

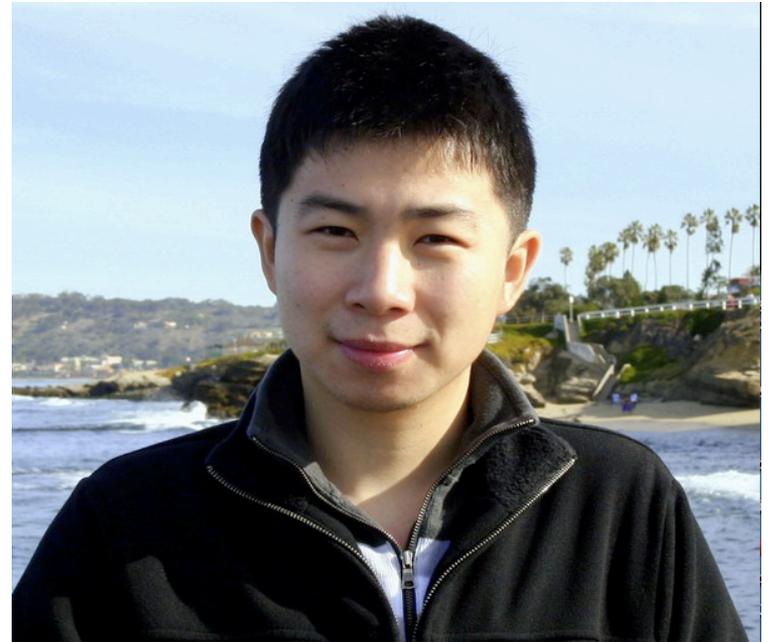
Arsenic and Old Lace: A Tale of Two Electrons in iron-based superconductors



F. Kruger



Antonio



W. Lv
1

Jiansheng Wu (not shown)

What are the pnictides Really?

Cuprates

manganites

What are the pnictides Really?



manganites

What are the pnictides Really?



manganites

multi-orbitals,
orbital order,
lattice distortions
magnetism, local/
itinerant electrons

What are the pnictides Really?

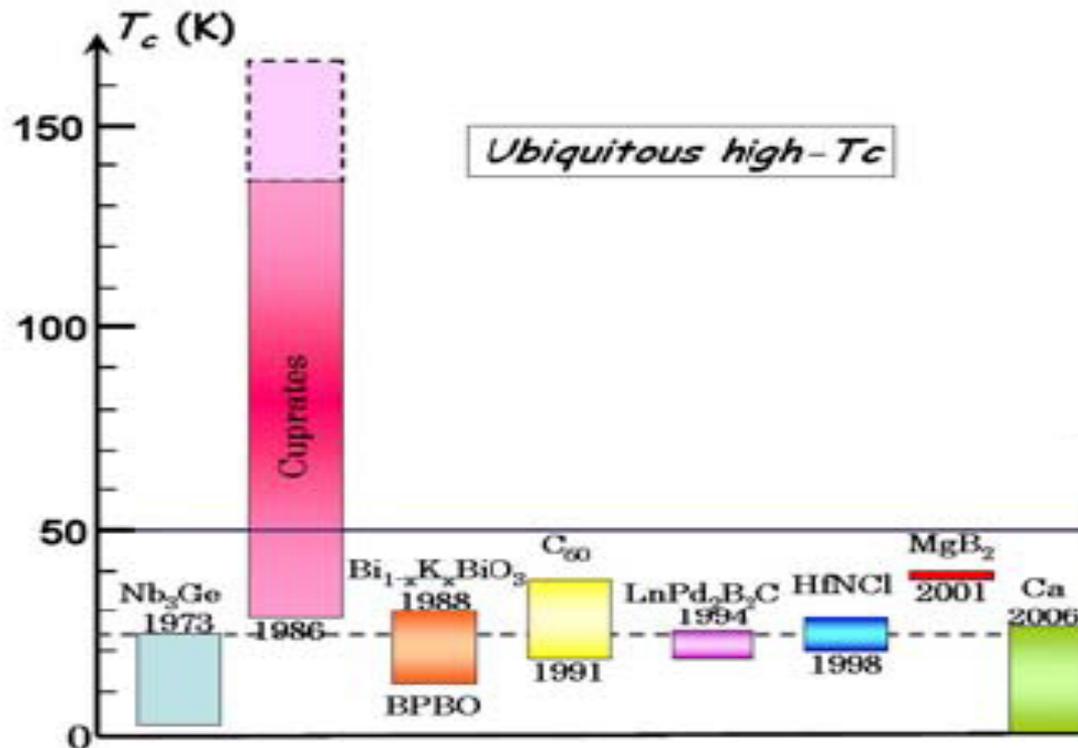


manganites

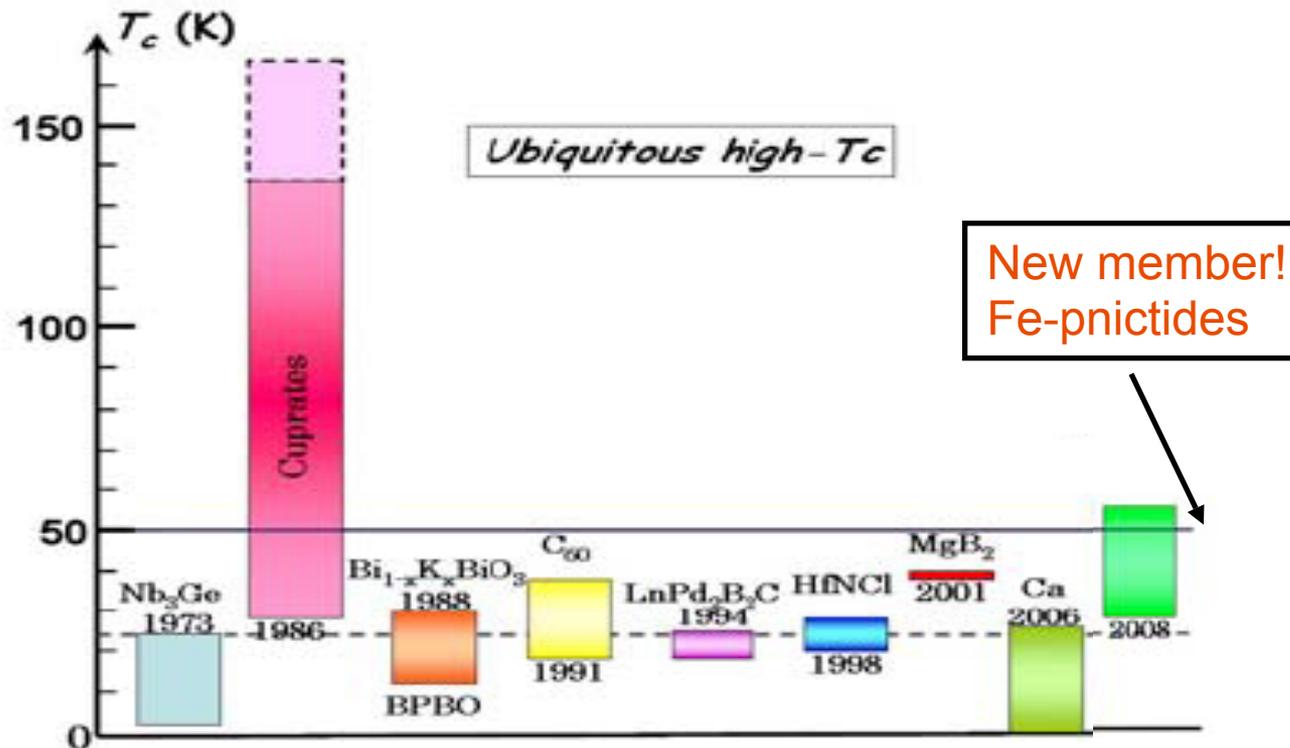
multi-orbitals,
orbital order,
lattice distortions
magnetism, local/
itinerant electrons

$$J_H S_c \ll \infty$$

Search for high T_c



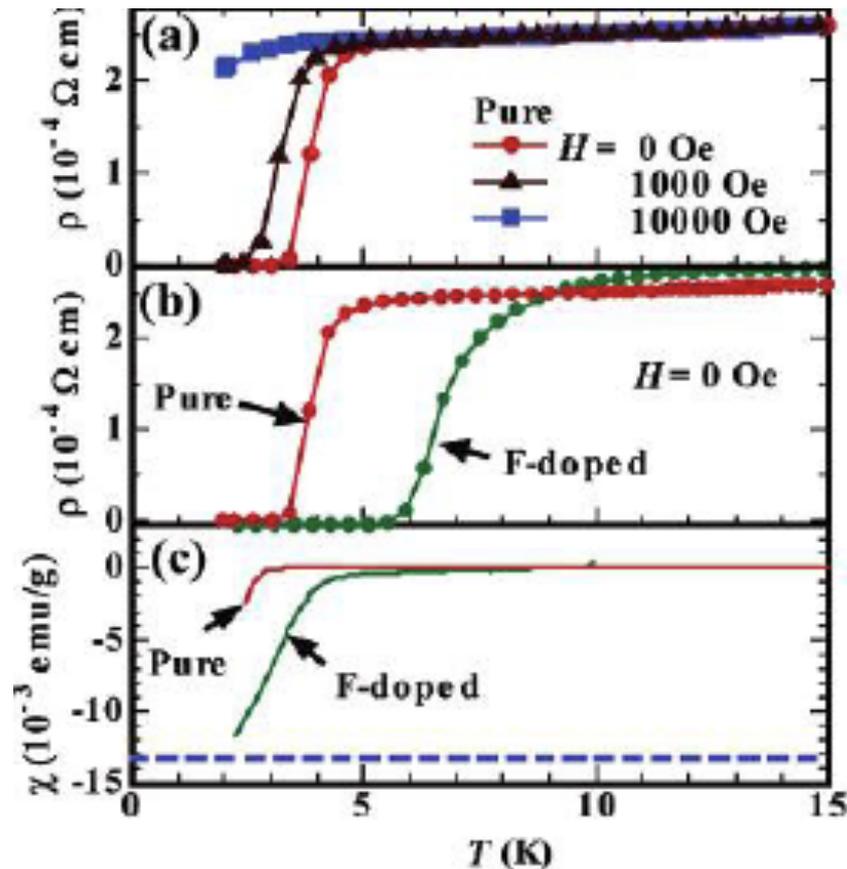
Search for high T_c



Iron-Based Layered Superconductor: LaOFeP

Yoichi Kamihara,[†] Hidenori Hiramatsu,[†] Masahiro Hirano,^{†,‡} Ryuto Kawamura,[§] Hiroshi Yanagi,[§]
Toshio Kamiya,^{†,§} and Hideo Hosono^{*,†,‡}

ERATO-SORST, JST, Frontier Collaborative Research Center, Tokyo Institute of Technology, Mail Box S2-13, 4259 Nagatsuta, Midori-ku, Yokohama 226-8503, Japan, Frontier Collaborative Research Center, Tokyo Institute of Technology, Mail Box S2-13, 4259 Nagatsuta, Midori-ku, Yokohama 226-8503, Japan, and Materials and Structures Laboratory, Tokyo Institute of Technology, Mail Box R3-4, 4259 Nagatsuta, Yokohama 226-8503, Japan

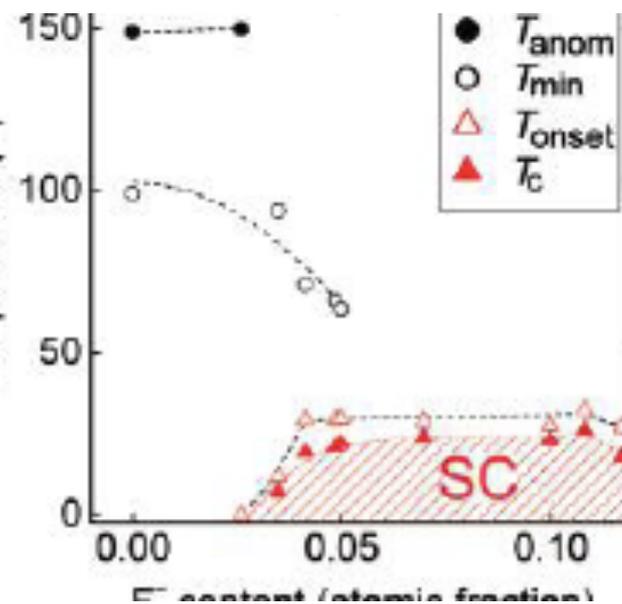
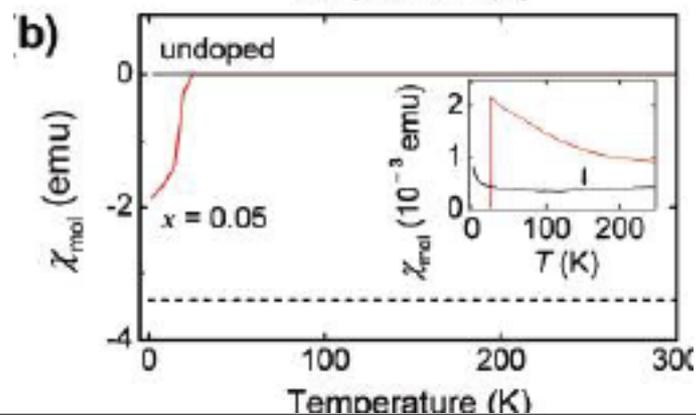
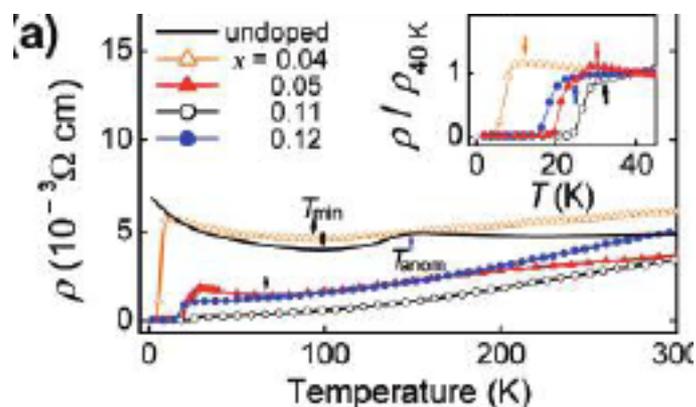


$T_C = 4$ K,
7 K (F doping)

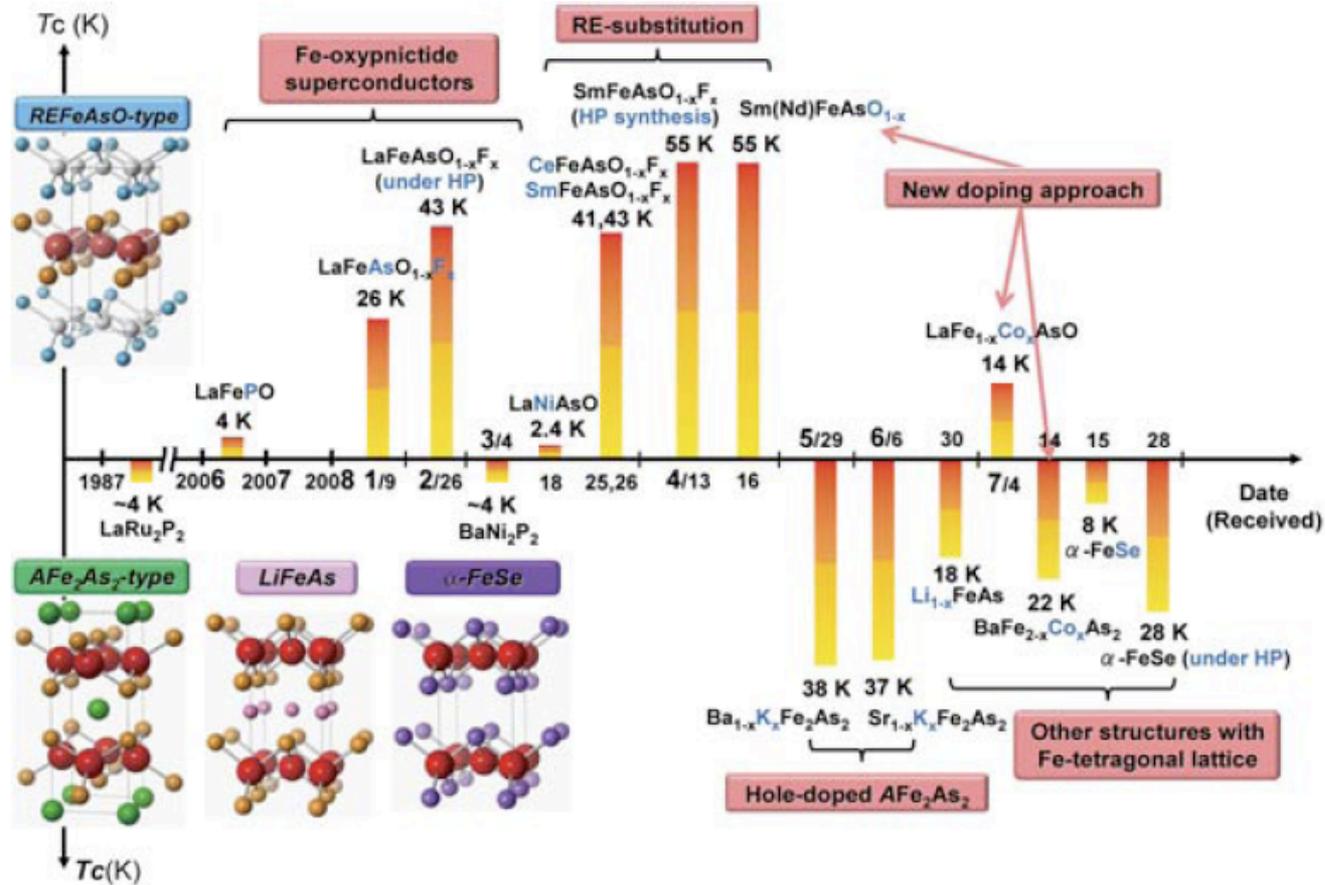
Iron-Based Layered Superconductor $\text{La}[\text{O}_{1-x}\text{F}_x]\text{FeAs}$ ($x = 0.05-0.12$) with $T_c = 26$ K

