Are the Cuprate High-Temperature Superconductors full of Unparticles?

Thanks to: NSF, EFRC (DOE)

Brandon Langley
Garrett Vanacore
Kridsangaphong Limtragool
Figure 1  Resistance in ohms of a specimen of mercury versus absolute temperature. This plot by Kamerlingh Onnes marked the discovery of superconductivity.
London Equation

\[ J \propto A \]
London Equation

\[ J \propto A \]

\[ B = B_0 e^{-r/\lambda} \]
strange metal

$\text{Y Ba}_2\text{Cu}_3\text{O}_7$

Cuprate Superconductors
strange metal

\[ J = d_U \]

Y Ba$_2$Cu$_{3}$O$_7$

Cuprate Superconductors
strange metal

\[ J = d_U \text{ fractional} \]
what is the strange metal?
Drude metal
Drude metal

\[ \dot{p} = e \left( E + \frac{p \times B}{m} \right) - \frac{p}{\tau} \]

momentum relaxation
Drude metal

\[ \dot{p} = e(E + \frac{p \times B}{m}) - \left(\frac{p}{\tau}\right) \]

momentum relaxation

\[ \sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau} \]
Drude metal

\[ \dot{p} = e \left( E + \frac{p \times B}{m} \right) - \frac{p}{\tau} \]

momentum relaxation

\[ \sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau} \quad \lim_{\tau \to 0} \Re \sigma \to \infty \]
\[ N_{\text{eff}} = \frac{2mV_{\text{cell}}}{\pi e^2} \int_0^\infty \sigma(\omega) d\omega \]
Growth of the optical conductivity in the Cu-O planes

AT&T Bell Laboratories, Murray Hill, New Jersey 07974

Z. Fisk
Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 7 March 1990)

\[
\text{La}_{2-x}\text{Sr}_x\text{CuO}_4
\]

optical gap
(Mott gap)
What is a Mott Insulator?

NiO insulates $d^8$?
What is a Mott Insulator?

NiO insulates $d^8$?

EMPTY STATES = METAL
What is a Mott Insulator?

NiO insulates $d^8$?

EMPTY STATES = METAL

band theory fails!
NiO insulates $d^8$? perhaps this costs energy

Mott mechanism
NiO insulates $d^8$? Perhaps this costs energy.

Mott mechanism

$U \gg t$
NiO insulates $d^8$? perhaps this costs energy

$U \gg t$

$\mu = 0$
NiO insulates $d^8$?
perhaps this costs energy

Mott mechanism

$U \gg t$

$\mu = 0$

no change in size of Brillouin zone

Monday, November 9, 15
doping dependence
modified f-sum rule

optical gap

\[ N_{\text{eff}}(\Omega) = \frac{2mV_{\text{cell}}}{\pi e^2} \int_{0}^{\Omega} \sigma(\omega) d\omega \]
modified f-sum rule

optical gap

$$N_{\text{eff}}(\Omega) = \frac{2mV_{\text{cell}}}{\pi e^2} \int_{0}^{\Omega} \sigma(\omega) d\omega$$

$$N_{\text{eff}} \propto x$$
Uchida, et al.

Cooper, et al.

La_{2-x}Sr_{x}CuO_{4}

N_{eff} = x

N_{eff}

R_{H} > 0, R_{H} < 0

Holes

Electrons

M(x(Sr))

M(x(Ce))

x
low-energy model for $N_{\text{eff}} > x$
excess carriers?
excess carriers?

charge carriers have no particle content?
Mott insulator
Mott insulator

\[ U = \infty \]
Mott insulator

\[ U = \infty \]
Mott insulator

\[ U = \infty \]

PES \[ \varepsilon F \] IPES

no hopping:
Mott insulator

\[ U = \infty \]

no hopping:

\[ \epsilon_F \]

PES \[ \rightarrow \] IPES

N \[ \rightarrow \] N

N-1

PES

Monday, November 9, 15
Mott insulator

PES \[\varepsilon_F\] IPES

no hopping:

PES \[N-1\] IPES \[N-1\]

\[U = \infty\]
Mott insulator

\[ U = \infty \]

No hopping:

\[ \varepsilon_F \]

\[ N - 1 \]

\[ PES \]

\[ \varepsilon_F \]

\[ IPES \]

\[ N - 1 \]

\[ PES \]

\[ \varepsilon_F \]

\[ IPES \]
Mott insulator

$U = \infty$

no hopping:

