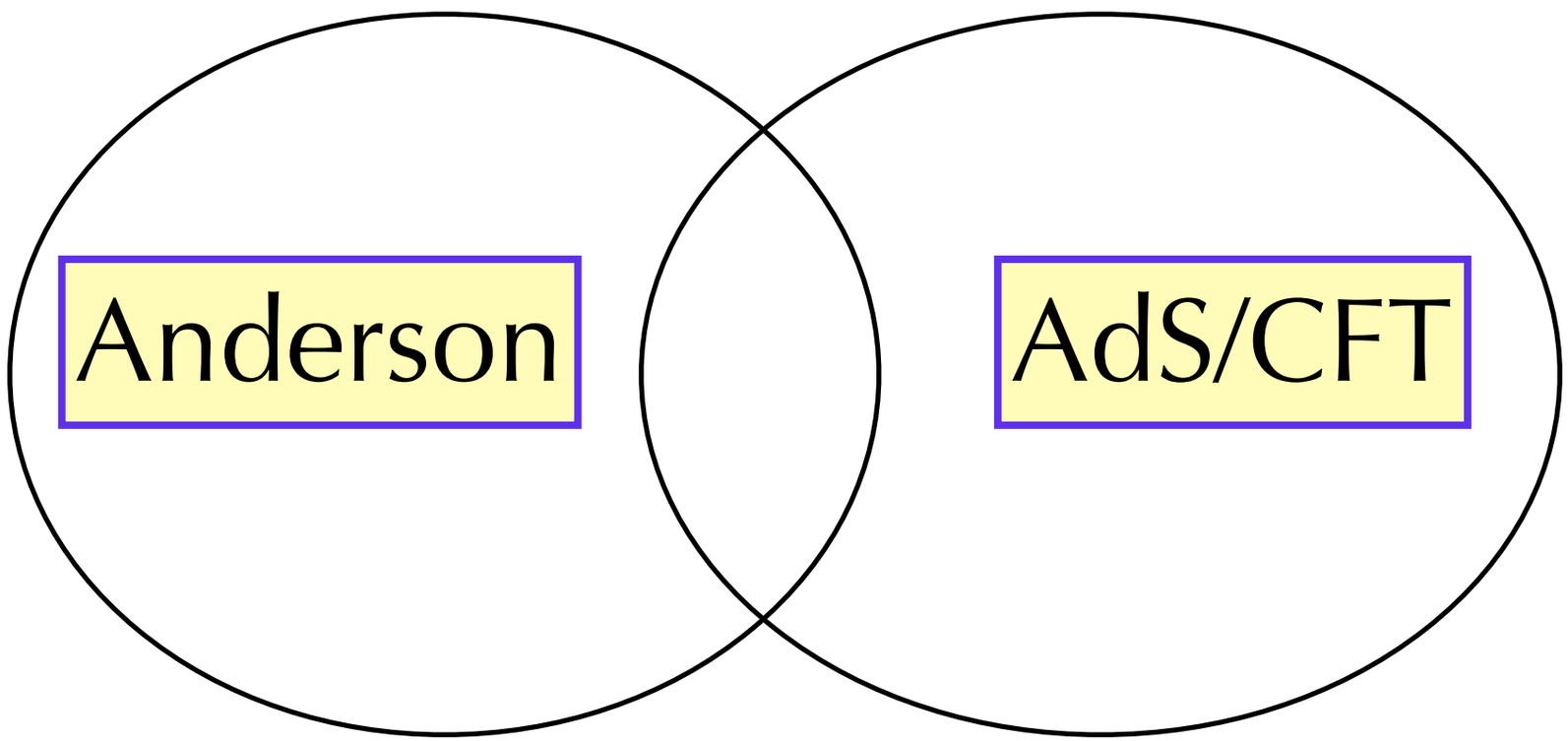


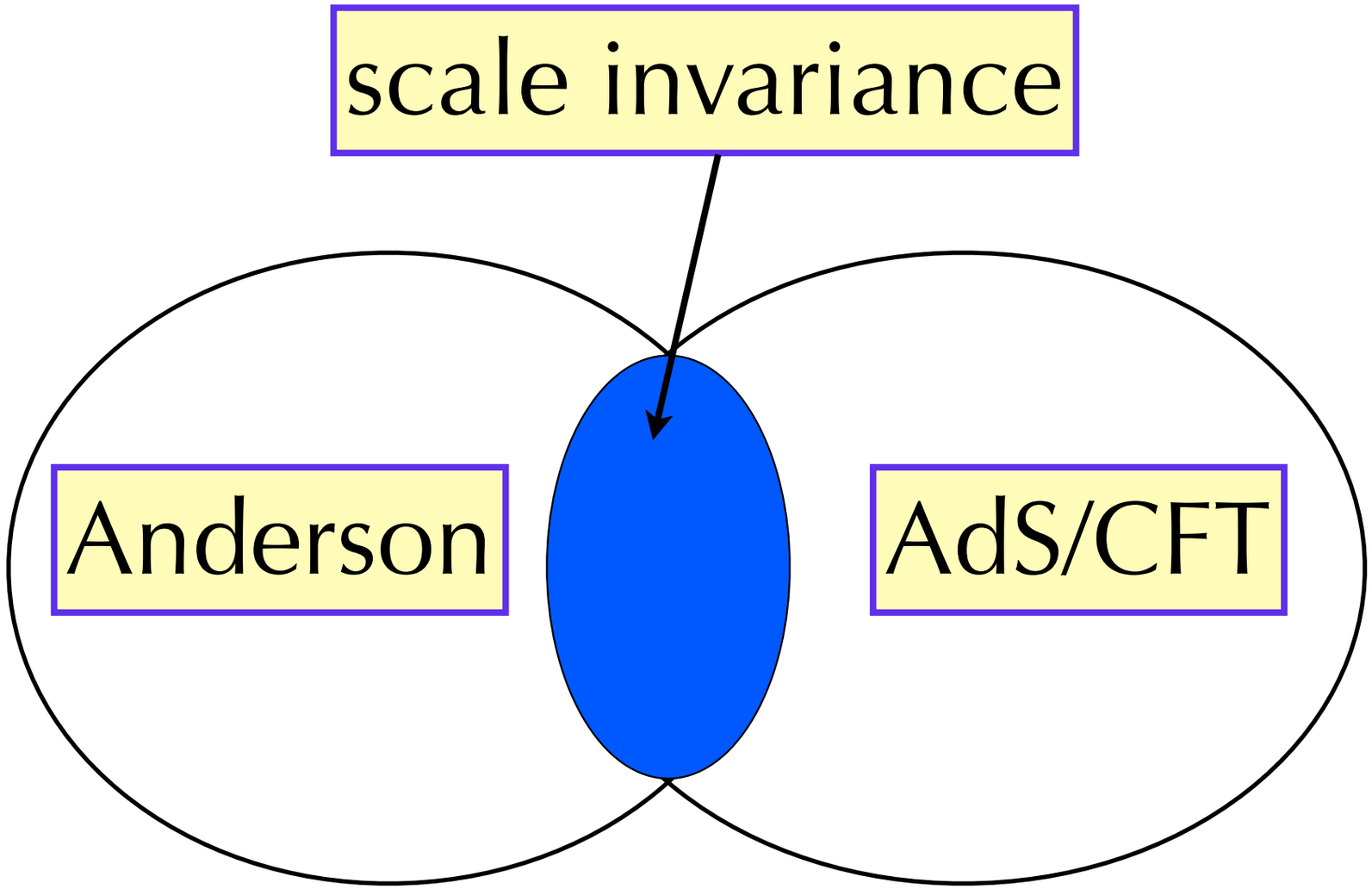
Results

$$A_1 = 0.75, k_1 = 2, k_2 = 2, \theta = 0, \mu = 1.4, T/\mu = 0.115$$

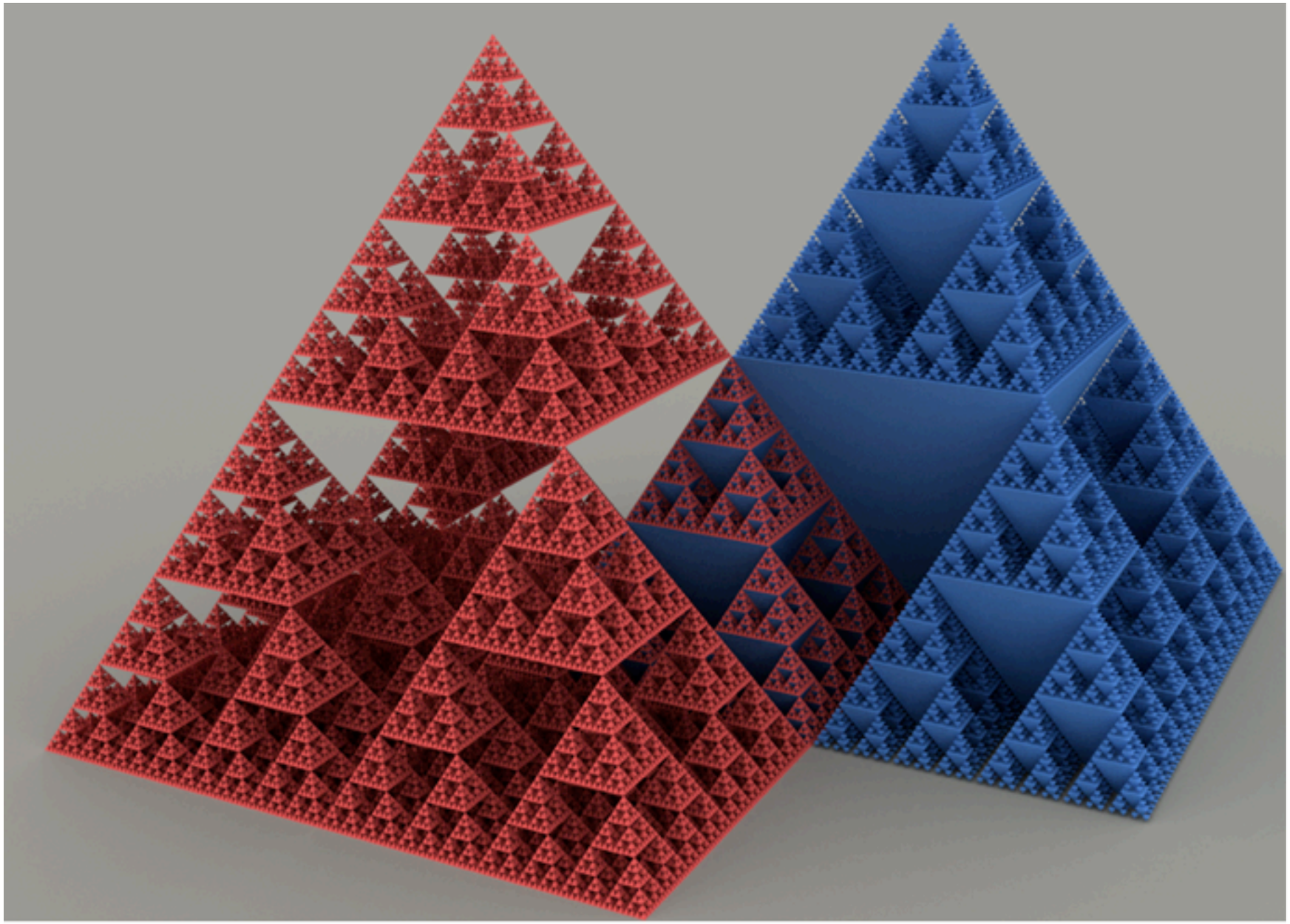


No


Anderson
AdS/CFT


scale invariance
Anderson
AdS/CFT

what is scale invariance?



invariance on all length scales

Properties all particles share?

charge

fixed mass!

Properties all particles share?

charge

fixed mass!

conserved (gauge invariance)

Properties all particles share?

charge

fixed mass!

mass sets a scale

conserved (gauge invariance)

Properties all particles share?

charge

fixed mass!

mass sets a scale

conserved (gauge invariance)

not conserved

Properties all particles share?

charge

fixed mass!

s sets a s

conserved (gauge invariance)

not conserved

no well-defined mass

no well-defined mass

incoherent stuff (all energies)

no well-defined mass

what has an ill-defined mass?

incoherent stuff (all energies)

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi + m^2 \phi^2$$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

$$x \to x/\Lambda$$
$$\phi(x) \to \phi(x)$$

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$$m^2\phi^2$$

no scale invariance

$$\mathcal{L} = \quad \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right)$$

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

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theory with all possible mass!

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$$m^2/\Lambda^2 \to m^2$$

$$\mathcal{L} \to \Lambda^4 \mathcal{L}$$

scale invariance is restored!!

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not particles

unparticles

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not particles

propagator