Yoichi Kamihara,^{*,†} Takumi Watanabe,[‡] Masahiro Hirano,^{†,§} and Hideo Hosono^{†,‡,§}

ERATO-SORST, JST, Frontier Research Center, Tokyo Institute of Technology, Mail Box S2-13, Materials and Structures Laboratory, Tokyo Institute of Technology, Mail Box R3-1, and Frontier Research Center, Tokyo Institute of Technology, Mail Box S2-13, 4259 Nagatsuta, Midori-ku, Yokohama 226-8503, Japan



Many iron-based superconductors with T_c up to 56K

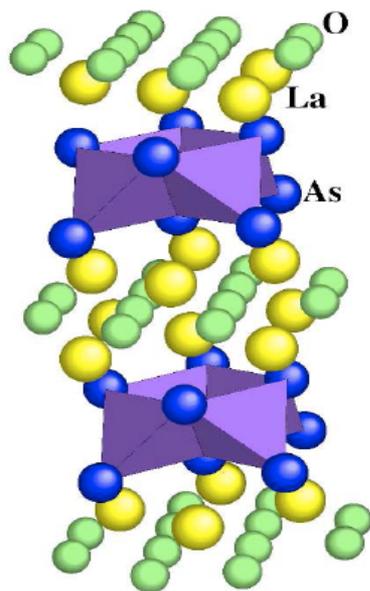


Tour of iron village



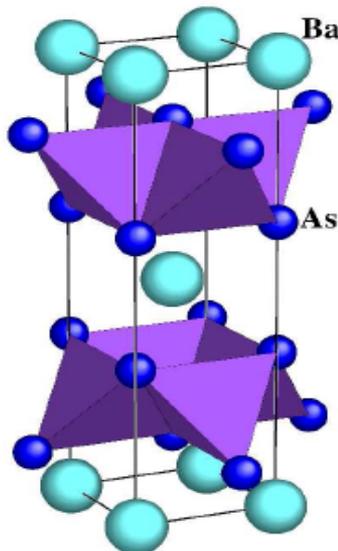
Taxonomy

- 1111-family



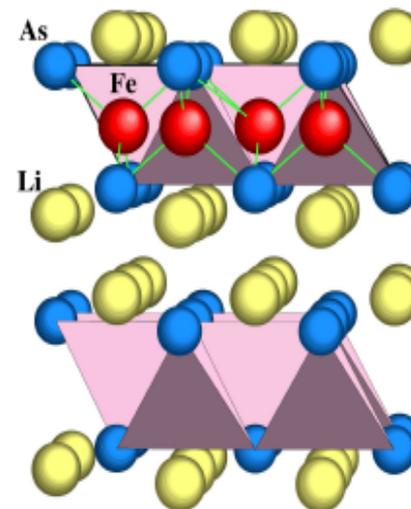
LaOFeAs

- 122-family



BaFe₂As₂

- 111-family

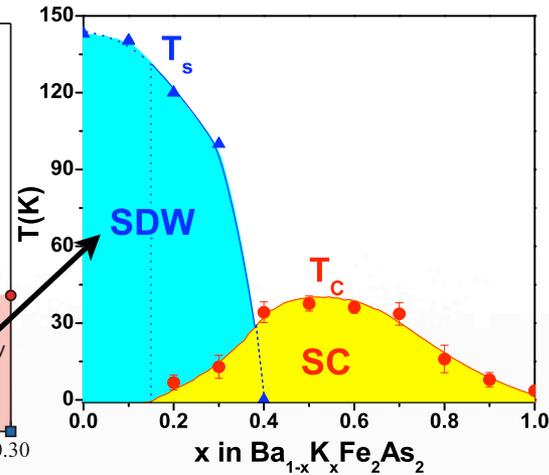
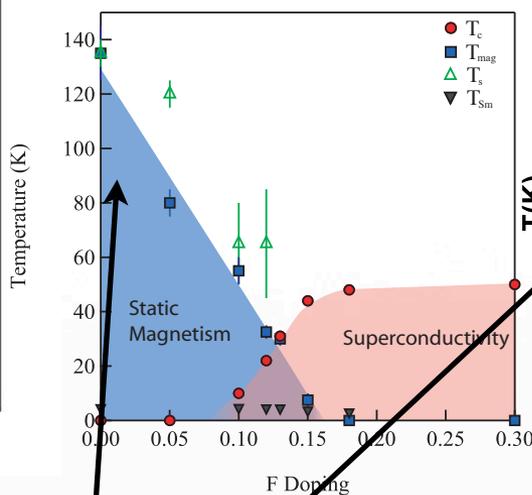
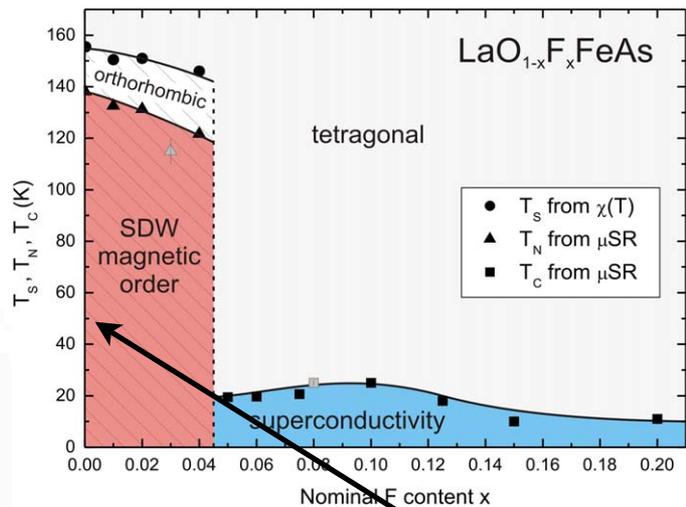
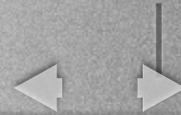


LiFeAs

11-family FeSe, 32522-family (Sr₃Sc₂O₅)Fe₂As₂ and many

4 As: tetrahedron

Key Structure:
8
Fe As-trilayers

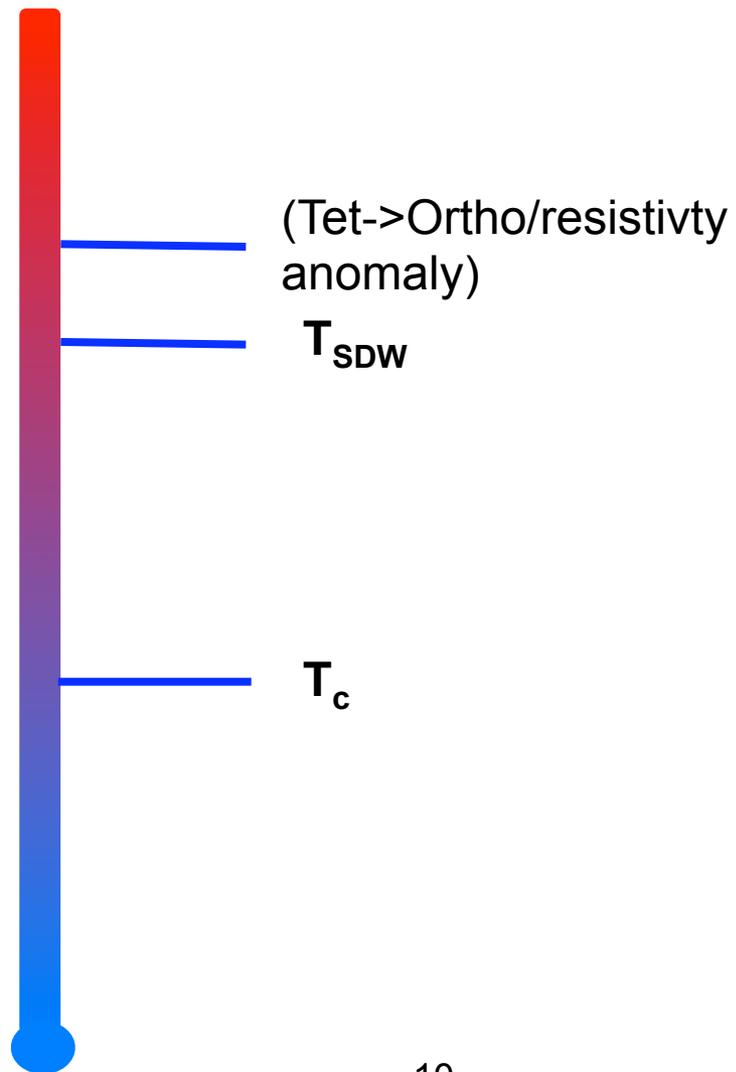


||||

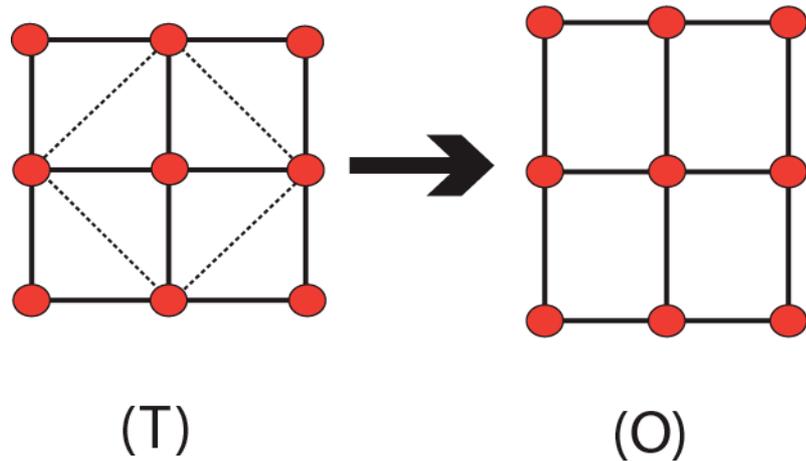
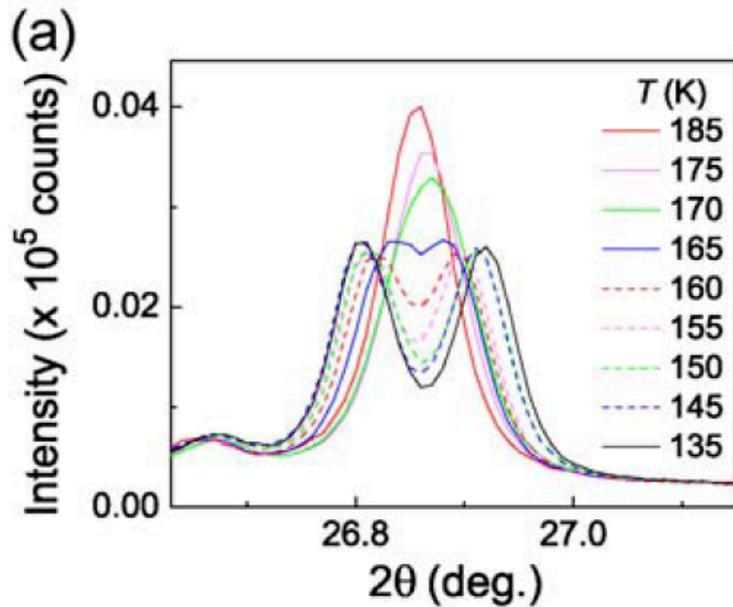
122

Why?

To explain??



Structural Phase Transition



$$T_{\text{SPT}} = 150 \text{ K}$$

Fe-Fe Bond Length:

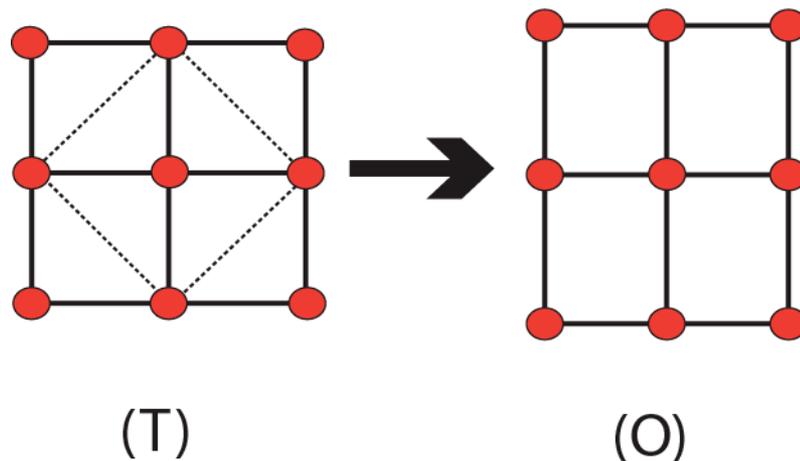
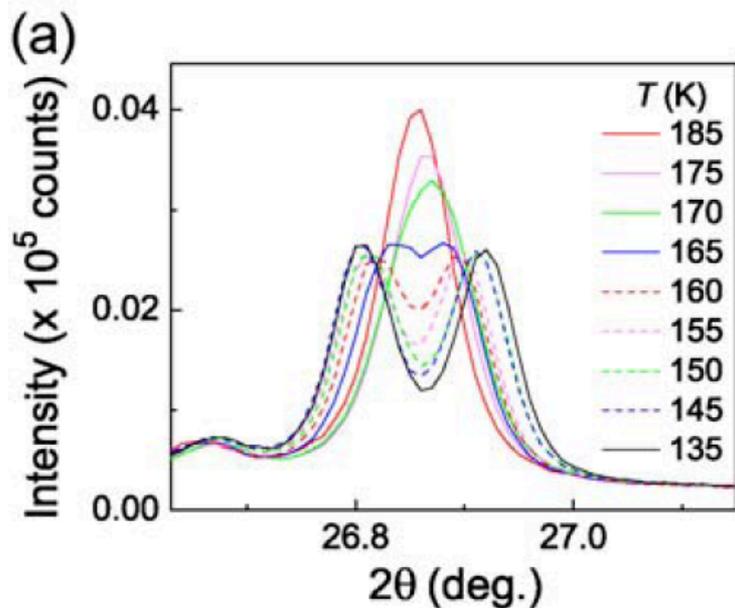
$$a = b = 2.85 \text{ \AA} \quad (\text{T})$$