Sawatzky
Mott insulator

\[ U = \infty \]

Sawatzky

PES \[ \varepsilon_F \] IPES

no hopping:

1-x

2x

1-x
spectral function (dynamics)
spectral function (dynamics)
low-energy states\[> 1 + x\]
spectral function (dynamics)

low-energy states \( > 1 + x \)  
Harris/Lange 1967
why is this a problem?
counting electron states

need to know: $N$ (number of sites)
counting electron states

\[ x = \frac{n_h}{N} \]

need to know: \( N \) (number of sites)
counting electron states

removal states

\[ x = \frac{n_h}{N} \]

1 - x

need to know:  N (number of sites)
counting electron states

\[ x = \frac{n_h}{N} \]

removal states \( 1 - x \)
addition states \( 1 + x \)

need to know: \( N \) (number of sites)
counting electron states

2x

removal states
1 - x

addition states
1 + x

x = \frac{n_h}{N}

need to know: N (number of sites)
counting electron states

\[ 2x \]

removal states \[ 1 - x \]

addition states \[ 1 + x \]

low-energy electron states

\[ x = n_h / N \]

need to know: \( N \) (number of sites)
counting electron states

\[ 2x \]

removal states \[ 1 - x \]

addition states \[ 1 + x \]

low-energy electron states

\[ 1 - x + 2x = 1 + x \]

need to know: \( N \) (number of sites)

\( x = n_h/N \)
counting electron states

removal states

addition states

\[ 2x \]

\[ x = \frac{n_h}{N} \]

low-energy electron states

\[ 1 - x + 2x = 1 + x \]

high energy

\[ 1 - x \]

need to know: \( N \) (number of sites)
dynamics are not exhausted by counting electrons alone
dynamics are not exhausted by counting electrons alone

what’s the extra stuff?
what else is weird?
Quantum critical behaviour in a high-$T_c$ superconductor

D. van der Marel$^1$, H. J. A. Molegraaf$^1$, J. Zaanen$^2$, Z. Nussinov$^2$, F. Carbone$^1$, A. Damascelli$^3$, H. Elsafi$^1$, M. Greven$^1$, P. H. Kes$^2$ & M. Li$^2$

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$^2$Leiden Institute of Physics, Leiden University, 2300 RA Leiden, The Netherlands
$^3$Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA

Drude conductivity

\[
\frac{n\tau e^2}{m} \frac{1}{1 - i\omega\tau}
\]
Quantum critical behaviour in a high-$T_c$ superconductor

D. van der Marel$^{1,2}$, H. J. A. Molegraaf$^1$, J. Zaanen$^2$, Z. Nussinov$^{2,3}$, F. Carbone$^1$, A. Damascelli$^{1,4}$, H. Eisaki$^{1,5}$, M. Greven$^1$, P. H. Kes$^1$ & M. Li$^2$

$^1$Materials Science Centre, University of Groningen, 9747 AG Groningen, The Netherlands
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Drude conductivity

\[
\frac{n\tau e^2}{m} \left( \frac{1}{1 - i\omega \tau} \right)
\]

\[
\sigma(\omega) = C\omega^{-\frac{2}{3}}
\]
YBCO thin films

![Graph showing conductivity vs energy for different films](image)

**TABLE II.** Values of the exponent $\alpha$ describing the best fit $\sigma \sim \omega^{-\alpha}$ for the various films labeled A, B, D, E, and F films. $\sigma_a$ and $\sigma_b$ stand for the conductivities measured along the $a$ and $b$ directions in an untwinned single crystal (Ref. 14). $\sigma_{\text{MFL}}$ stands for the computed conductivity at 100 and 300 K for the marginal-Fermi-liquid model (MFL) (Ref. 16). The exponent is given within $\pm 0.05$.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_A$ (300 K)</th>
<th>$\sigma_B$ (300 K)</th>
<th>$\sigma_D$ (300 K)</th>
<th>$\sigma_E$ (300 K)</th>
<th>$\sigma_F$ (300 K)</th>
<th>$\sigma_a$ (100 K)</th>
<th>$\sigma_b$ (100 K)</th>
<th>$\sigma_{\text{MFL}}$ (300 K)</th>
<th>$\sigma_{\text{MFL}}$ (100 K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.77</td>
<td>0.70</td>
<td>0.77</td>
<td>0.76</td>
<td>0.76</td>
<td>0.67</td>
<td>none</td>
<td>0.74</td>
<td>0.68</td>
</tr>
</tbody>
</table>
criticality