$$\left(\int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon}\right)^{-1} \propto p^{2|\gamma|}$$

propagator

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$$d_U - 2$$

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$$d_U - 2$$

continuous mass

$\phi(x, m^2)$

flavors

propagator

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$$d_U - 2$$

continuous mass

$\phi(x, m^2)$ ⟷ $e^2(m)$

Karch, 2015

flavors

multi-bands

take experiments seriously

take experiments seriously

$$\sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega\tau_i}$$

take experiments seriously

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continuous mass $d\sigma(m) = \sigma(m) dm$

take experiments seriously

$$\sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega\tau_i}$$

continuous mass $d\sigma(m) = \sigma(m) dm$

$$\sigma(\omega) = \int_0^M \frac{\rho(m) e^2(m) \tau(m)}{m} \frac{1}{1 - i\omega\tau(m)} dm$$

variable masses for everything

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

$$e(m) = e_0 \frac{m^b}{M^b}$$

$$\tau(m) = \tau_0 \frac{m^c}{M^c}$$

Karch, 2015

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Karch, 2015

perform integral

$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

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$$\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M} \frac{1}{\omega^{\frac{2}{3}}} \int_0^{\omega\tau_0} dx \; \frac{x^{-\frac{1}{3}}}{1-ix}$$

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$$\Big\downarrow \; \omega\tau_0 \to \infty$$

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$$\omega\tau_0 \to \infty$$

$$\boxed{\sigma(\omega) = \frac{1}{3}(\sqrt{3}+3i)\pi \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M\omega^{\frac{2}{3}}}}$$