$$a = 2.84 \text{ \AA}$$

$$b = 2.86 \text{ \AA} \quad (\text{O})$$

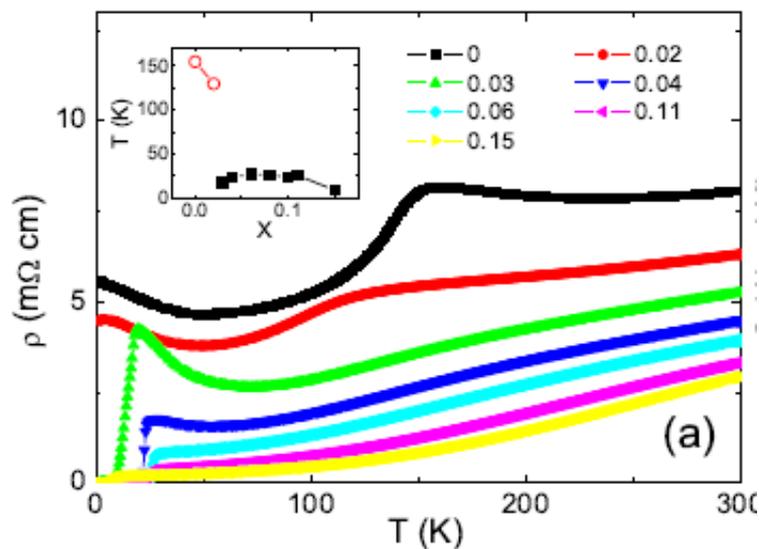
$$\delta = 0.01 \text{ \AA}$$

Structural Phase Transition



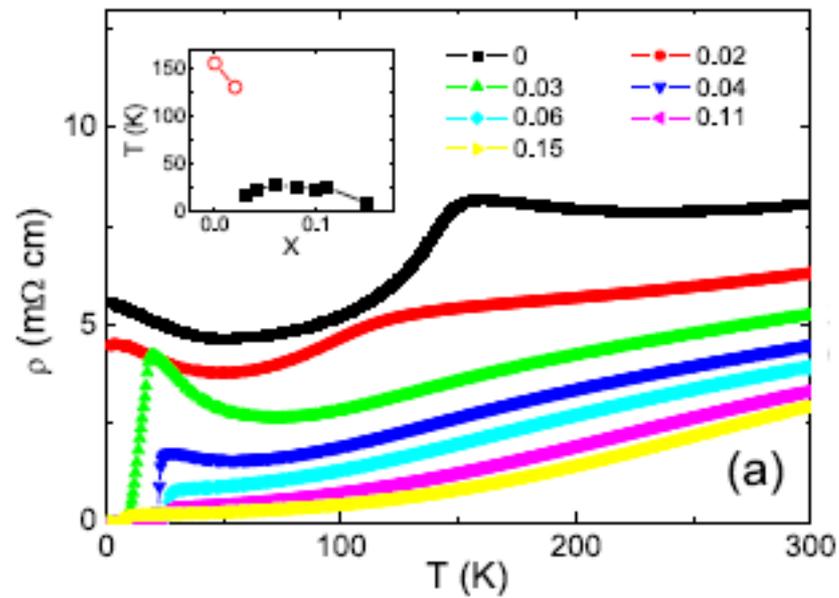
● Resistivity Anomaly $T_{RA} = T_{SPT}$

$T_{SPT} = 150$ K
 Fe-Fe Bond Length:
 $a = b = 2.85$ Å (T)
 $a = 2.84$ Å
 $b = 2.86$ Å (O)
 $\delta = 0.01$ Å



arXiv:0803.3426

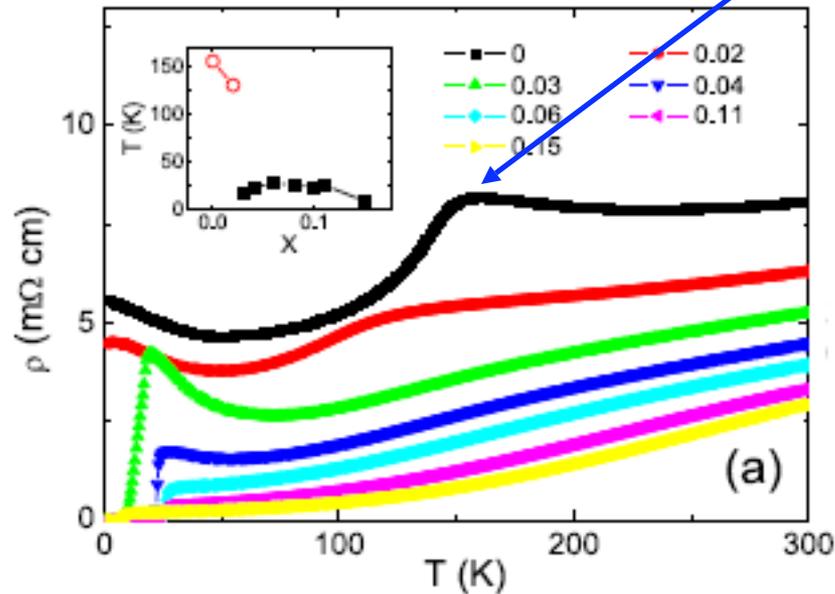
LaO_{1-x}F_xFeAs



arXiv:0803.3426

Structural phase transition (1) And Resistivity Anomaly(150K)

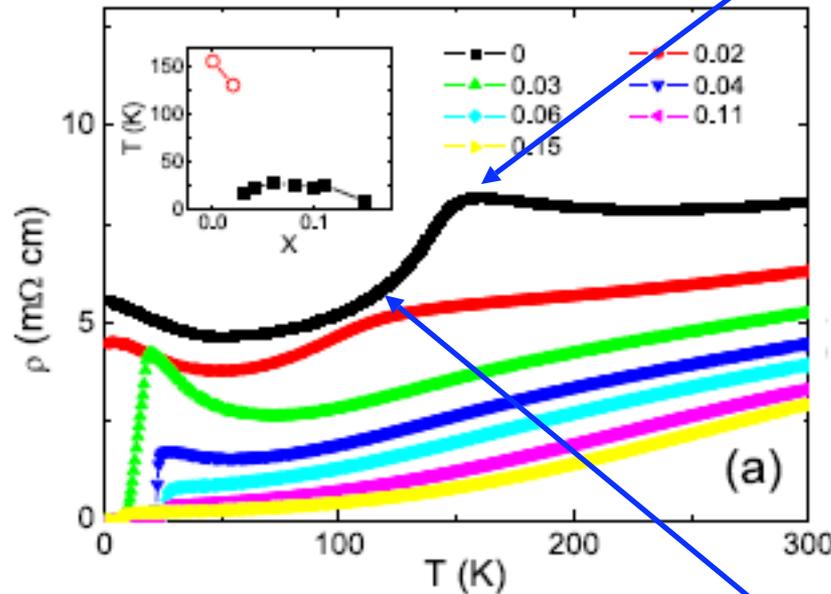
$\text{LaO}_{1-x}\text{F}_x\text{FeAs}$



arXiv:0803.3426

Structural phase transition (1) And Resistivity Anomaly(150K)

$\text{LaO}_{1-x}\text{F}_x\text{FeAs}$

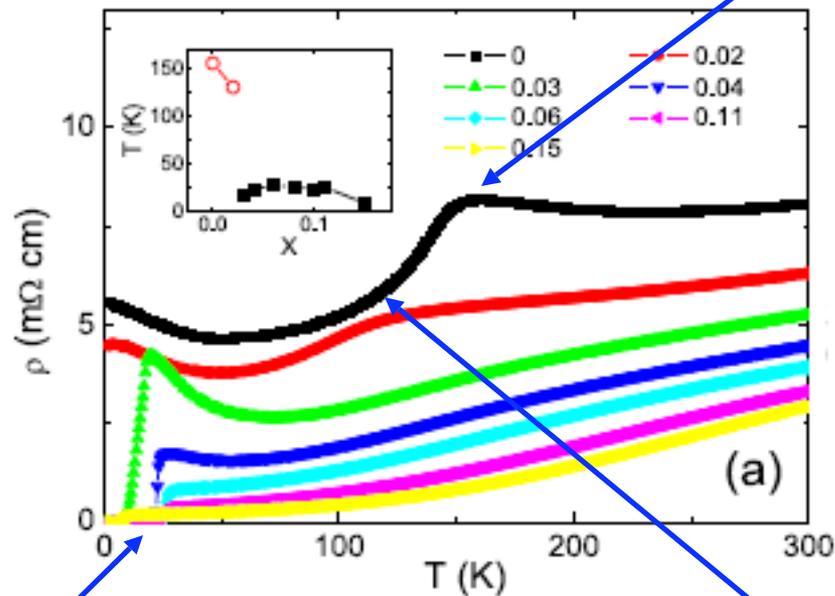


Spin-density-wave
Transition (137K) (2)

arXiv:0803.3426

Structural phase transition (1) And Resistivity Anomaly(150K)

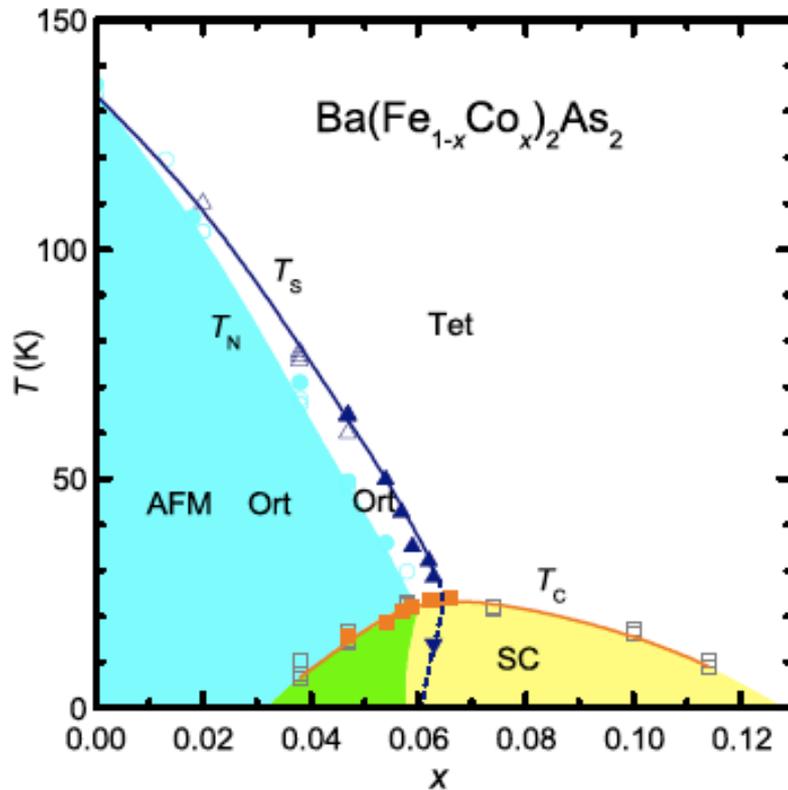
$\text{LaO}_{1-x}\text{F}_x\text{FeAs}$



$T_c = 26\text{ K}$
Superconductivity (3)

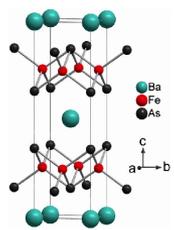
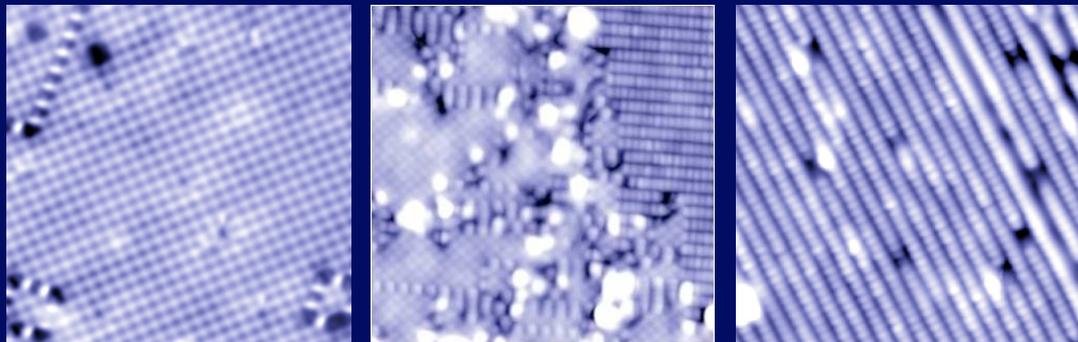
Spin-density-wave
Transition (137K) (2)

Where does the structural transition end?

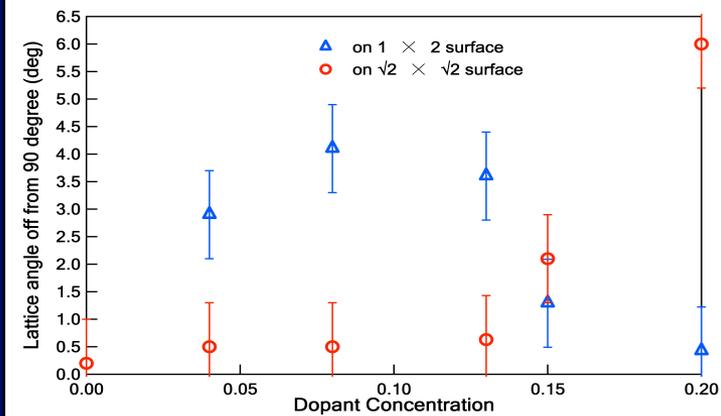
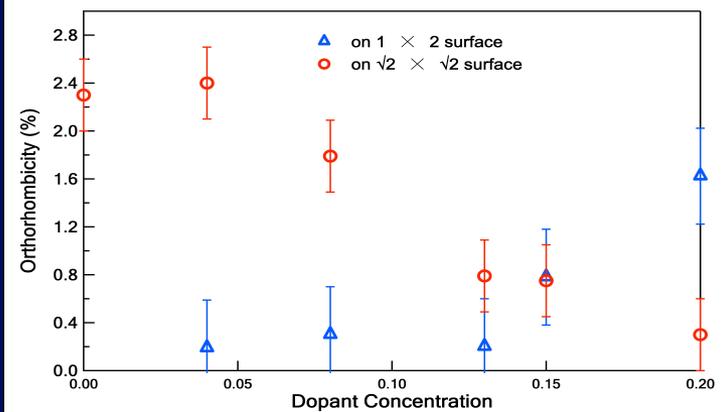
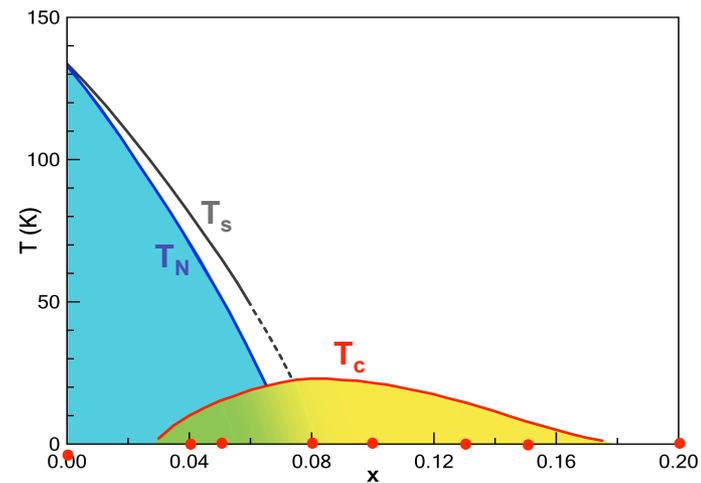
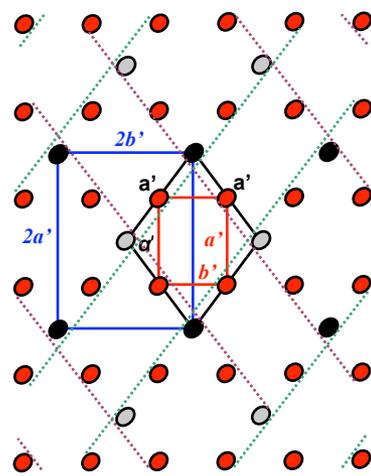
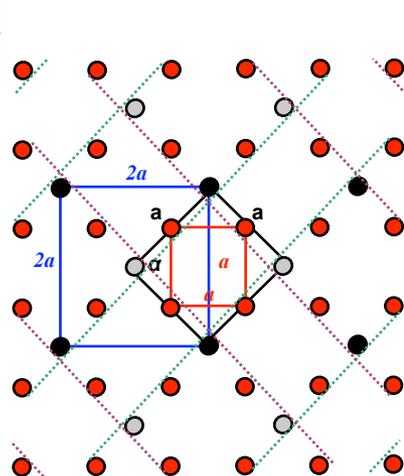


Phys. Rev. Lett. 104, 057006 (2010)