scale invariance

power law correlations
Anderson: use scale-invariant propagators

\[ G^R \propto \frac{1}{(\omega - v_sk)^\eta} \]
Anderson: use scale-invariant propagators

\[ G^R \propto \frac{1}{(\omega - v_sk)^\eta} \]

d=1+1 system
Luttinger liquid
compute conductivity without vertex corrections (PWA)

is flawed. In fact, in the Luttinger liquid such direct calculations are not to be trusted very firmly, since it is the nature of the Luttinger liquid that vertex corrections, if they must be included, will be singular; conventional transport theory is not applicable, and special methods such as the above are necessary.

\[ \sigma(\omega) \propto \frac{1}{\omega} \int dx \int dt G^e(x,t)G^h(x,t)e^{i\omega t} \propto (i\omega)^{-1+2\eta} \]
1997
Anderson
cuprates d=2+1, vertex corrections matter
1997 Anderson

cuprates d=2+1, vertex corrections matter

2012
string theory
AdS/CFT
optical conductivity from a gravitational lattice

log-log plots for various parameters

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

for $0.2 \lesssim \omega \tau \lesssim 0.8$

G. Horowitz et al., Journal of High Energy Physics, 2012
log-log plots for various parameters

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a remarkable claim! replicates features of the strange metal? how?

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\[ |\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C \]

for \(0.2 \lesssim \omega_T \lesssim 0.8\)

a remarkable claim! replicates features of the strange metal? how?

momentum relaxation??

G. Horowitz et al., Journal of High Energy Physics, 2012
Could string theory be the answer?

power law?

momentum relaxation?
IR

UV

QFT

coupling constant $g = 1/\text{ego}$
IR

$\frac{dg(E)}{d\ln E} = \beta(g(E))$

locality in energy

coupling constant

$g = 1/\text{ego}$
gauge-gravity duality
(Maldacena, 1997)

locality in energy

g = \frac{1}{\text{ego}}

d\frac{g(E)}{d\ln E} = \beta(g(E))

coupling constant

IR

gravity

UV

QFT

(Monday, November 9, 15)
what’s the geometry?
what's the geometry?

\[
\frac{dg(E)}{d\ln E} = \beta(g(E)) = 0
\]

scale invariance
(continuous)
what’s the geometry?

\[
\frac{dg(E)}{dlnE} = \beta(g(E)) = 0
\]

\[E \rightarrow \lambda E\]

\[x^\mu \rightarrow x^\mu / \lambda\]

scale invariance (continuous)
what’s the geometry?

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solve Einstein equations
what’s the geometry?

\[ \frac{dg(E)}{dlnE} = \beta(g(E)) = 0 \]

scale invariance (continuous)

\[ E \rightarrow \lambda E \]

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solve Einstein equations

\[ ds^2 = \left( \frac{u}{L} \right)^2 \eta_{\mu\nu} dx^\mu dx^\nu + \left( \frac{L}{E} \right)^2 dE^2 \]

anti de-Sitter space
replace coupled theory with geometry

scale-invariant anti de-Sitter

$R \propto \sqrt{g}$

$g(\bar{\psi}\psi)^2$
replace coupled theory with geometry

scale-invariant anti de-Sitter

$\text{AdS}_2 \times \mathbb{R}^2$ $\quad R \quad \text{AdS}_4$

$g(\bar{\psi}\psi)^2$

$R \propto \sqrt{g}$
replace coupled theory with geometry

scale-invariant anti de-Sitter

\( \text{AdS}_2 \times R^2 \)

\( \text{AdS}_4 \)

\( R \propto \sqrt{g} \)

\( g(\bar{\psi}\psi)^2 \)

\( \text{RG}=\text{GR} \)
replace coupled theory with geometry

scale-invariant anti de-Sitter

$R \propto \sqrt{g}$

cannot describe systems at $g=0$!
certain strongly coupled theories have gravity duals

absence of locality
Einstein-Maxwell equations + non-uniform charge density = $B\omega^{-2/3}$
not so fast!
Drude conductivity

\[ \frac{n \tau e^2}{m} \frac{1}{1 - i\omega \tau} \]
Donos and Gauntlett (gravitational crystal)