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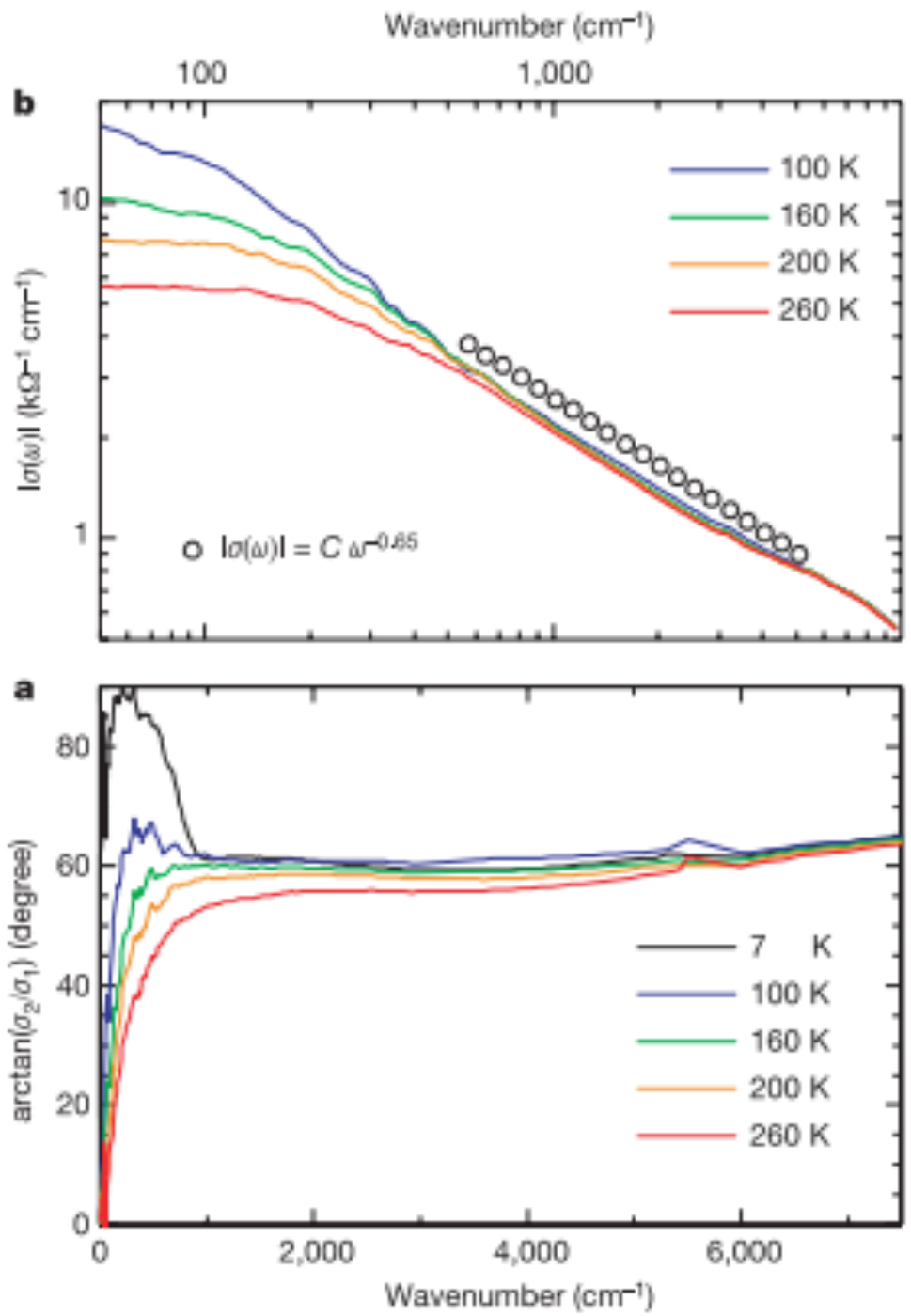
$$\boxed{\begin{array}{c} \tan\sigma = \sqrt{3} \\ 60^\circ \end{array}}$$