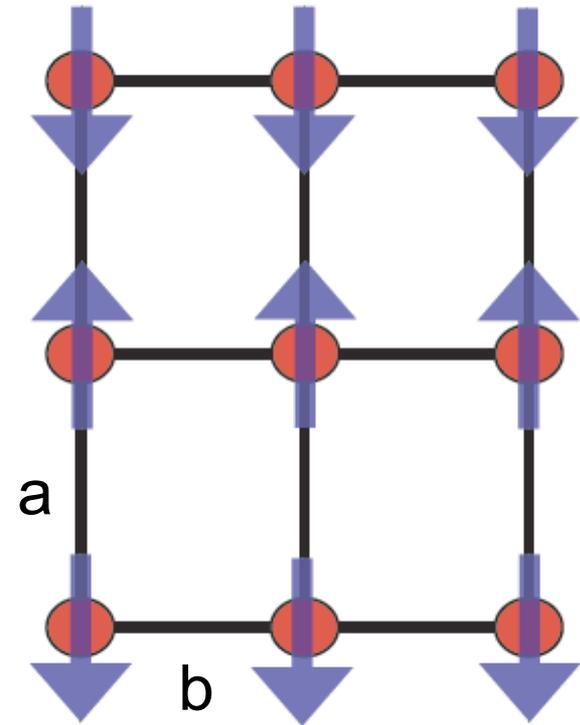
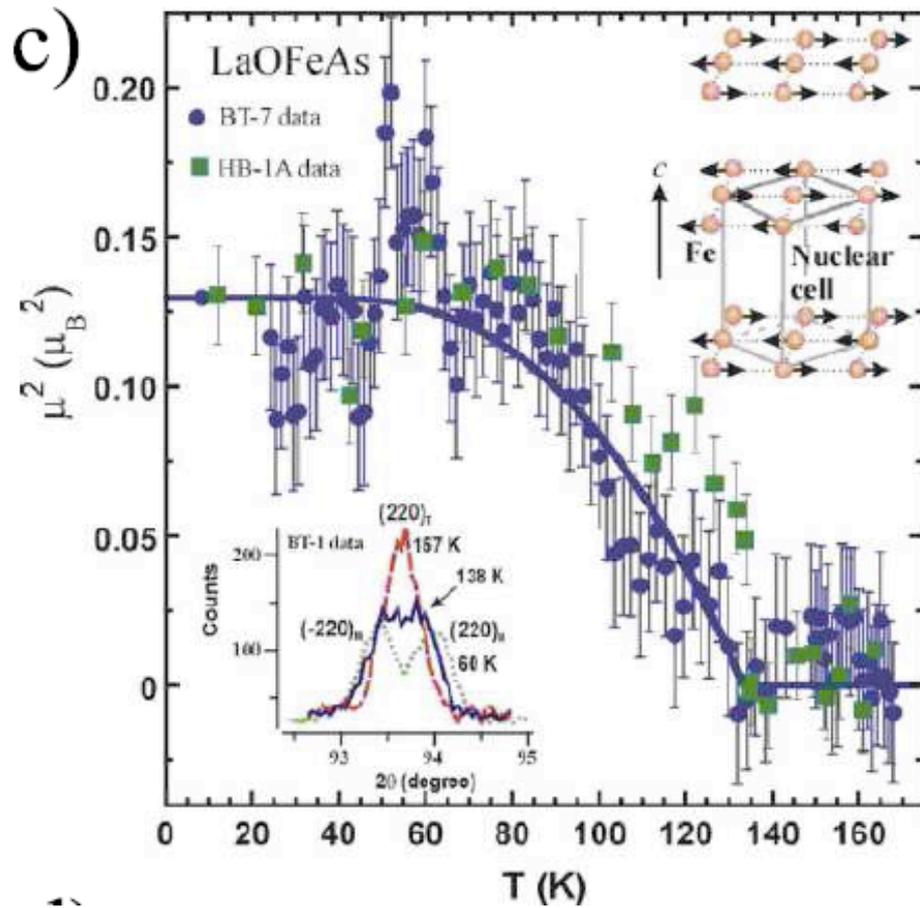
Doping Dependence of Lattice Orthorhombicity



$a = b = 3.96 \text{ \AA}$
 $c = 12.98 \text{ \AA}$



Collinear Anti-ferromagnetism (SDW)



1111 family:

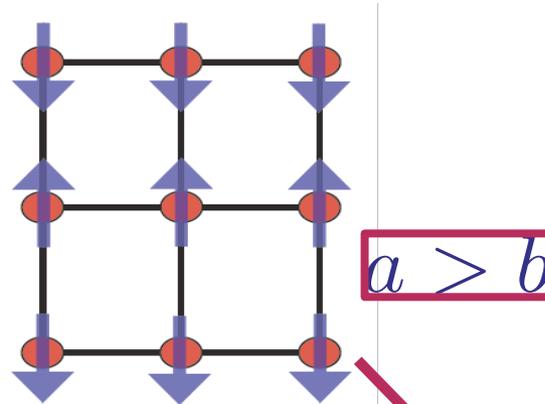
$$T_{SDW} < T_{SPT}$$

122 family:

$$T_{SDW} = T_{SPT}$$

C. de la Cruz, Nature

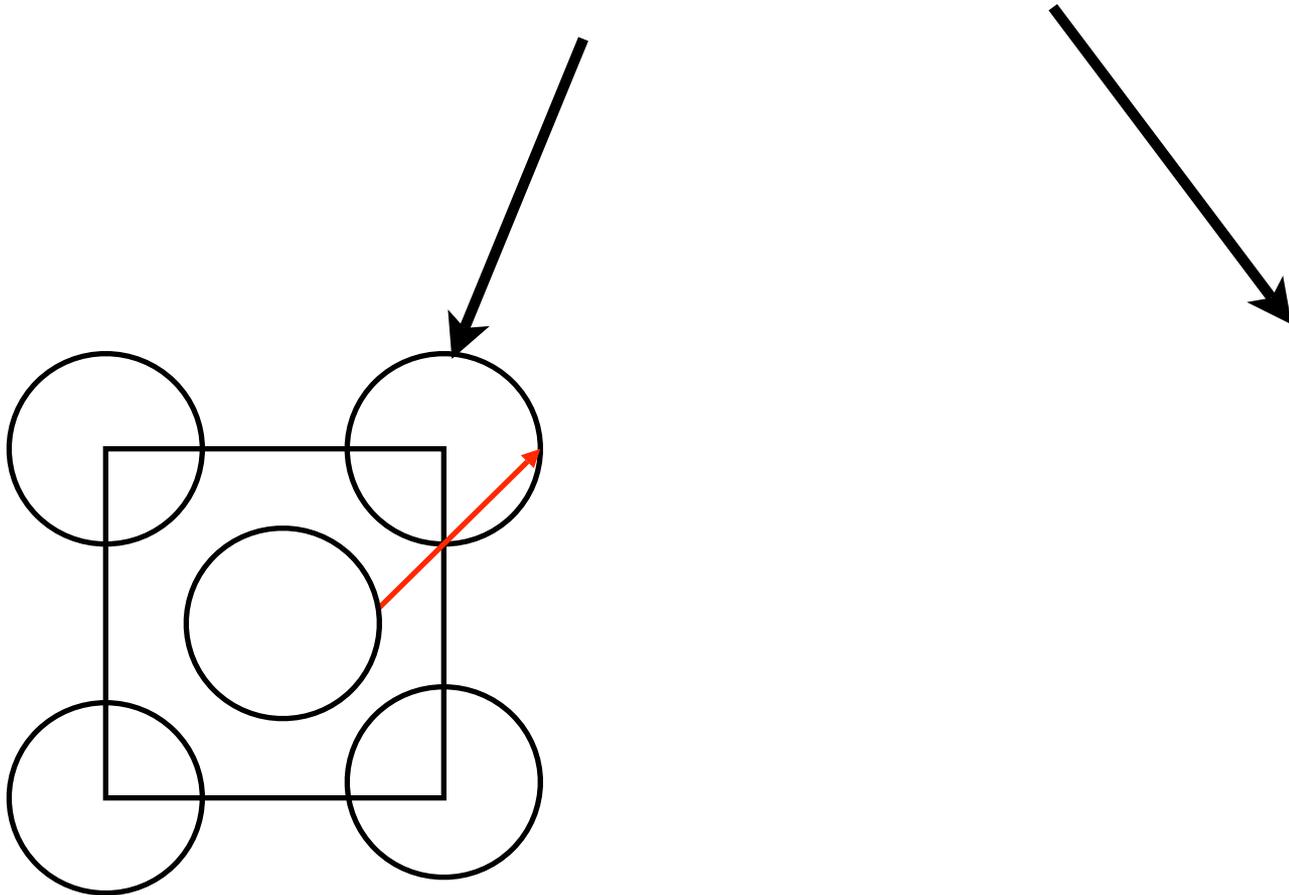
Magnetism



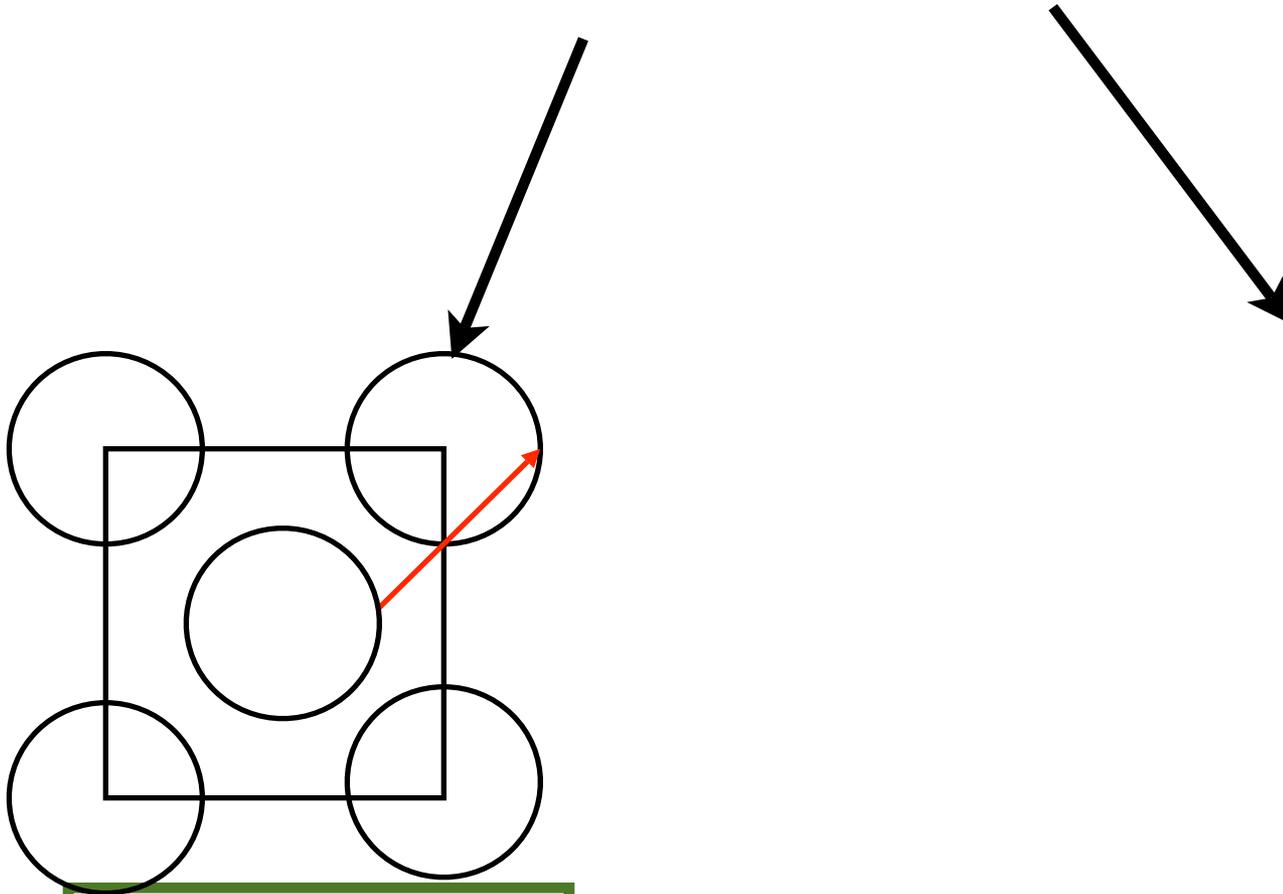
- 1111 (LaOFeAs)
- 2 second-order phase transitions
 $T_S=157, T_M=137\text{K}$

- 122 (LaOFeAs)
- 1 first-order transition
 $T_S=T_M=144\text{K}$

Simple Model for Magnetism

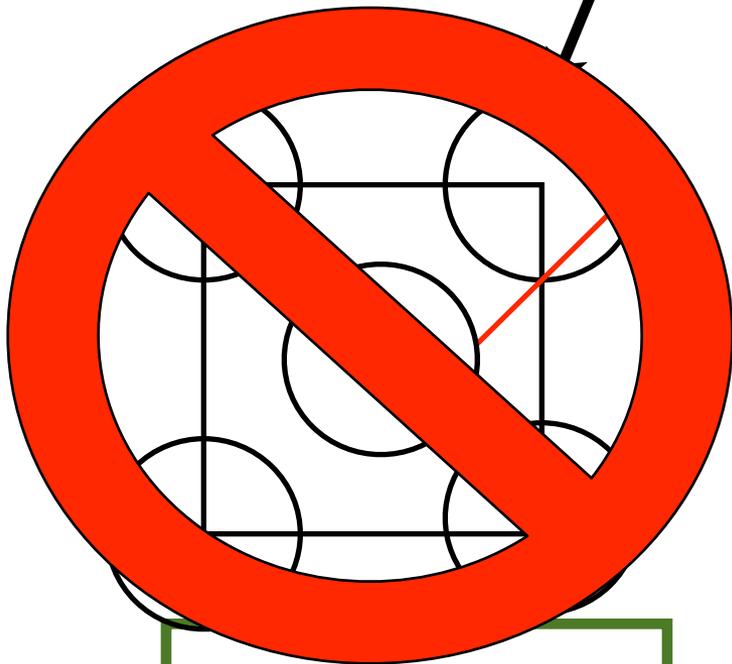


Simple Model for Magnetism



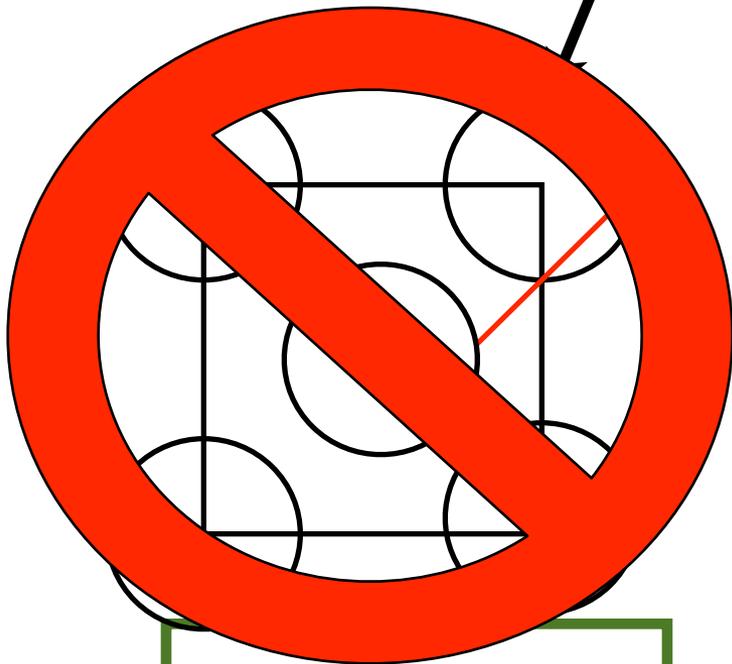
monoclinic
distortion

Simple Model for Magnetism



monoclinic
distortion

Simple Model for Magnetism

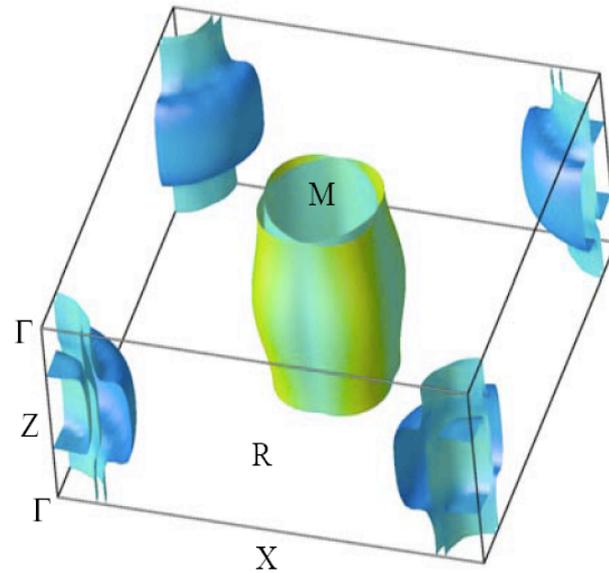
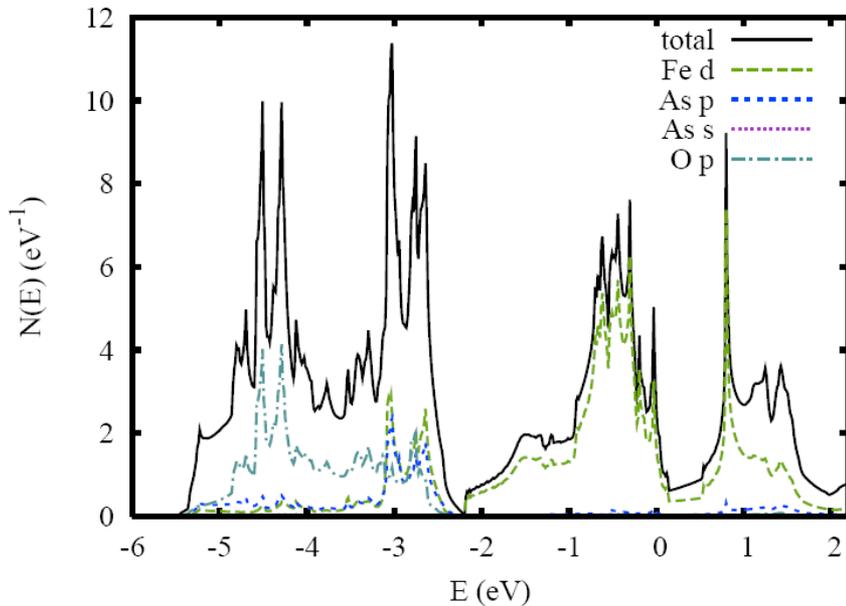


monoclinic
distortion

local moments

Are the pnictides strongly correlated?