Drude conductivity

\[ \frac{n\tau e^2}{m} \frac{1}{1 - i\omega\tau} \]

\[ \frac{-2}{3} = 1 + \omega \frac{|\sigma|''}{|\sigma|'} \]
Donos and Gauntlett
(gravitational crystal)

Drude conductivity

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Drude conductivity

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\[ -\frac{2}{3} = 1 + \omega \frac{|\sigma|''}{|\sigma|'} \]

no power law!!
who is correct?
who is correct?

let’s redo the calculation
model
model

\[
\text{action} = \text{gravity} + \text{EM} + \text{lattice}
\]
model

action = gravity + EM + lattice

\[ S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} F^2 \right) , \]
$S = \frac{1}{16\pi G_N} \int d^4 x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} F^2 \right)$,
model

\[ \text{action} = \text{gravity} + \text{EM} + \text{lattice} \]

\[ S = \frac{1}{16\pi G_N} \int d^4 x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} F^2 \right), \]

\[ \mathcal{L}(\phi) = \sqrt{-g} \left[ -|\partial\phi|^2 - V(|\phi|) \right] \]
conductivity within AdS

\((g_{ab}, V(\Phi), A_t)\)

(metric, potential, gaugefield)
conductivity within AdS

\( (g_{ab}, V(\Phi), A_t) \)

(metric, potential, gaugefield)
conductivity within AdS

\[(g_{ab}, V(\Phi), A_t)\]

(metric, potential, gaugefield)

perturb with electric field
conductivity within AdS

\((g_{ab}, V(\Phi), A_t)\)
(metric, potential, gaugefield)

perturb with electric field

\(Q\)

\(\Phi\)

\(\vec{E}\)
conductivity within AdS

\[(g_{ab}, V(\Phi), A_t)\]
(metric, potential, gaugefield)

perturb with electric field

\[g_{ab} = \bar{g}_{ab} + h_{ab}\]
\[A_a = \bar{A}_a + b_a\]
\[\Phi_i = \bar{\Phi}_i + \eta_i\]

\[\sigma = J_x(x, \omega)/E\]
conductivity within AdS

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solve equations of motion with gauge invariance (without mistakes)

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conductivity within AdS

$(g_{ab}, V(\Phi), A_t)$
(metric, potential, gaugefield)

perturb with electric field

$g_{ab} = \bar{g}_{ab} + h_{ab}$

$A_a = \bar{A}_a + b_a$

$\Phi_i = \bar{\Phi}_i + \eta_i$

solve equations of motion with gauge invariance (without mistakes)

$\sigma = J_x(x, \omega)/E$
gravitational crystals

\[ g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu} \]
gravitational crystals

\[ g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu} \]

\[ \frac{1}{2} \Box h_{\mu\nu} - 2m^2 h_{\mu\nu} + \cdots \]

\[ m^2 \propto V(\Phi) \]

graviton has a mass!
gravitational crystals

\[ g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu} \]

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momentum relaxation
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\[ \partial_t T^{ti} \neq 0 = -\tau^{-1} T^{ti} \]
gravitational crystals

\[ g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu} \]

\[ \frac{1}{2} \Box h_{\mu\nu} - 2m^2 h_{\mu\nu} + \cdots \]

\[ m^2 \propto V(\Phi) \]

graviton has a mass!

momentum relaxation

\[ \partial_t T^{ti} \neq 0 = -\tau^{-1} T^{ti} \]

in Drude formula
Horowitz, Santos, Tong (HST)

\[ V(\Phi) = -\frac{\Phi^2}{L^2} \]

\[ \Phi(x) = A_0 \cos(kx) \]

inhomogeneous in \( x \)
Horowitz, Santos, Tong (HST)

\[ V(\Phi) = -\frac{\Phi^2}{L^2} \]

\[ \Phi(x) = A_0 \cos(kx) \]

inhomogeneous in \( x \)

HST vs. DG

DG

\[ V(|\Phi|^2) \]

\[ \Phi(z, x) = \phi(z)e^{ikx} \]

no inhomogeneity in \( x \)
Our Model

\[ \mathcal{L}_\Phi = (\nabla \Phi_1)^2 + (\nabla \Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2) \]
Our Model

\[ \mathcal{L}_\Phi = (\nabla \Phi_1)^2 + (\nabla \Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2) \]

\[ \Phi_1(x) = A_0 \cos \left( kx - \frac{\theta}{2} \right), \]
Our Model

\[ \mathcal{L}_\Phi = (\nabla \Phi_1)^2 + (\nabla \Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2) \]

\[ \Phi_1(x) = A_0 \cos \left(kx - \frac{\theta}{2}\right), \quad \Phi_2 = A_0 \cos \left(kx + \frac{\theta}{2}\right) \]
Our Model