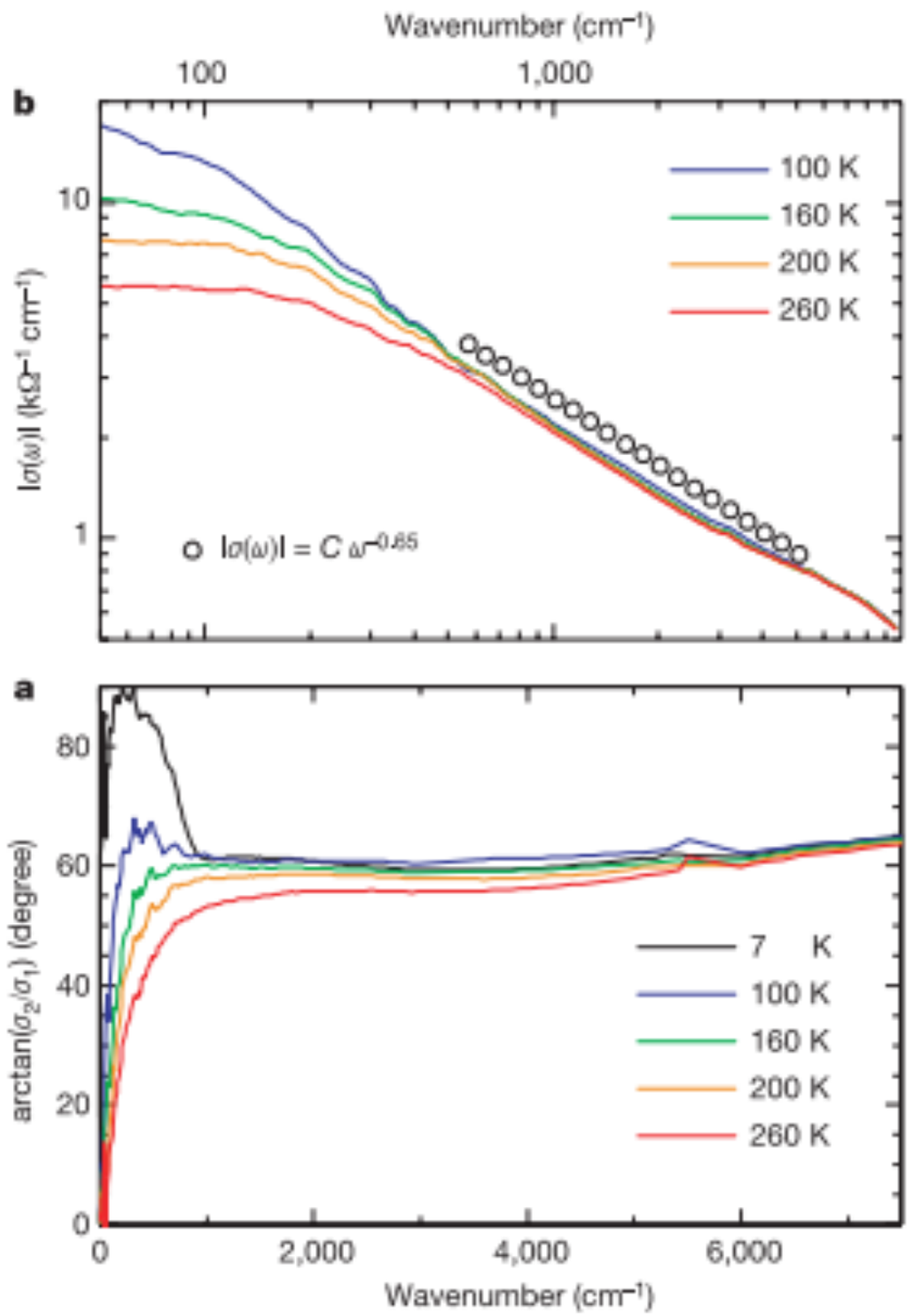
experiments



$$\sigma(\omega) = C\omega^{\gamma-2}e^{i\pi(1-\gamma/2)}$$

$$\gamma = 1.35$$

experiments



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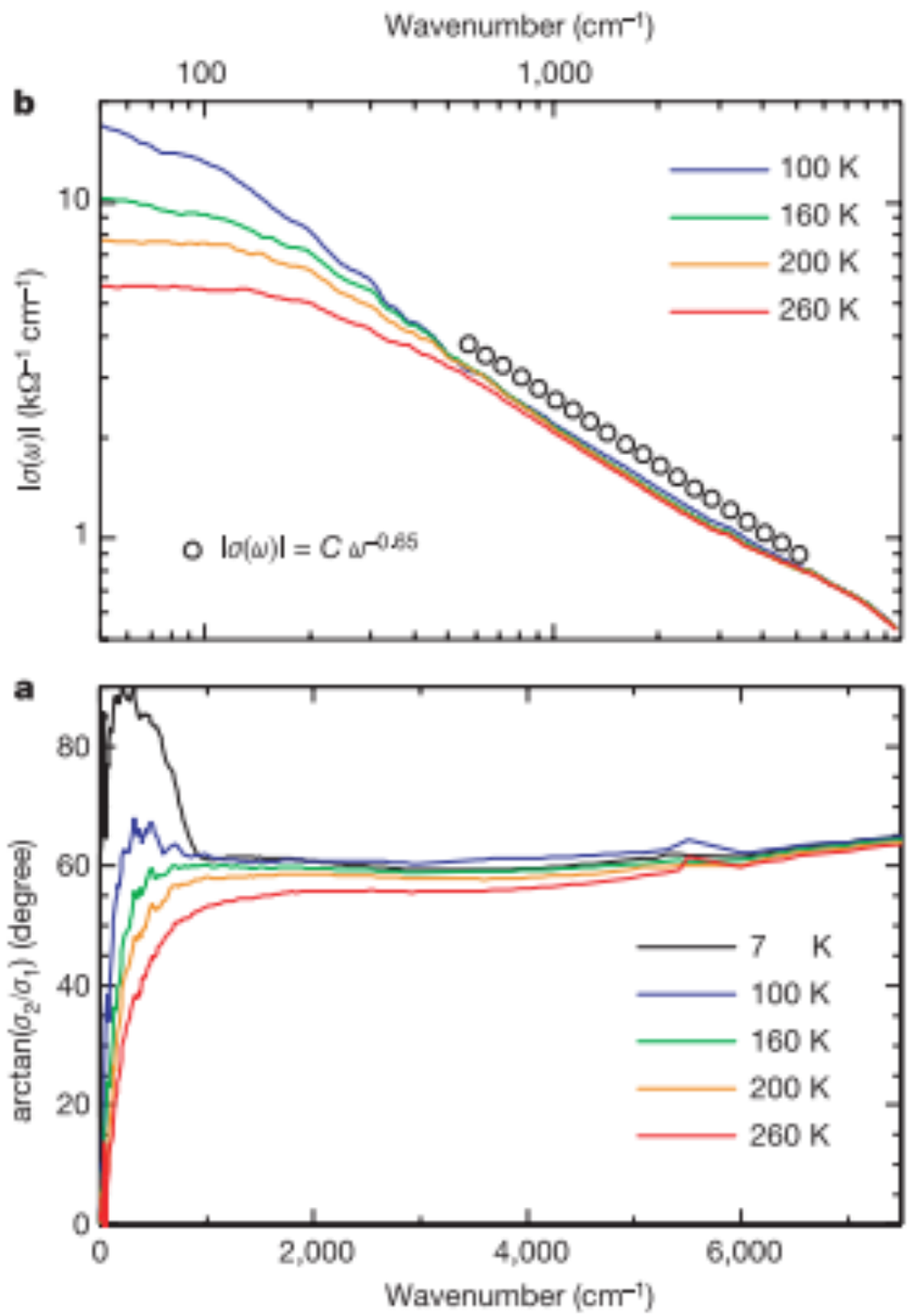

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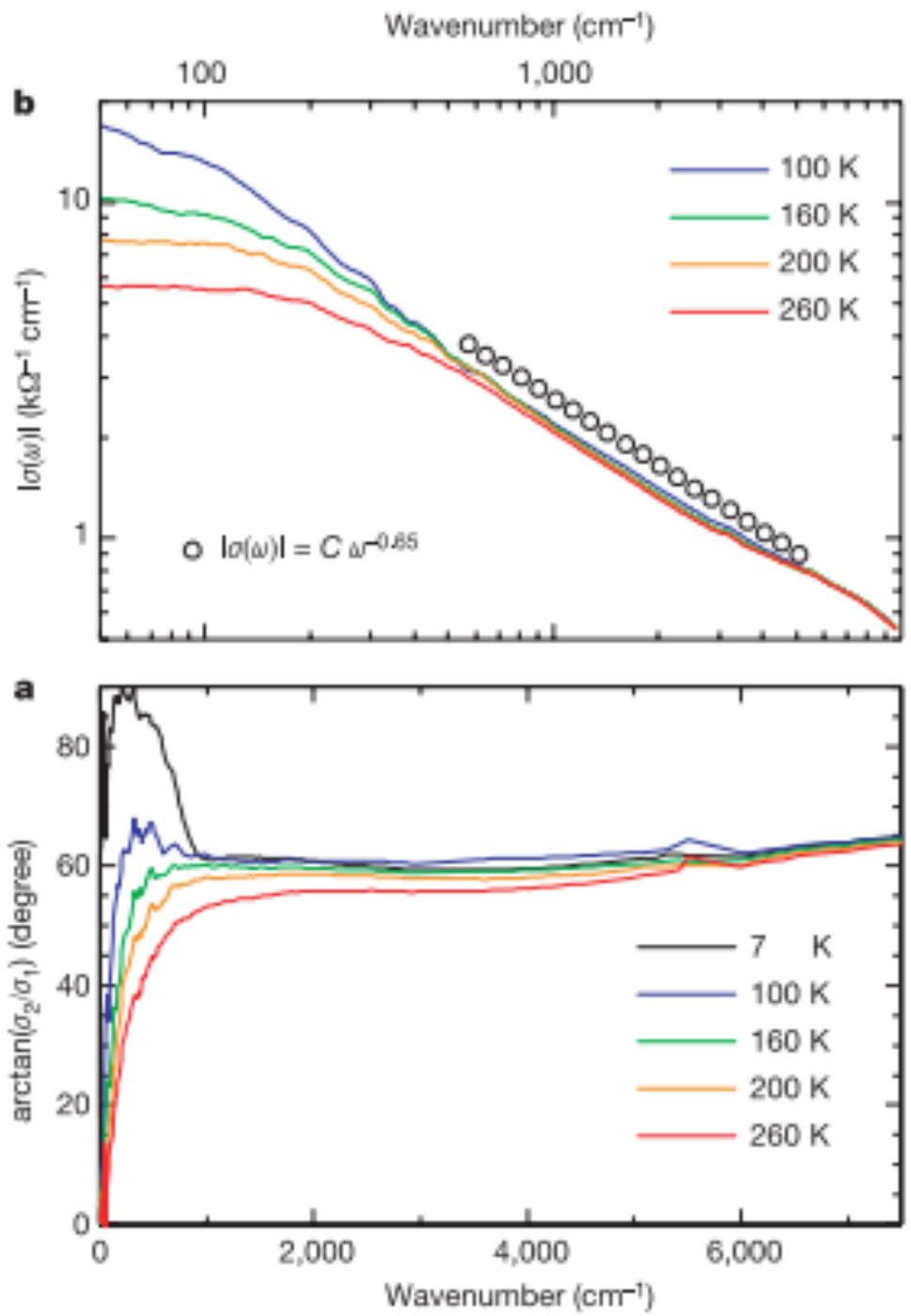