Multibands



PRL **100**, 237003 (2008)

PHYSICAL REVIEW LETTERS

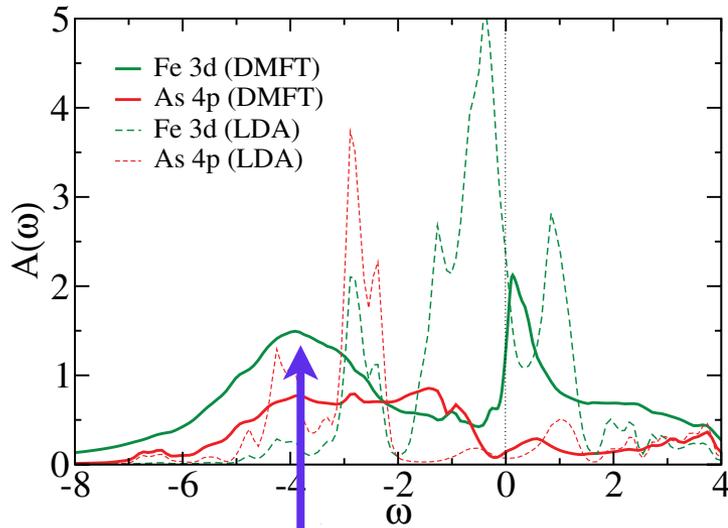
week ending
13 JUNE 2008

Density Functional Study of $\text{LaFeAsO}_{1-x}\text{F}_x$: A Low Carrier Density Superconductor Near Itinerant Magnetism

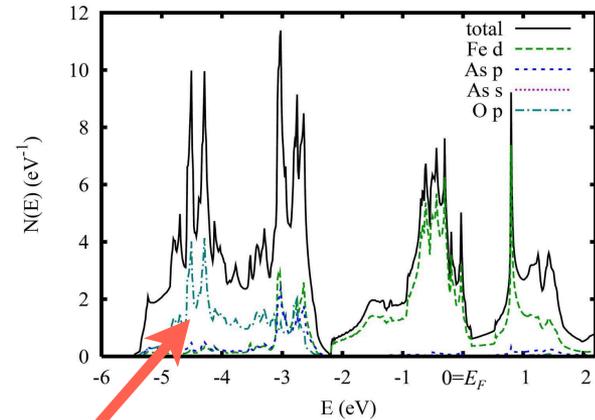
D. J. Singh and M.-H. Du

Materials Science and Technology Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-6114, USA
(Received 4 March 2008; published 12 June 2008)

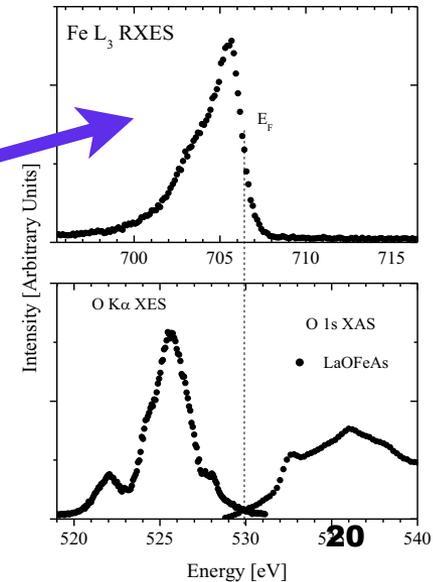
Are the pnictides strongly correlated?



Fe 3d peak at 4eV below E_F :
Lower Hubbard band

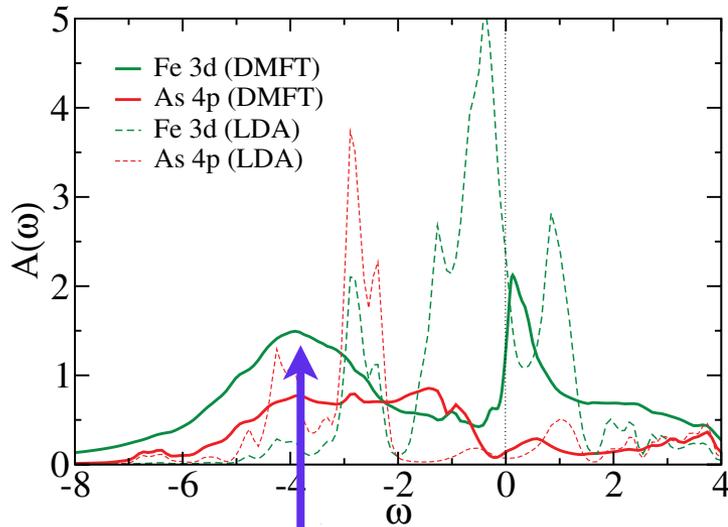


LDA and Experiment show no Fe 3d lower Hubbard band

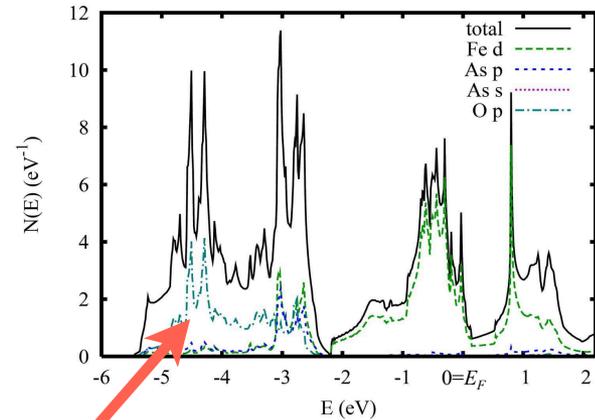


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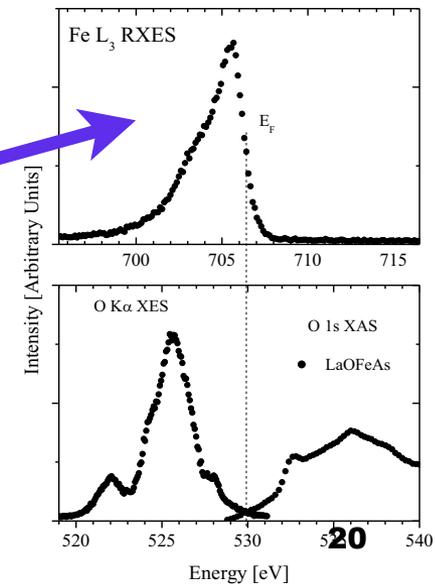
Where's Mott?



Fe 3d peak at 4eV below E_F :
Lower Hubbard band



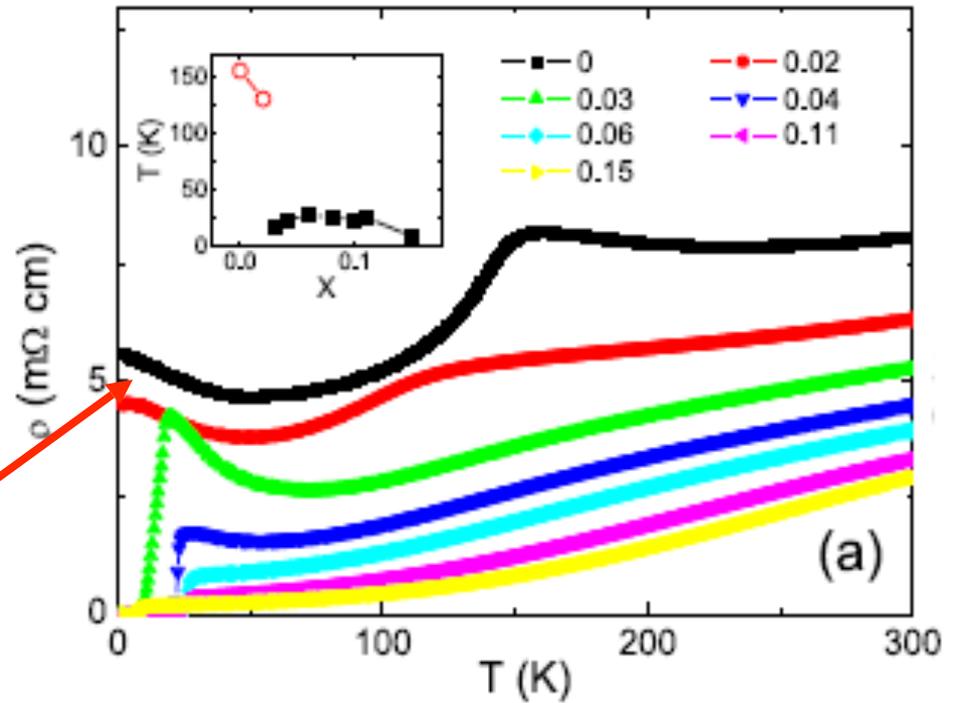
LDA and Experiment show no Fe 3d lower Hubbard band



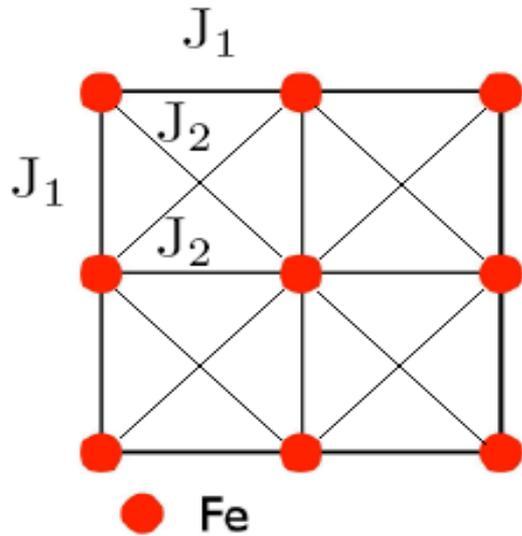
No

Intermediate Coupling

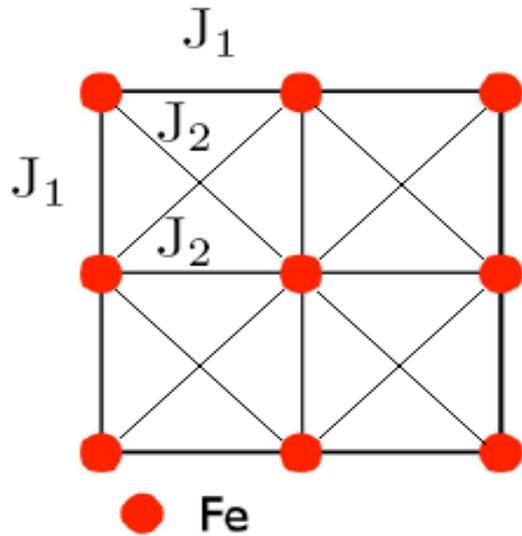
Fe-pnictides
Parent compound:
Bad metal



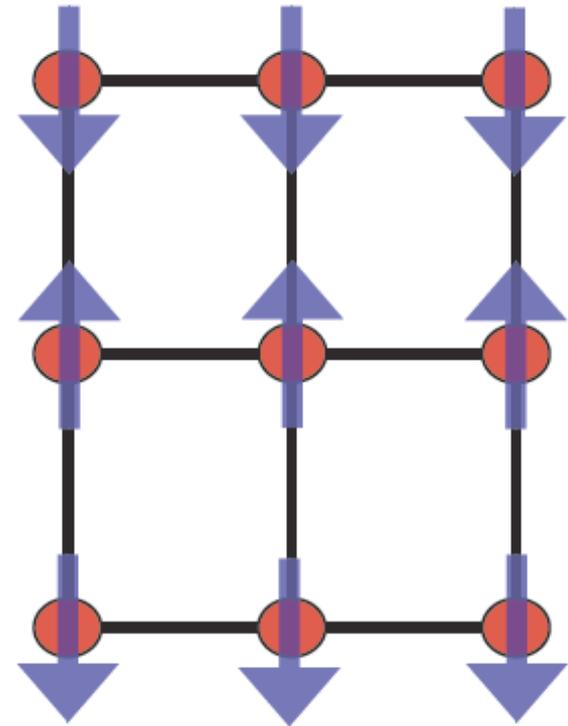
how far can one get with local moments?



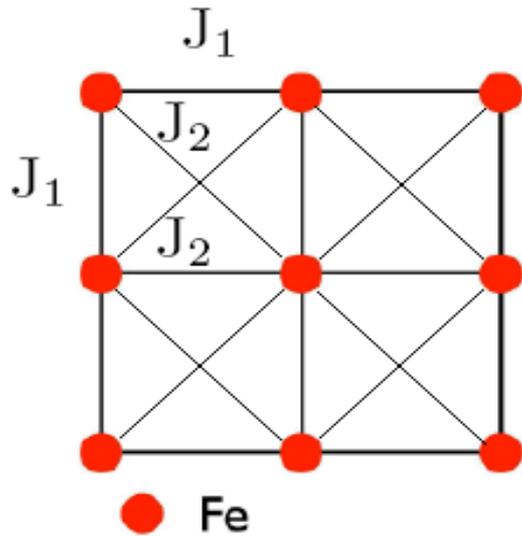
how far can one get with local moments?



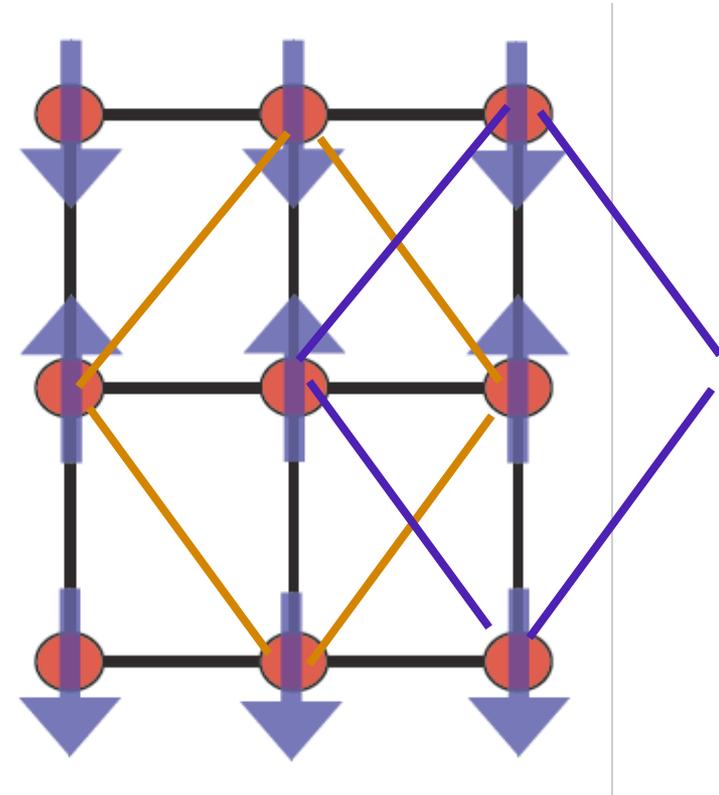
$$J_2 > J_1/2$$



how far can one get with local moments?