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\[ \theta = 0 \]
Our Model

\[ \mathcal{L}_\Phi = (\nabla \Phi_1)^2 + (\nabla \Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2) \]

\[ \Phi_1(x) = A_0 \cos \left( kx - \frac{\theta}{2} \right), \quad \Phi_2 = A_0 \cos \left( kx + \frac{\theta}{2} \right) \]

\[ \theta = 0 \quad \text{HST} \]

\[ \theta = \frac{\pi}{2} \quad \text{DG} \]
$A_0 = 0.75, \ k = 1, \ \mu = 1.4, \ T/\mu = 0.115$

$\theta = 0$

$\theta = \frac{\pi}{4}$

$\theta = \frac{\pi}{2}$
translational invariance is broken in metric in multiples of $2k$
charge density

\[ \rho = \lim_{z \to 0} \sqrt{-gF^{tz}} \]
- high-frequency behavior is identical
- low-frequency RN has $\text{Re}(\sigma) \sim \delta(\omega)$, $\text{Im}(\sigma) \sim 1/\omega$
- low-frequency lattice has Drude form
is there a power law?
is there a power law?

Results

HST

DG

\(-\frac{2}{3}\)
Results
$A_1 = 0.75, k_1 = 2, k_2 = 2, \theta = 0, \mu = 1.4, T/\mu = 0.115$
No
Anderson

AdS/CFT
what is scale invariance?

invariance on all length scales
Properties all particles share?

charge

fixed mass!
Properties all particles share?

- charge
- fixed mass!
- conserved (gauge invariance)
Properties all particles share?

- charge
- fixed mass!
  - mass sets a scale

conserved (gauge invariance)
Properties all particles share?

charge

conserved (gauge invariance)

fixed mass!

mass sets a scale

not conserved
Properties all particles share?

- charge
- fixed mass!

Conserved (gauge invariance)

Not conserved
no well-defined mass
no well-defined mass

incoherent stuff (all energies)
no well-defined mass

what has an ill-defined mass?

incoherent stuff (all energies)
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]
\[ \phi(x) \rightarrow \phi(x) \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

mass
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \]

\[ x \rightarrow \frac{x}{\Lambda} \]

\[ \phi(x) \rightarrow \phi(x) \]

\[ m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \]

mass

\[ x \to x/\Lambda \]
\[ \phi(x) \to \phi(x) \]

\[ m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \to x/\Lambda \]

\[ \phi(x) \to \phi(x) \]

no scale invariance

mass

\[ \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \]

\[ m^2 \phi^2 \]
\[ \mathcal{L} = \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \]
\[ \mathcal{L} = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]
\[ \mathcal{L} = \int_0^{\infty} \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!
\[ L = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]
\[ x \rightarrow x / \Lambda \]
\[ m^2 / \Lambda^2 \rightarrow m^2 \]
\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial_{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]
\[ x \rightarrow x / \Lambda \]
\[ m^2 / \Lambda^2 \rightarrow m^2 \]

\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!
\[ \mathcal{L} = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]

\[ x \rightarrow x / \Lambda \]

\[ m^2 / \Lambda^2 \rightarrow m^2 \]

\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!

not particles
unparticles

\[\mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2\]

theory with all possible mass!

\[\phi \rightarrow \phi(x, m^2/\Lambda^2)\]
\[x \rightarrow x/\Lambda\]
\[m^2/\Lambda^2 \rightarrow m^2\]

\[\mathcal{L} \rightarrow \Lambda^4 \mathcal{L}\]

scale invariance is restored!!