$$\sigma(\omega) = \frac{1}{3}(\sqrt{3}+3i)\pi\frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M\omega^{\frac{2}{3}}}$$

victory!!

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$$\theta = 60^\circ$$

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$$\gamma = 1.35$$

are anomalous dimensions necessary

$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

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hyperscaling violation

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momentum loss

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$$c = 1$$

$$a + 2b = 2/3$$

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hyperscaling violation

anomalous dimension

momentum loss

$$c = 1$$

$$a + 2b = 2/3$$

$$\Rightarrow$$

$$b = 0$$

$$a = 2/3$$

No

No

but the Lorenz ratio is not a constant

$$L_H = \frac{\kappa_{xy}}{T\sigma_{xy}} \sim T \equiv T^{-2\Phi/z}$$

No

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Hartnoll/Karch

$$\Phi = bz = -2/3$$

combine AC
+DC transport

fixes all exponents
a,b,c

$$[J] = d_U$$

combine AC
+DC transport

fixes all exponents
a,b,c

probe with noise
measurements

$$[J] = d_U$$

what really is the summation over mass?

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$$\mathcal{L}_{\text{eff}} = \int_0^\infty \mathcal{L}(x, m^2) dm^2$$

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but $m \propto 1/L$ → hidden extra dimension

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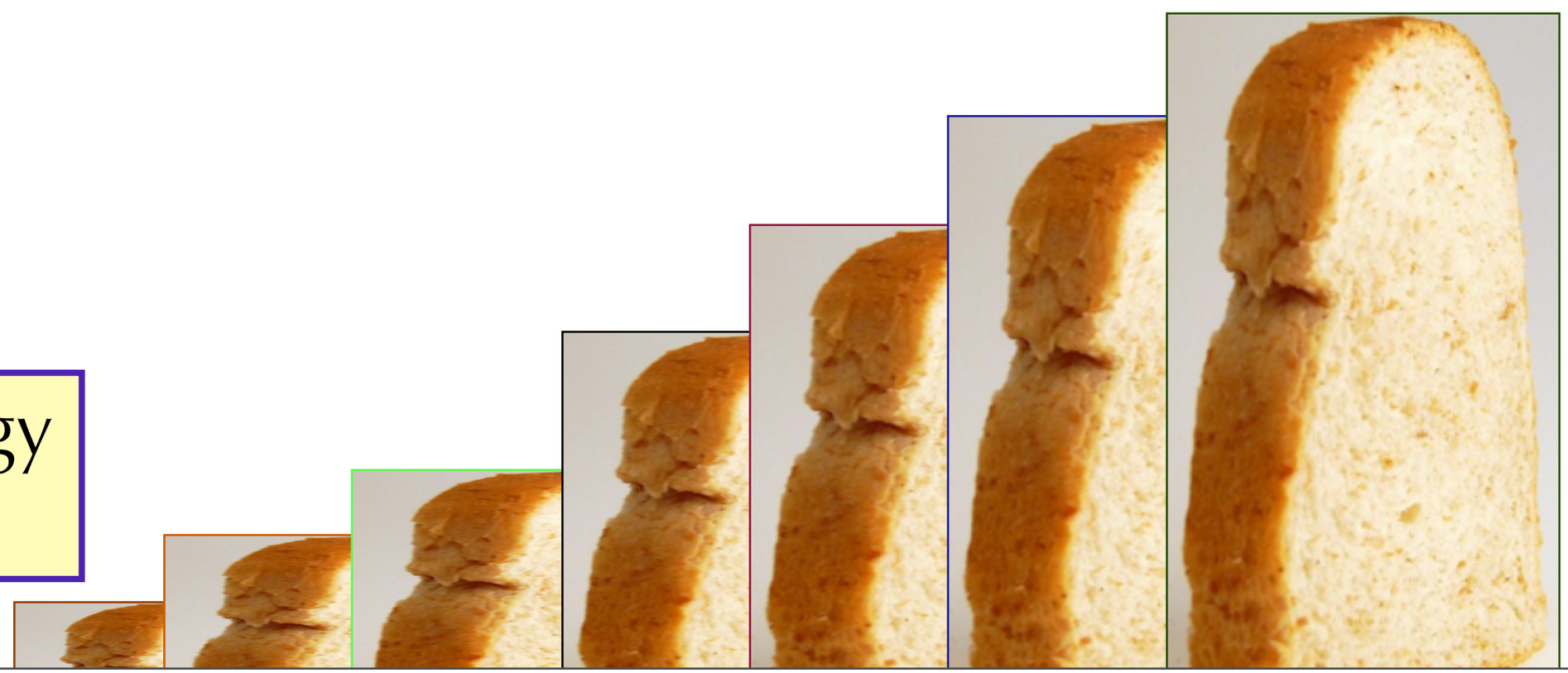
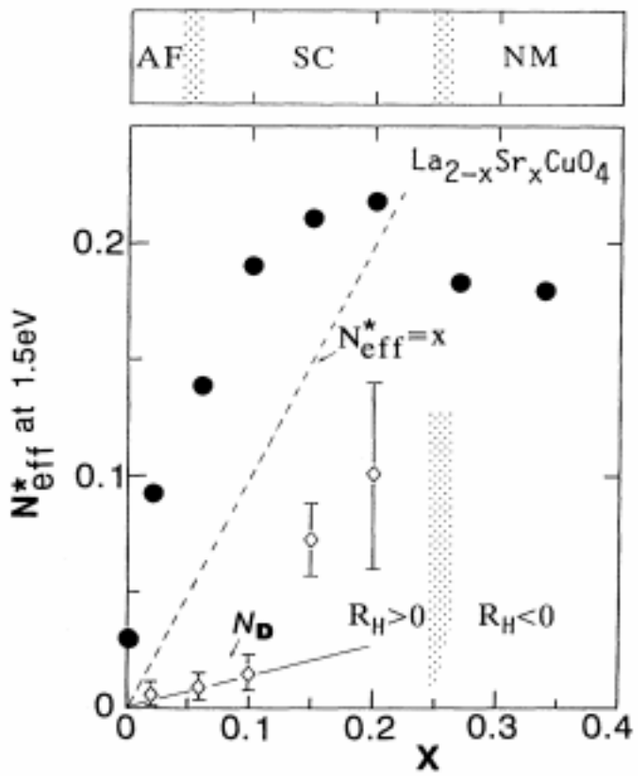
high energy (UV)