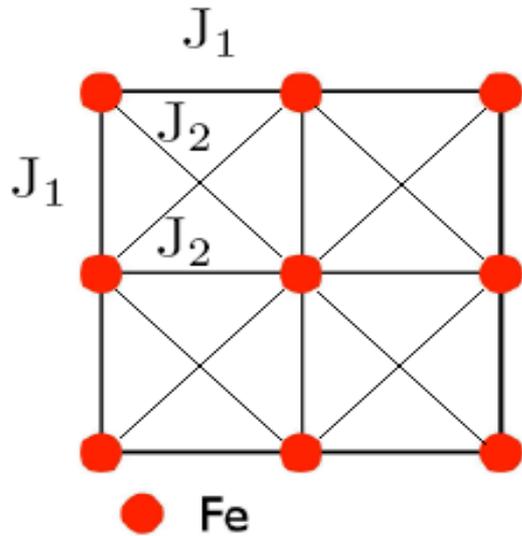


$$J_2 > J_1/2$$



Two interpenetrating
AF sublattices

how far can one get with local moments?



$$J_2 > J_1/2$$

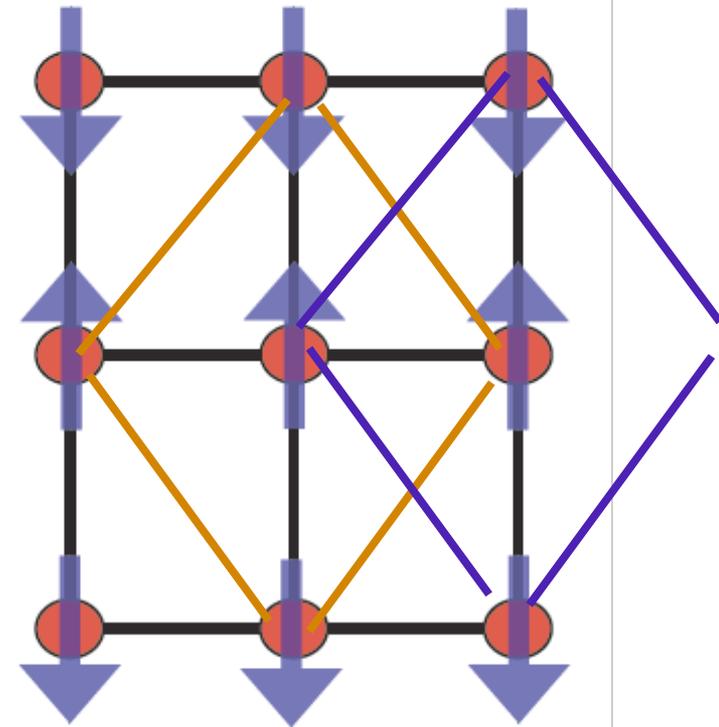
$$\varphi = \hat{n}_1 \cdot \hat{n}_2$$

$$\langle \varphi \rangle \neq 0$$

$$\langle n_1 \rangle = \langle n_2 \rangle \neq 0$$

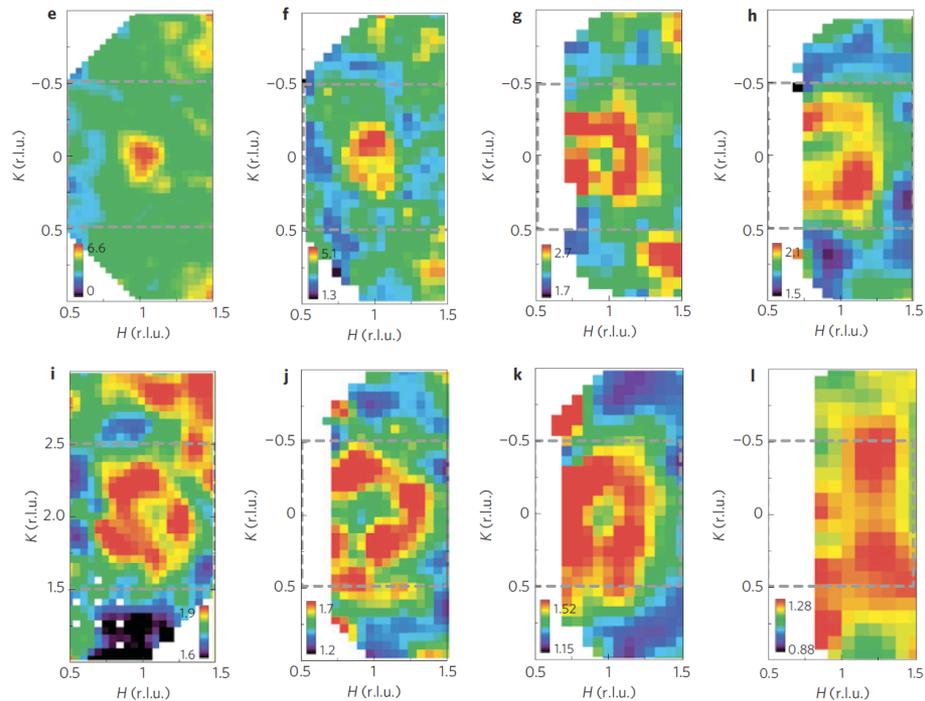
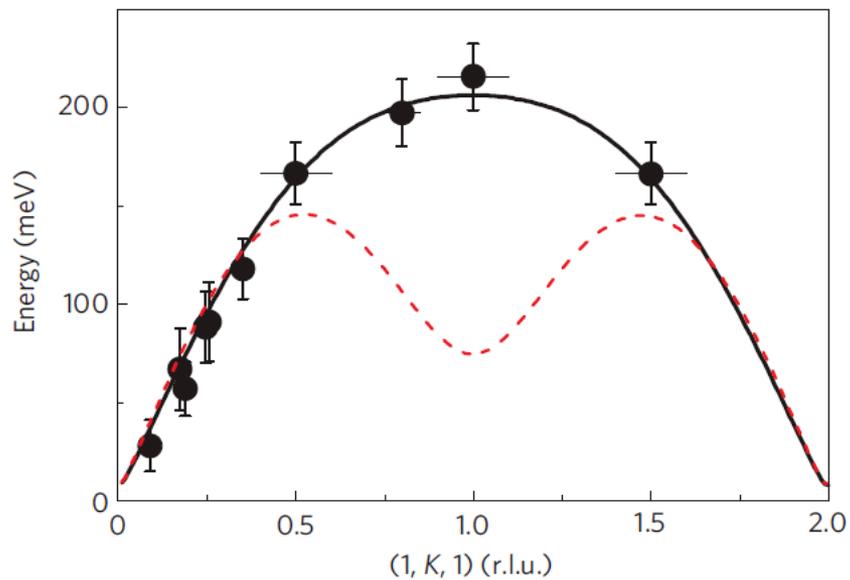
'nematic' $\langle \varphi \rangle \neq 0$

$$\langle n_1 \rangle = \langle n_2 \rangle = 0$$



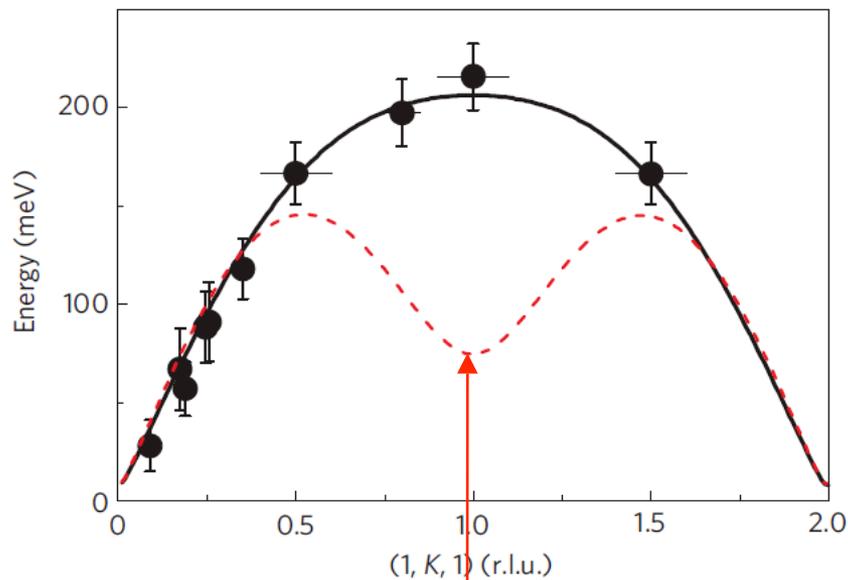
Two interpenetrating
AF sublattices

• *Inelastic neutron scattering*

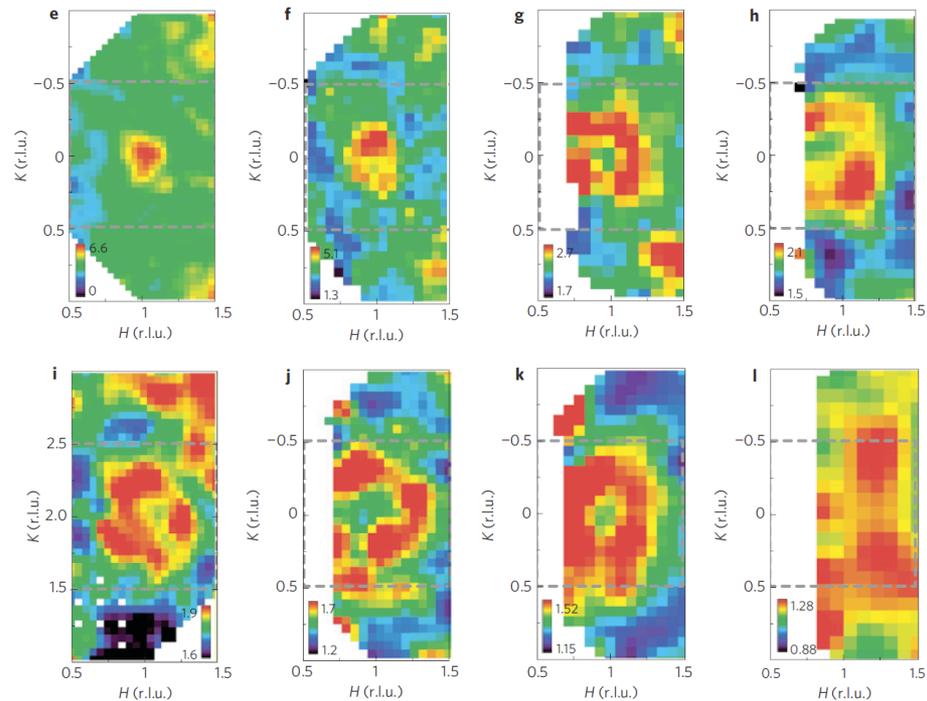


J. Zhao, *et al.* Nature Physics (2009)

• *Inelastic neutron scattering*

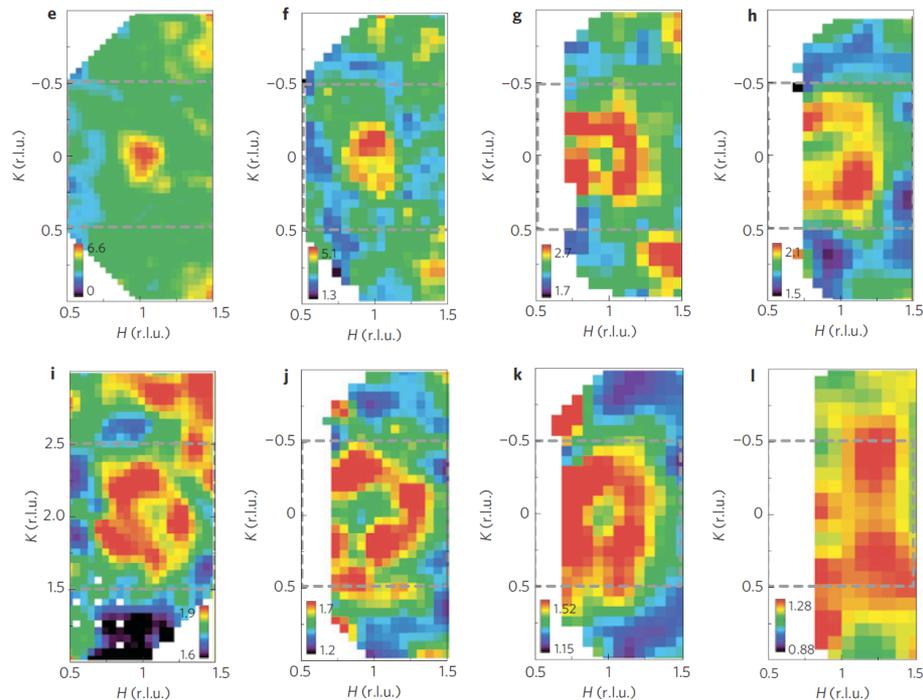
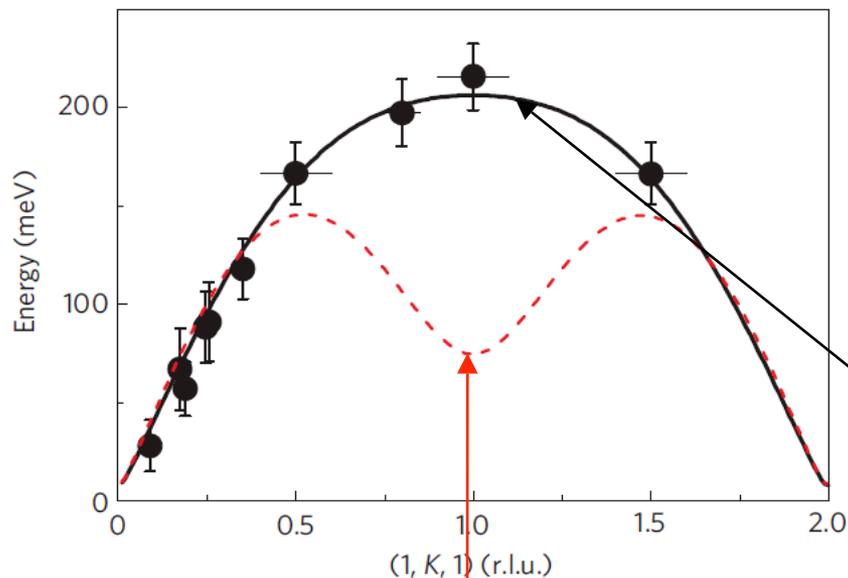


Isotropic $J_{1a} \approx J_{1b}$
 $SJ_1^a = 27 \text{ meV}$
 $SJ_1^b = 25 \text{ meV}$
 $SJ_2 = 36 \text{ meV}$