not particles

Monday, November 9, 15
\[ \left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^{2|\gamma|} \]
\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^{2|\gamma|} 
\]

\[d_U - 2\]
propagator

\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^{2|\gamma|}
\]

continuous mass

\[ \phi(x, m^2) \]

flavors

Monday, November 9, 15
propagator

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\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^{2|\gamma|}
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continuous mass

\[\phi(x, m^2)\]

flavors

Monday, November 9, 15
(\int_0^\infty d\!m^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon})^{-1} \propto p^{2|\gamma|}

\begin{align*}
\phi(x, m^2) &\quad \leftrightarrow \quad e^2(m) \\
\text{flavors} &\quad \text{Karch, 2015} \\
\text{continuous mass} &\quad \text{multi-bands}
\end{align*}
take experiments seriously
take experiments seriously

\[ \sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega \tau_i} \]
take experiments seriously

\[ \sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega\tau_i} \]

continuous mass \[ d\sigma(m) = \sigma(m) dm \]
take experiments seriously

\[ \sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega \tau_i} \]

continuous mass \[ d\sigma(m) = \sigma(m) dm \]

\[ \sigma(\omega) = \int_0^M \frac{\rho(m) e^2(m) \tau(m)}{m} \frac{1}{1 - i\omega \tau(m)} dm \]
variable masses for everything

\[ \rho(m) = \rho_0 \frac{m^{a-1}}{M^a} \]
\[ e(m) = e_0 \frac{m^b}{M^b} \]
\[ \tau(m) = \tau_0 \frac{m^c}{M^c} \]

Karch, 2015
variable masses for everything

\[ \rho(m) = \rho_0 \frac{m^{a-1}}{M^a} \]
\[ e(m) = e_0 \frac{m^b}{M^b} \]
\[ \tau(m) = \tau_0 \frac{m^c}{M^c} \]

perform integral

Karch, 2015
\[
\frac{a + 2b - 1}{c} = -\frac{1}{3}
\]
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\frac{a + 2b - 1}{c} = -\frac{1}{3}
\]

\[
\sigma(\omega) = \frac{\rho_0 e^{2\tau_0}}{M} \frac{1}{\omega^{\frac{2}{3}}} \int_0^{\omega \tau_0} dx \frac{x^{-\frac{1}{3}}}{1 - ix}
\]
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\frac{a + 2b - 1}{c} = -\frac{1}{3}
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\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M} \frac{1}{\omega^{\frac{2}{3}}} \int_0^{\omega \tau_0} dx \frac{x^{-\frac{1}{3}}}{1 - ix}
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\[\omega \tau_0 \rightarrow \infty\]
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\frac{a + 2b - 1}{c} = -\frac{1}{3}
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\]

\[
\omega \tau_0 \rightarrow \infty
\]

\[
\sigma(\omega) = \frac{1}{3} (\sqrt{3} + 3i) \pi \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M \omega^{\frac{2}{3}}}
\]
\[
\frac{a + 2b - 1}{c} = -\frac{1}{3}
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\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0^{1/3}}{M} \frac{1}{\omega^{2/3}} \int_0^{\omega \tau_0} dx \frac{x^{-1/3}}{1 - ix}
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\sigma(\omega) = \frac{1}{3} (\sqrt{3} + 3i) \pi \frac{\rho_0 e_0^2 \tau_0^{1/3}}{M \omega^{2/3}}
\]

\[
\tan \sigma = \sqrt{3}
\]

\[
60^\circ
\]
\[ \sigma(\omega) = C \omega^{\gamma-2} e^{i\pi(1-\gamma/2)} \]
\[ \gamma = 1.35 \]
experiments

\[
\sigma(\omega) = \frac{1}{3}(\sqrt{3} + 3i)\pi \rho_0 e_0^2 \tau_0^{1/3} \frac{1}{M \omega^{2/3}}
\]

\[
\sigma(\omega) = C \omega^{\gamma-2} e^{i\pi(1-\gamma/2)}
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\[
\gamma = 1.35
\]
Experiments

\[ \sigma(\omega) = \frac{1}{3}(\sqrt{3} + 3i)\pi \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M \omega^{\frac{2}{3}}} \]

\[ \tan \frac{\sigma_2}{\sigma_1} = \sqrt{3} \]

\[ \theta = 60^\circ \]

\[ \sigma(\omega) = C \omega^{\gamma-2} e^{i\pi(1-\gamma/2)} \]

\[ \gamma = 1.35 \]
experiments

\[ \sigma(\omega) = \frac{1}{3} (\sqrt{3} + 3i) \pi \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M \omega^{\frac{2}{3}}} \]

victory!!