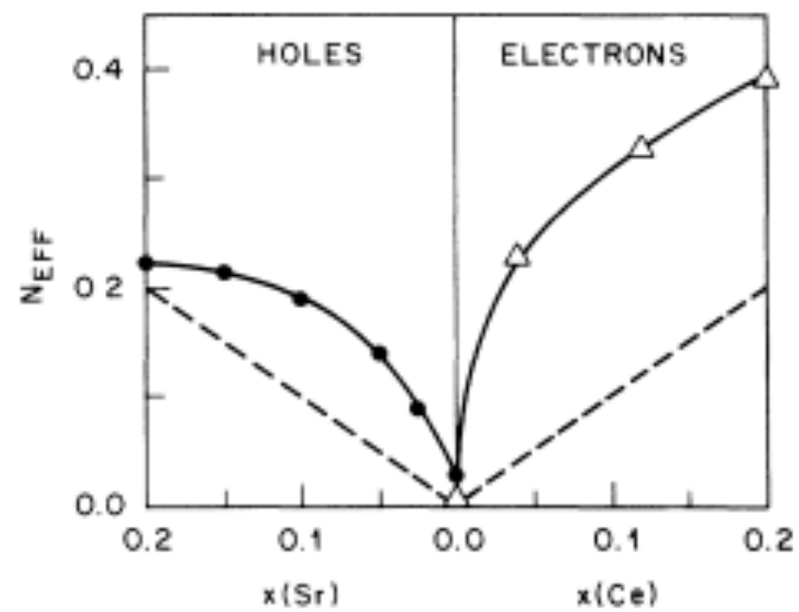


low energy (IR)

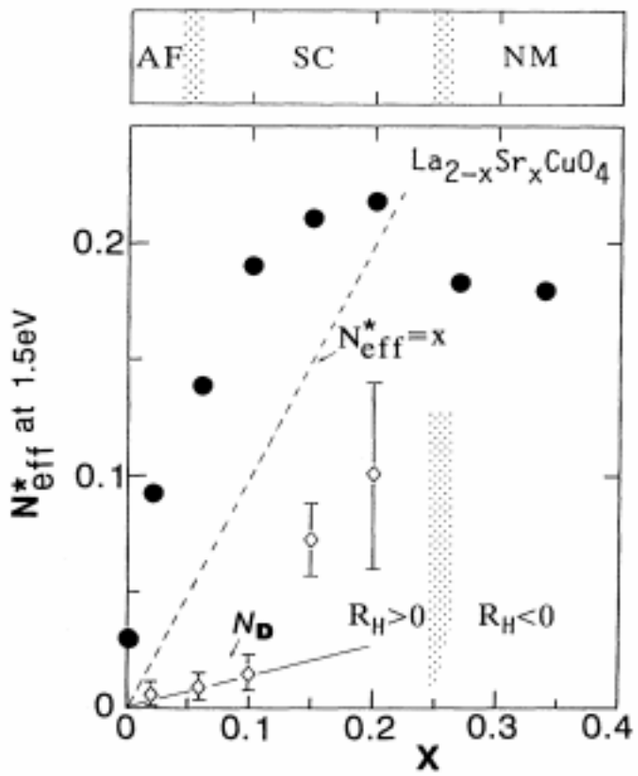
no gravity?

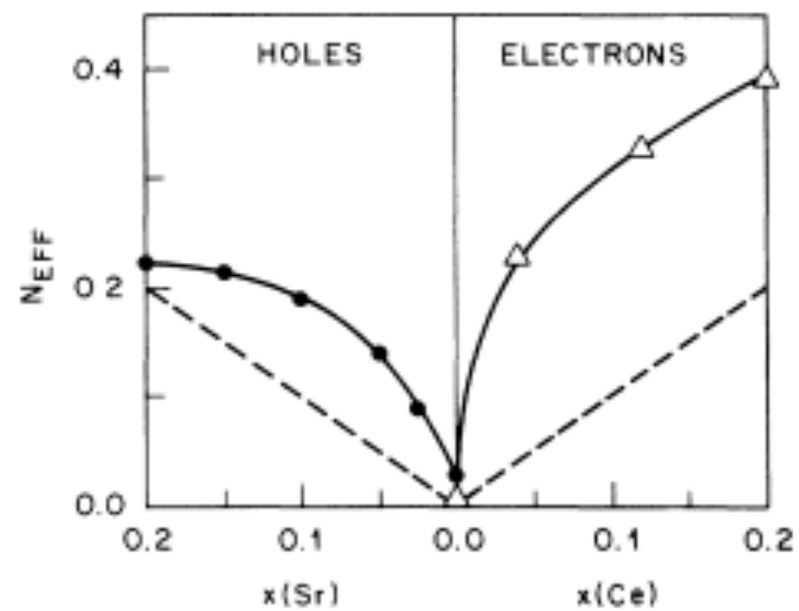
Cooper, et al.


AF
SC
NM
$La_{2-x}Sr_xCuO_4$
N^*_{eff} at 1.5eV
$N^*_{eff}=x$
N_D
$R_H>0$
$R_H<0$
0.2
0.1
0
0
0.1
0.2
0.3
0.4
x


HOLES
ELECTRONS
N_{EFF}
0.4
0.2
0.0
0.2
0.1
0.0
0.1
0.2
x(Sr)
x(Ce)

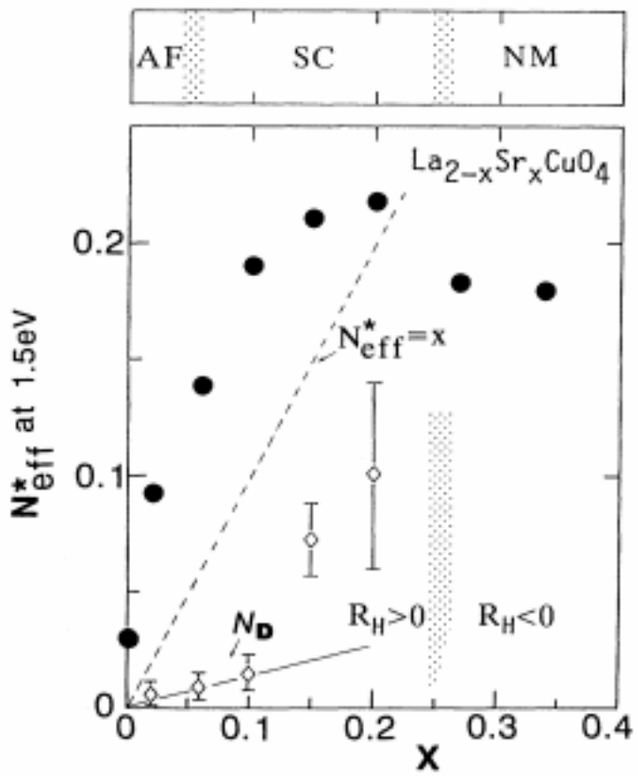
Cooper, et al.