J. Zhao, *et al.* Nature Physics (2009)

• *Inelastic neutron scattering*



J. Zhao, *et al.* Nature Physics (2009)

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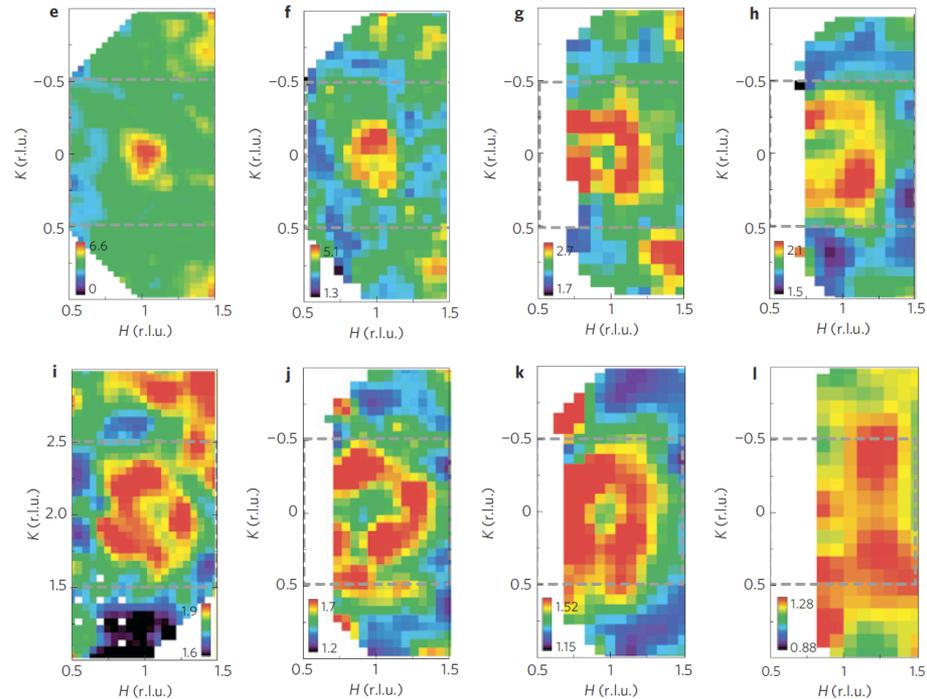
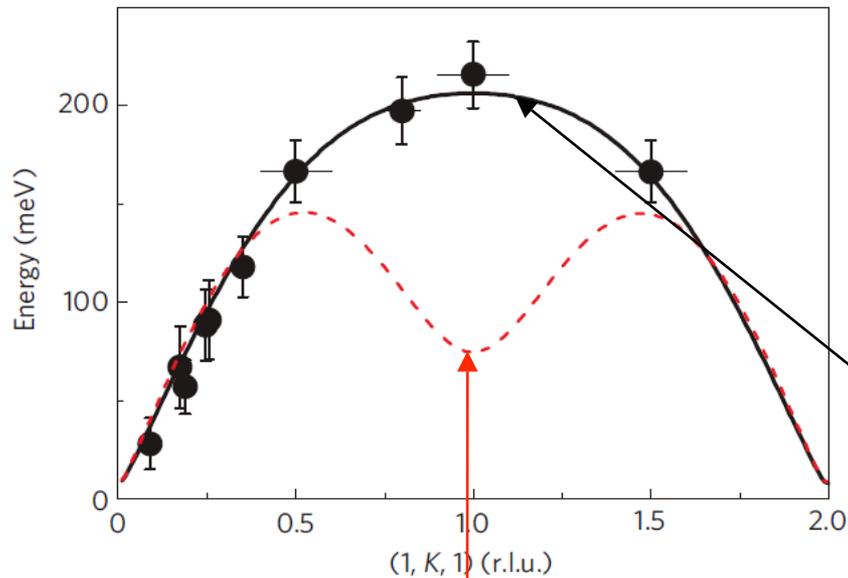
Anisotropic $J_{1a} \gg J_{1b}$

$$SJ_1^a = 49.9 \pm 9.9 \text{ meV}$$

$$SJ_1^b = -5.7 \pm 4.5 \text{ meV}$$

$$SJ_2 = 18.9 \pm 3.4 \text{ meV}$$

• *Inelastic neutron scattering*



J. Zhao, *et al.* Nature Physics (2009)

Isotropic $J_{1a} \approx J_{1b}$

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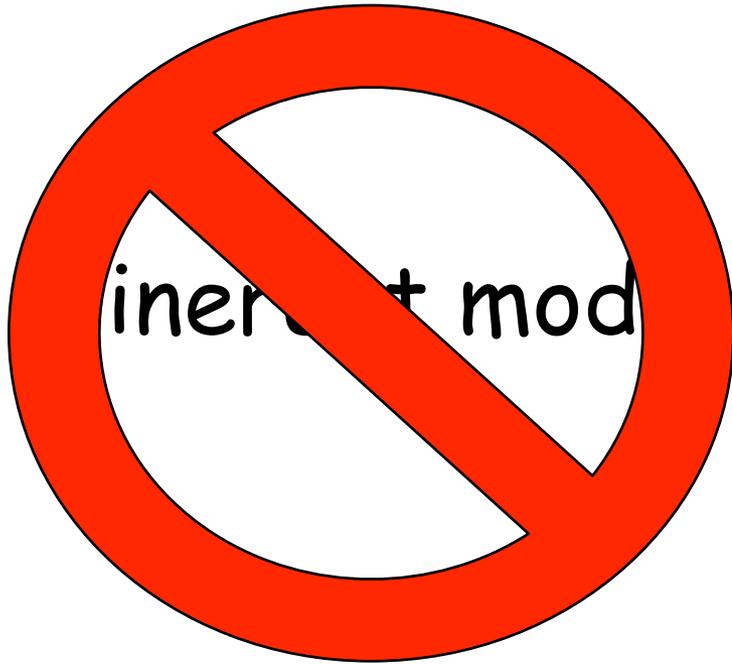
$$SJ_2 = 18.9 \pm 3.4 \text{ meV}$$

Problem?

itinerant model

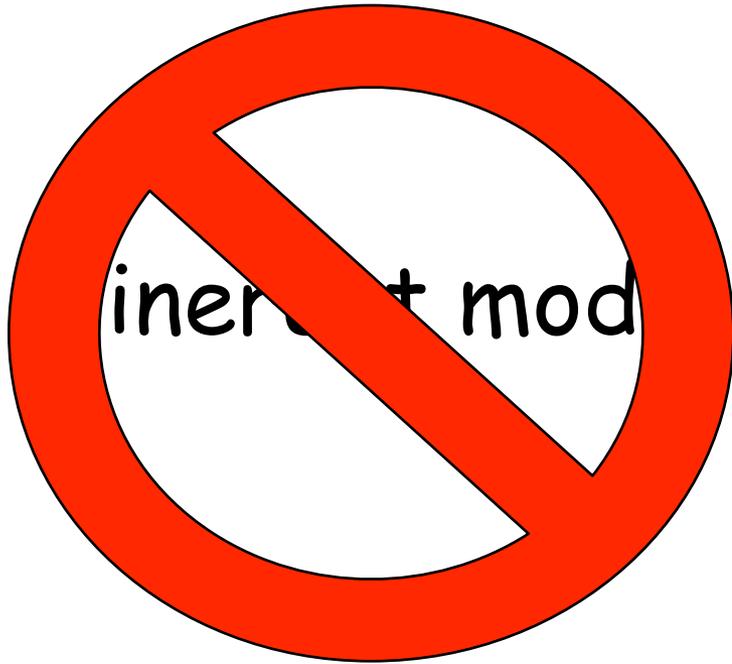
local picture
(different
signs of J along
 x and y axes???)

Problem?



local picture
(different
signs of J along
 x and y axes???)

Problem?



Is there a simple model
that captures this physics?

localized/extended electrons+Hund's coupling

unfrustrated
magnetism,
SPT, RA

orbital
ordering

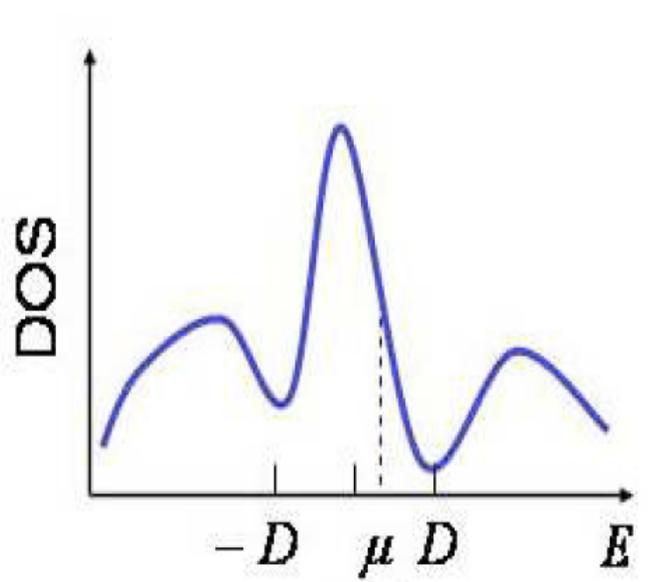
A new scenario: itinerant-localized dichotomy

J. Wu, P. Phillips, A. Castro-Neto, PRL(2008);

Z.-Y. Weng et.al., arXiv:0811.4111,

Q. Si, et.al. arXiv: 0901.4112.

J. Dai. et.al., arXiv: 0901.2787



A new scenario: itinerant-localized dichotomy

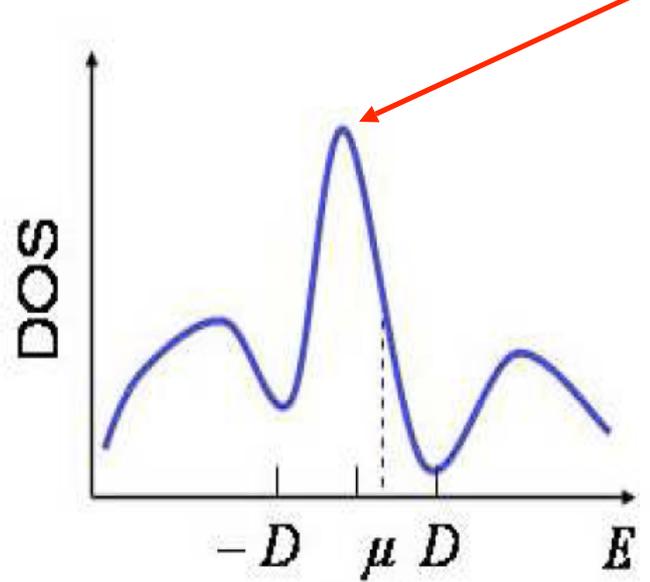
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**Itinerant electrons
(coherent, ungapped)
Metallic state**



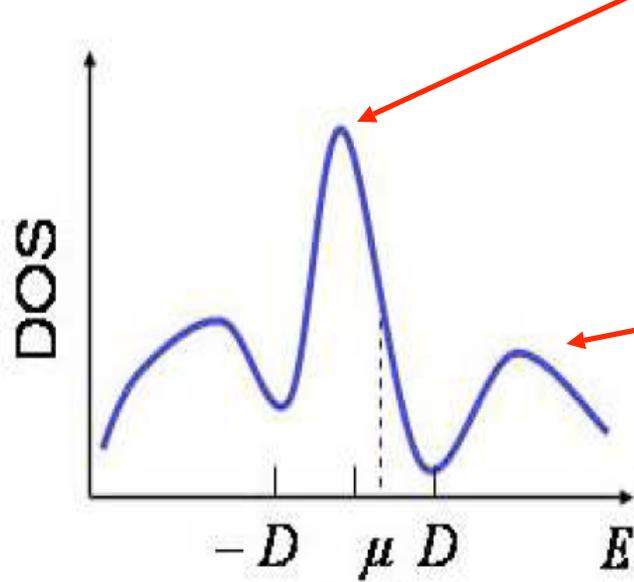
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**Localized electrons
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magnetic order**

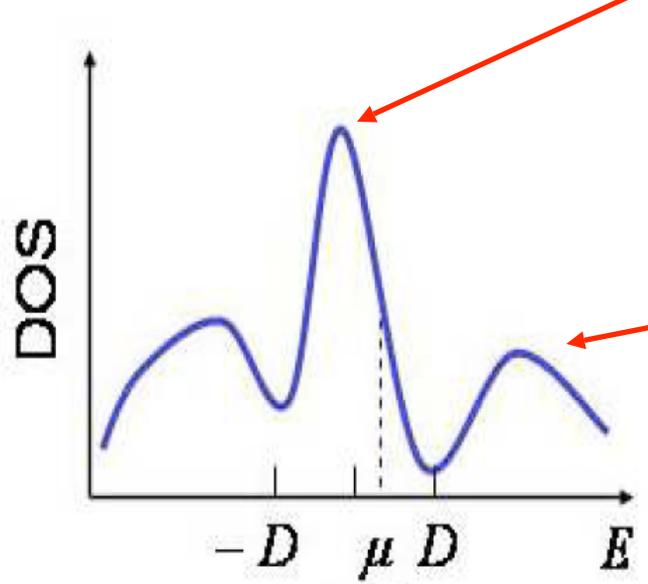
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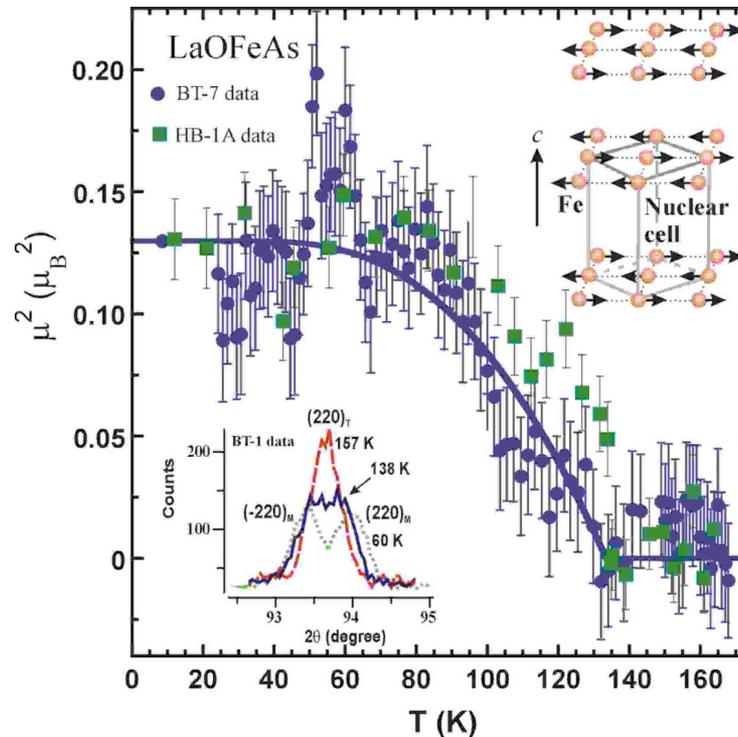
**Localized electrons
(incoherent, gapped)
magnetic order**

But how ?

1.) Why magnetic moments are so small?

J. Wu, P. Phillips, A. Castro-Neto, Phys.
Rev. Lett. 101, 126401 (2008)

Antiferromagnetic magnetic Order



$$.25\mu_B - .4\mu_B$$

Why?

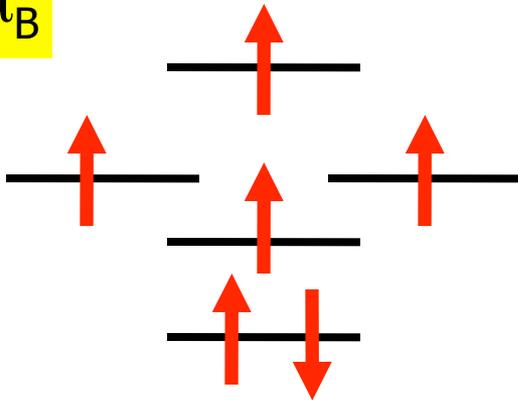
Figure 4 Temperature dependence of the order parameter at $Q = 1.53 \text{ \AA}^{-1}$ obtained on BT-7/HB-1A and our determined magnetic structure for LaOFeAs. The solid blue circles are data obtained on BT-7 and solid green squares are data taken on HB-1A. The solid line is a simple fit to mean field theory which gives $T_N = 134(1) \text{ K}$. The lower left inset shows the temperature dependence of the nuclear (220) peak obtained on BT-

Why small magnetic moments are so striking?

No way to get $0.36\mu_B$ given Hund's rules: Fe^{2+} 6 electrons on 5 orbitals (local moments)

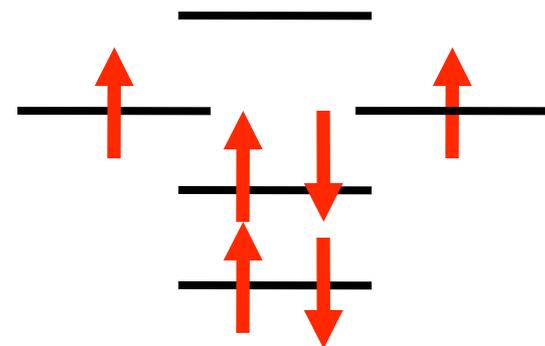
High spin configuration

$4\mu_B$

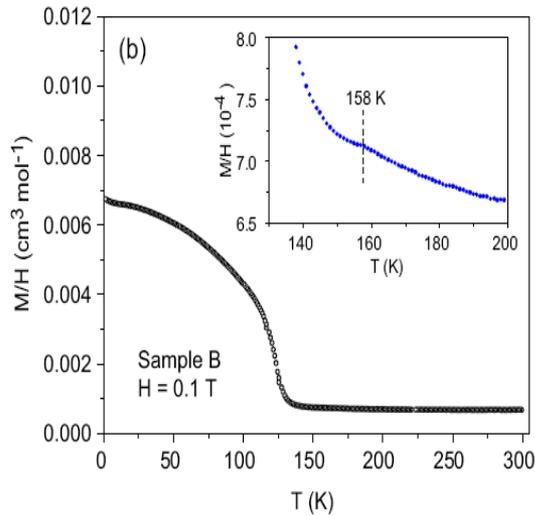


Low spin configuration

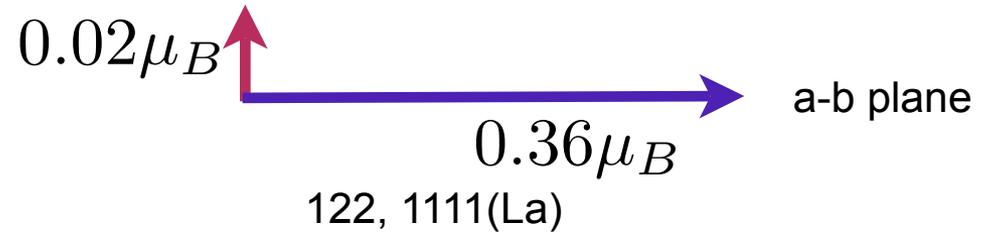
$2\mu_B$



Origin of residual moment along z-axis??