\[ \tan \sigma_2 / \sigma_1 = \sqrt{3} \]
\[ \theta = 60^\circ \]

\[ \sigma(\omega) = C \omega^{\gamma - 2} e^{i\pi (1 - \gamma/2)} \]
\[ \gamma = 1.35 \]
are anomalous dimensions necessary

\[ \frac{a + 2b - 1}{c} = -\frac{1}{3} \]

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\]

hyperscaling violation
anomalous dimension
momentum loss
are anomalous dimensions necessary

\[ \frac{a + 2b - 1}{c} = -\frac{1}{3} \]

\[ \rho(m) = \rho_0 \frac{m^{a-1}}{M^a} \]

\[ e(m) = e_0 \frac{m^b}{M^b} \]

\[ \tau(m) = \tau_0 \frac{m^c}{M^c} \]

\[ c = 1 \]

\[ a + 2b = \frac{2}{3} \]
are anomalous dimensions necessary

\[ \frac{a + 2b - 1}{c} = -\frac{1}{3} \]

\[ \rho(m) = \rho_0 \frac{m^{a-1}}{M^a} \]

\[ e(m) = e_0 \frac{m^b}{M^b} \]

\[ \tau(m) = \tau_0 \frac{m^c}{M^c} \]

\[ c = 1 \quad b = 0 \]

\[ a + 2b = \frac{2}{3} \quad a = \frac{2}{3} \]
No
No

but the Lorenz ratio is not a constant

\[ L_H = \frac{\kappa_{xy}}{T\sigma_{xy}} \sim T \equiv T^{-2\Phi/z} \]
No

but the Lorenz ratio is not a constant

\[ L_H = \frac{\kappa_{xy}}{T \sigma_{xy}} \sim T = T^{-2\Phi/z} \]

Hartnoll/Karch

\[ \Phi = bz = -2/3 \]
combine AC + DC transport

fixes all exponents a, b, c

\[ [J] = d_U \]
combine AC +DC transport

fixes all exponents $a,b,c$

probe with noise measurements

$[J] = d_U$
what really is the summation over mass?
what really is the summation over mass?

\[ \mathcal{L}_{\text{eff}} = \int_0^\infty \mathcal{L}(x, m^2) dm^2 \]
what really is the summation over mass?

\[ \mathcal{L}_{\text{eff}} = \int_{0}^{\infty} \mathcal{L}(x, m^2) dm^2 \]

but \( m \propto 1/L \)

hidden extra dimension
what really is the summation over mass?

\[ \mathcal{L}_{\text{eff}} = \int_0^\infty \mathcal{L}(x, m^2) dm^2 \]

but \( m \propto 1/L \)

hidden extra dimension

high energy (UV)

low energy (IR)
no gravity?
low-energy model for $N_{\text{eff}} > x$
Cooper, et al.

low-energy model for $N_{\text{eff}} > x$??

does such a theory include gravity?
f-sum rule

\[ K.E. = \frac{p^2}{2m} \]

\[ N_{\text{eff}} = x \]
what if?

K.E. \( \propto (\partial^2_\mu)\alpha \)

f-sum rule

\[
\frac{W(n, T)}{\pi c e^2} = A n^{1 + \frac{2(\alpha - 1)}{d}} + \cdots
\]
K.E. $\propto (\partial_\mu^2)^\alpha$

f-sum rule

$$\frac{W(n, T)}{\pi c e^2} = A n^{1+\frac{2(\alpha-1)}{d}} + \cdots$$

$W > n$ if $\alpha < 1$
what theories have $\alpha < 1$?
Unparticles as the Holographic Dual of Gapped AdS Gravity

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\[ S_{\text{eff}} = (-1)^{-\frac{\Delta + \frac{1}{2} + 1}{\Delta + 1}} \int d^dx \left[ \Phi(-\partial^2)^{-\Delta} \Phi + (-1)^{-\frac{3}{(\Delta + 1)(\Delta + 2)}} A_{\mu}^{\perp}(-\partial^2)^{-\Delta - 1} A_{\mu}^{\perp} + h_{\mu\nu}^{T}(\partial^2)^{-\Delta} h^{T\mu\nu} \right] . \]  

\[ \tilde{\Delta}_\Phi < 0 \]

non-local action with non-canonical kinetic energy
incoherent metals: Mottness

unparticles

momentum relaxation

f-sum rule violation

`massive gravity'
Are the Cuprate High-Temperature Superconductors full of Unparticles?
Are the Cuprate High-Temperature Superconductors full of Unparticles?

Yes!