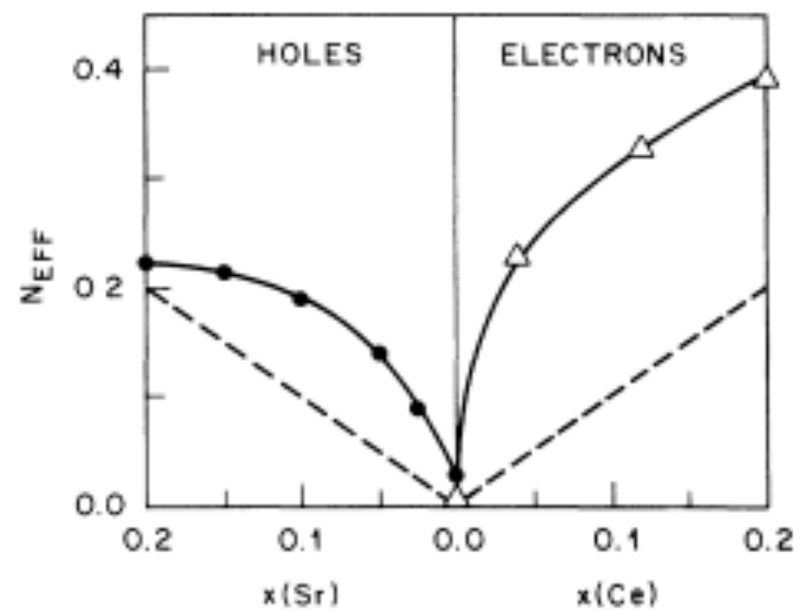




low-energy model for $N_{\text{eff}} > x$??

Cooper, et al.





low-energy model for $N_{\text{eff}} > x$??

does such a theory include gravity?

f-sum rule

$$\text{K.E.} = p^2/2m$$

$$N_{\text{eff}} = x$$

what if?

$$\text{K.E.} \propto (\partial_\mu^2)^\alpha$$

f-sum rule

$$\frac{W(n,T)}{\pi c e^2} = A n^{1+\frac{2(\alpha-1)}{d}} + \cdots$$

what if?

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f-sum rule

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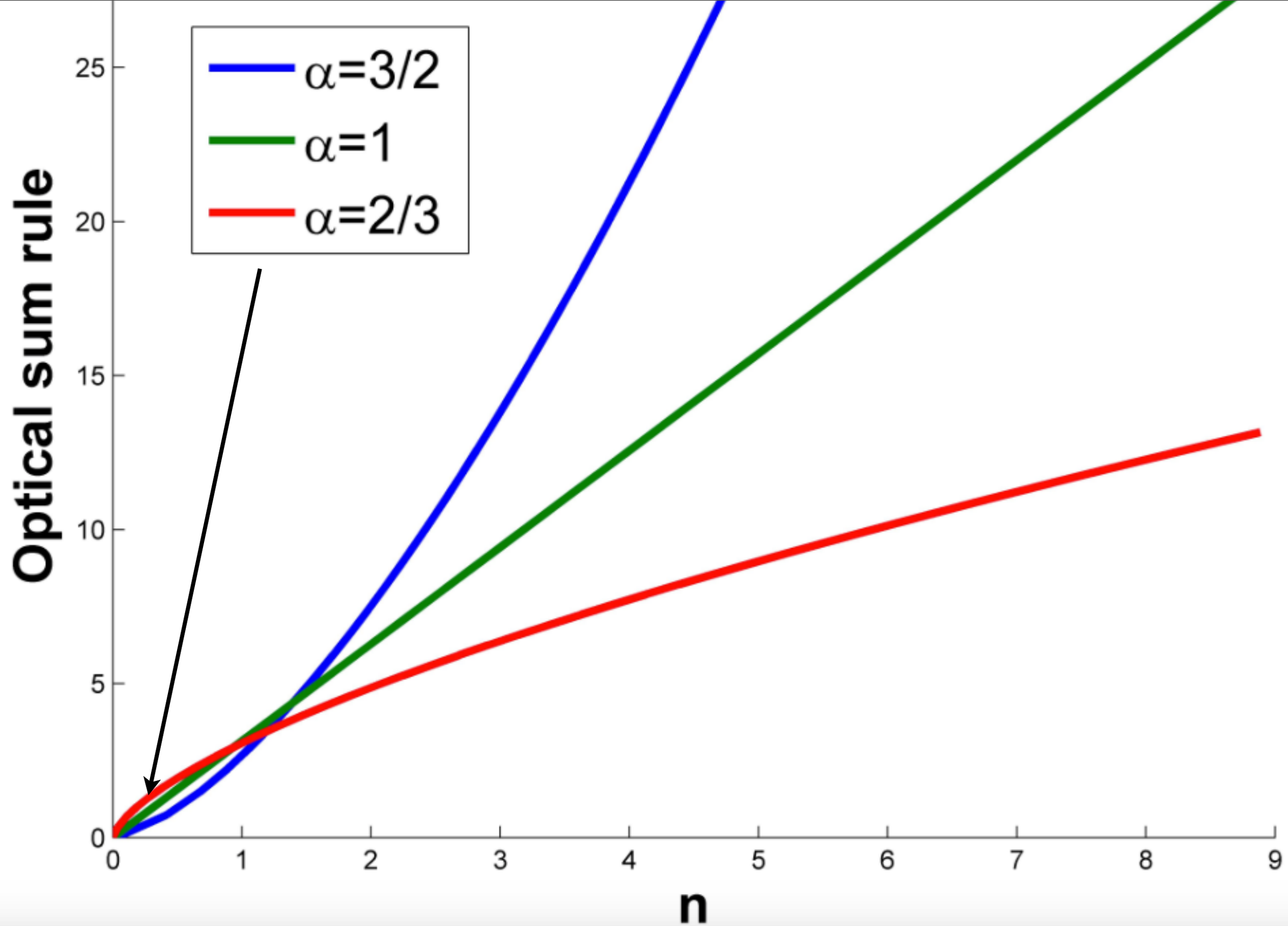
W>n if $\alpha < 1$


α=3/2
α=1
α=2/3
Optical sum rule
n
0
5
10
15
20
25
0
1
2
3
4
5
6
7
8
9

what theories have $\alpha < 1$?

Unparticles as the Holographic Dual of Gapped AdS Gravity

Sophia K. Domokos* and Gregory Gabadadze†

Center for Cosmology and Particle Physics
Department of Physics, New York University
New York, NY 10003, USA

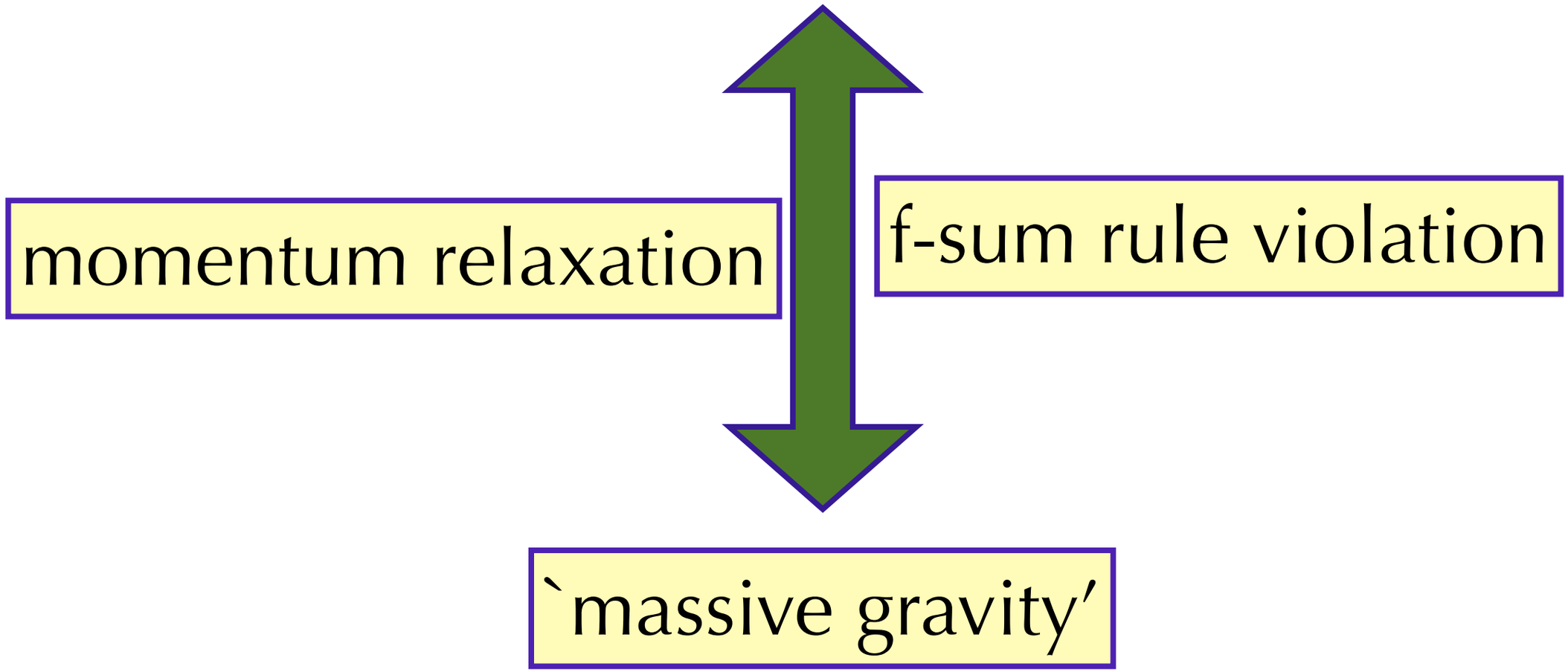
$$S_{eff} = (-1)^{-\frac{2\tilde{\Delta}+1}{\tilde{\Delta}+1}} \int d^d x \left[\Phi(-\partial^2)^{-\tilde{\Delta}} \Phi + (-1)^{-\frac{1}{(\tilde{\Delta}+1)(\tilde{\Delta}+2)}} A^{\perp}_{\mu}(-\partial^2)^{-\tilde{\Delta}-1} A^{\perp\mu} + h^{Tt}_{\mu\nu}(-\partial^2)^{-\tilde{\Delta}} h^{Tt\mu\nu} \right] . \tag{79}$$

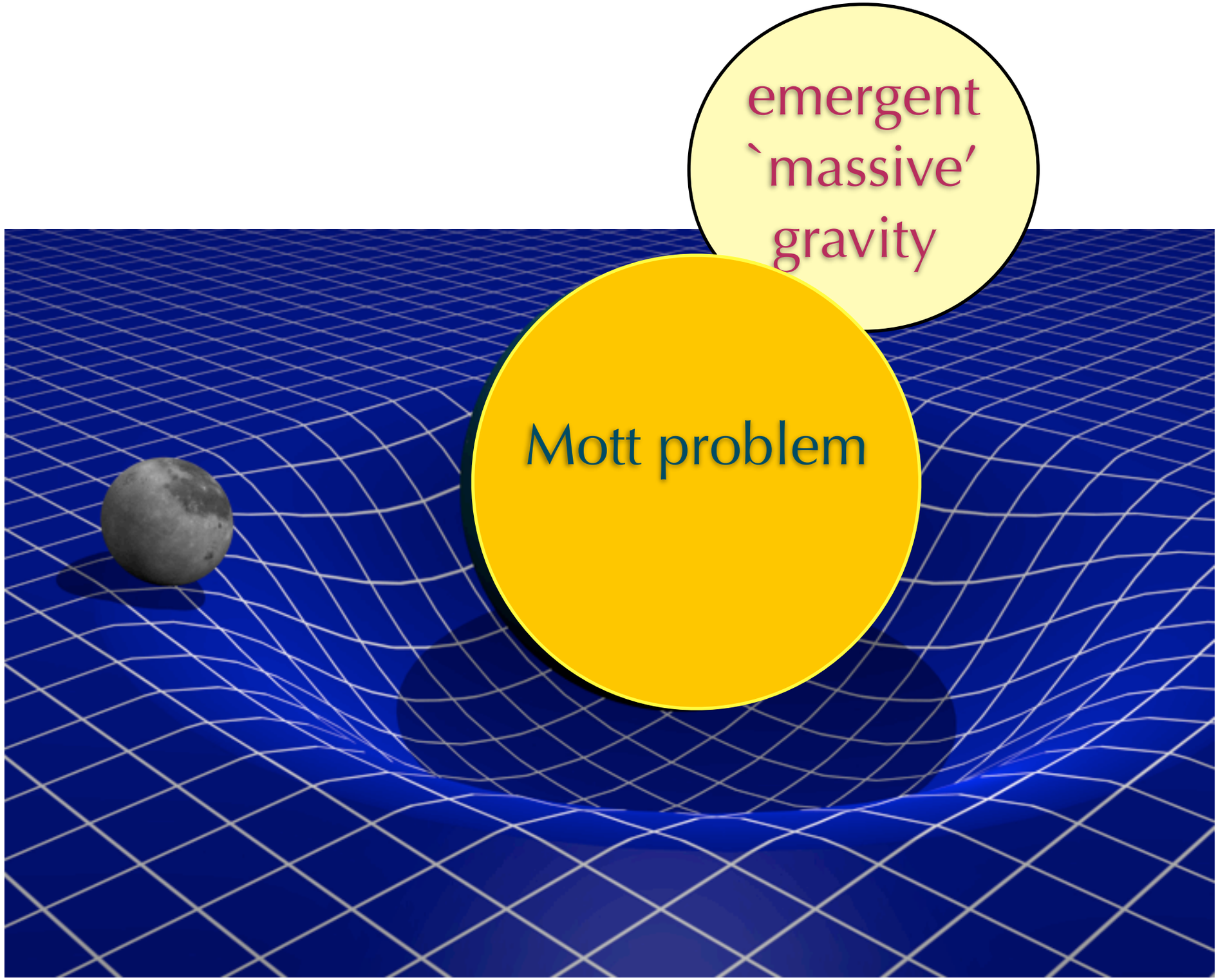
$\tilde{\Delta}_{\Phi} < 0$

non-local action with non-canonical kinetic energy

incoherent metals: Mottness

unparticles




emergent
`massive'
gravity
Mott problem

Are the Cuprate High-Temperature Superconductors full of Unparticles?

Are the Cuprate High-Temperature Superconductors full of Unparticles?

Yes!