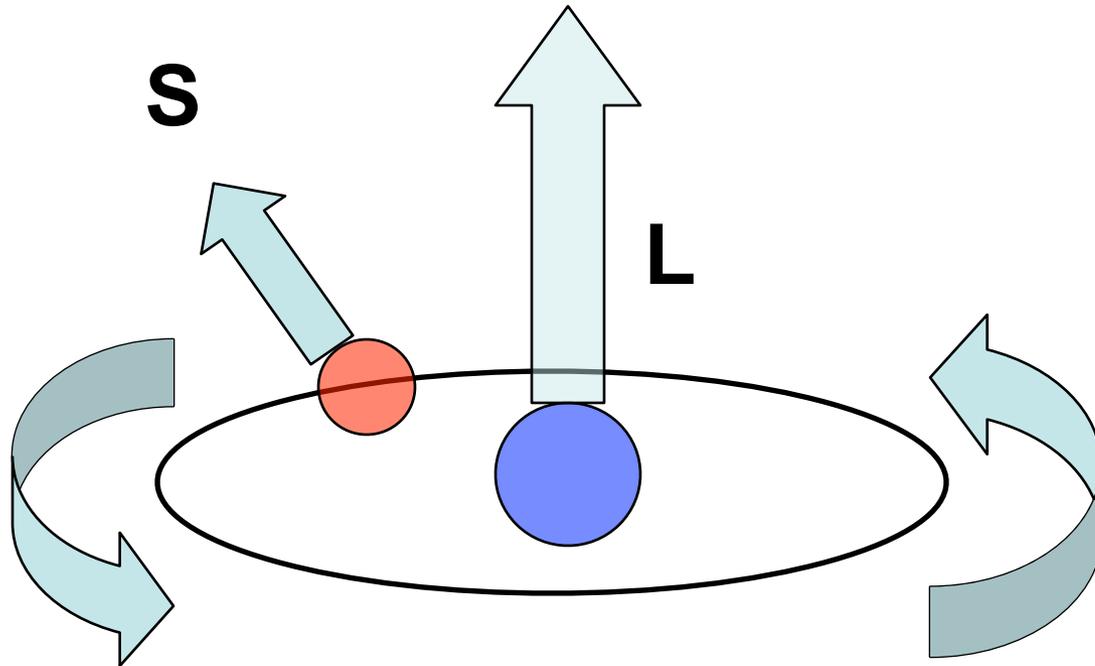


Why?



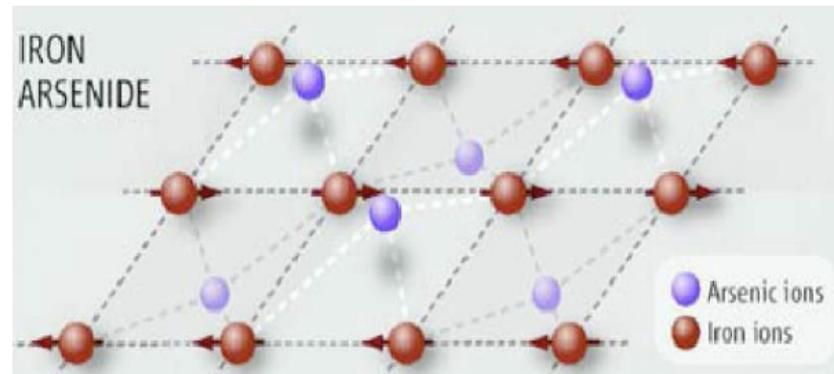
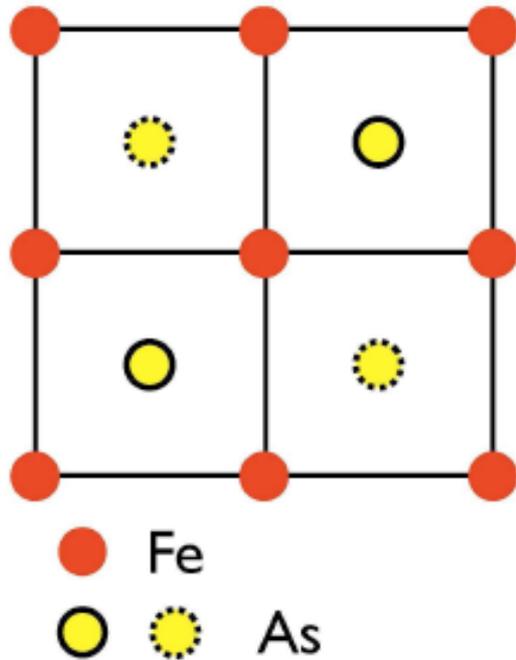
PRB, vol. 78, 094517 (2008).

Spin-Orbit interaction



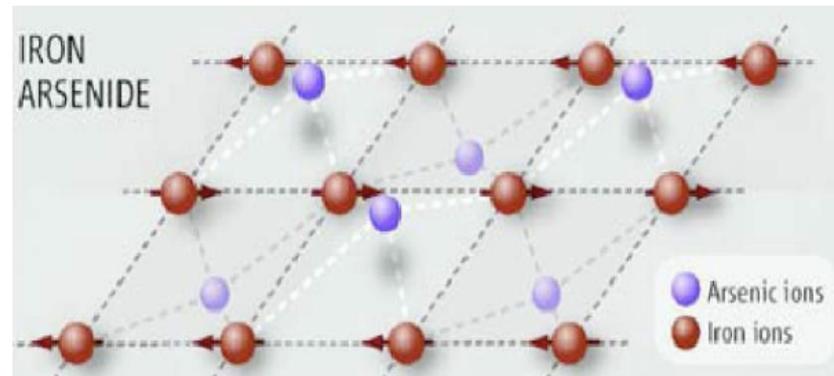
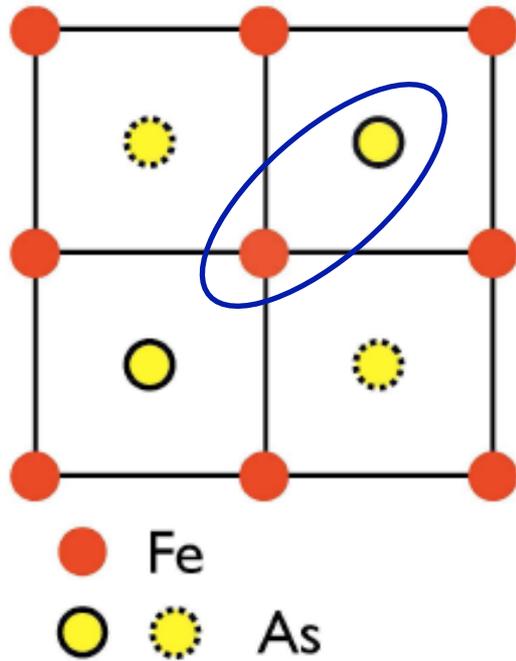
As: $V_{\text{spin-orbit}} = 0.4 \text{ eV}$

Key structure: Fe-As plane



arXiv: 0812.0302

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First-principles calculations of Fe on GaAs(100)

S. Mirbt, B. Sanyal, C. Isheden, and B. Johansson
 Department of Physics, Uppsala University, Uppsala, Sweden
 (Received 10 December 2002; published 30 April 2003)

A hint

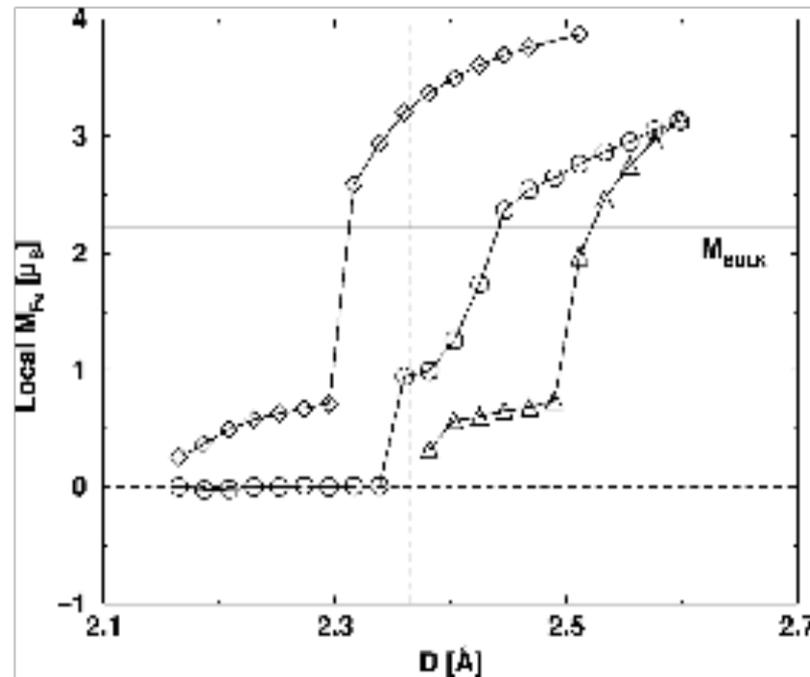


FIG. 8. Bulk magnetic moment per Fe atom for zinc-blende FeAs (circles), zinc-blende FeSe (diamonds), and zinc-blende FeTe (triangles) as a function of the Fe-anion distance D . The dashed vertical line is the same as in Fig. 7 shown here for comparison.

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A hint

As 4p-Fe
 3d
 hybridisation
 is
 crucial

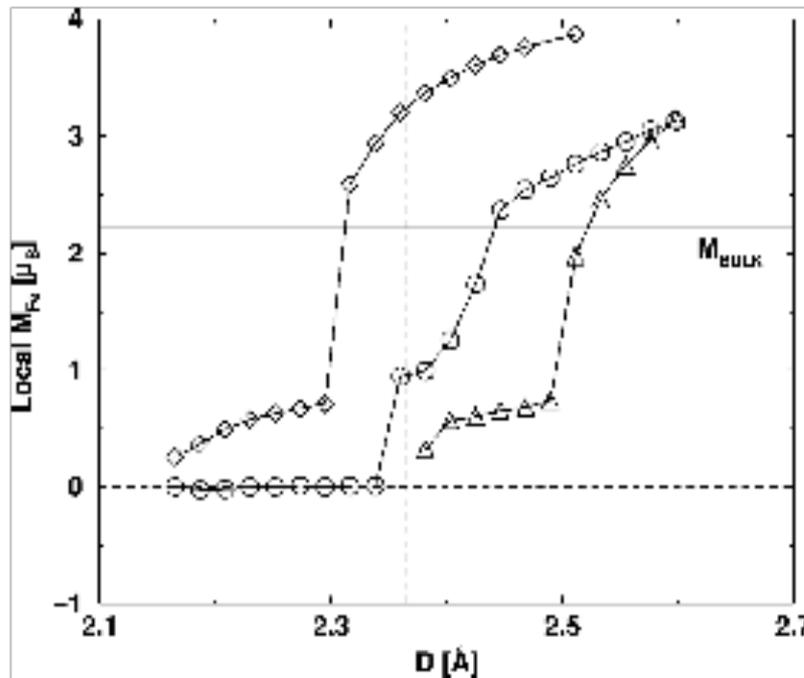


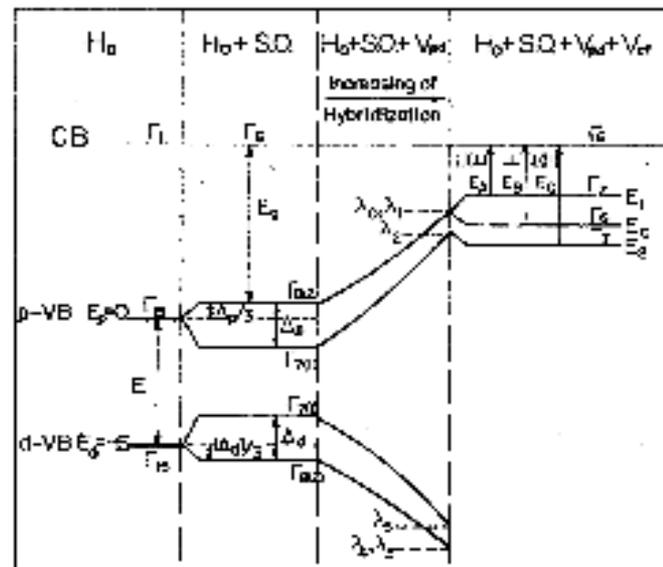
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Effects of *p-d* hybridization on the valence band of I-III-VI₂ chalcopyrite semiconductors

Kujurnyud Yoodee¹ and John C. Woolley
 Physics Department, University of Ottawa, Ottawa, Ontario, Canada K1N 6N5

Viruh Sa-yakauit
 Physics Department, Chulalongkorn University, Bangkok 10500, Thailand
 (Received 15 June 1984)

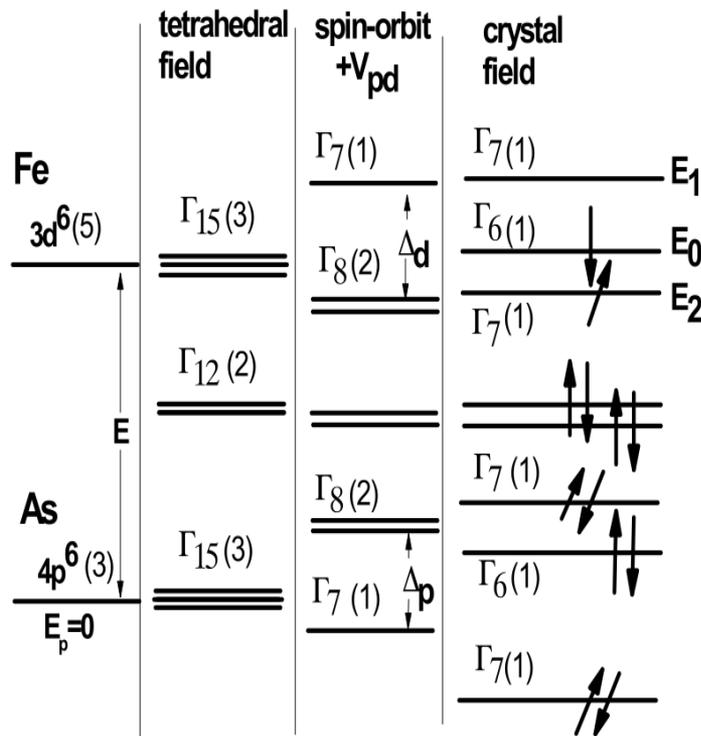
The electronic structure of the valence band at the Brillouin-zone center of the I-III-VI₂ chalcopyrite compounds has been calculated using a model developed by adding the effects of *p-d* hybridization and the crystal field to the Hamiltonian of the Kane model. Two important parameters in the model are the energy separation *E* between the *p* and *d* levels and the interaction *M* between these levels. It is shown that three previous models (Fall and Bridenbush, Kaldal, and the linear hybridization model) can be derived as special cases of the present model. The model has been used to analyze the available data on some 13 compounds. It is shown that the dimensionless parameters *M/E* and $\Delta E_g/E_g$, where ΔE_g is the band-gap anomaly, show a smooth variation with the fractional *d* character of the valence band and appear to be characteristic of the structure. Values of the deformation potentials *b₁* and *b₂* averaged over all of the compounds have been determined and found to be *b₁* = (-0.8 ± 0.2) eV and *b₂* = (-4.3 ± 1.5) eV.



spin-orbit+
 pd hybridization+
 crystal field

New way to arrange electrons in As-Fe band:

(by diagonalizing the above Hamiltonian)



Sketch of the energy levels of the Fe 3p and As 4p hybridized levels after the inclusion of spin-orbit coupling, p-d hybridization, and the monoclinic crystal field distortion.

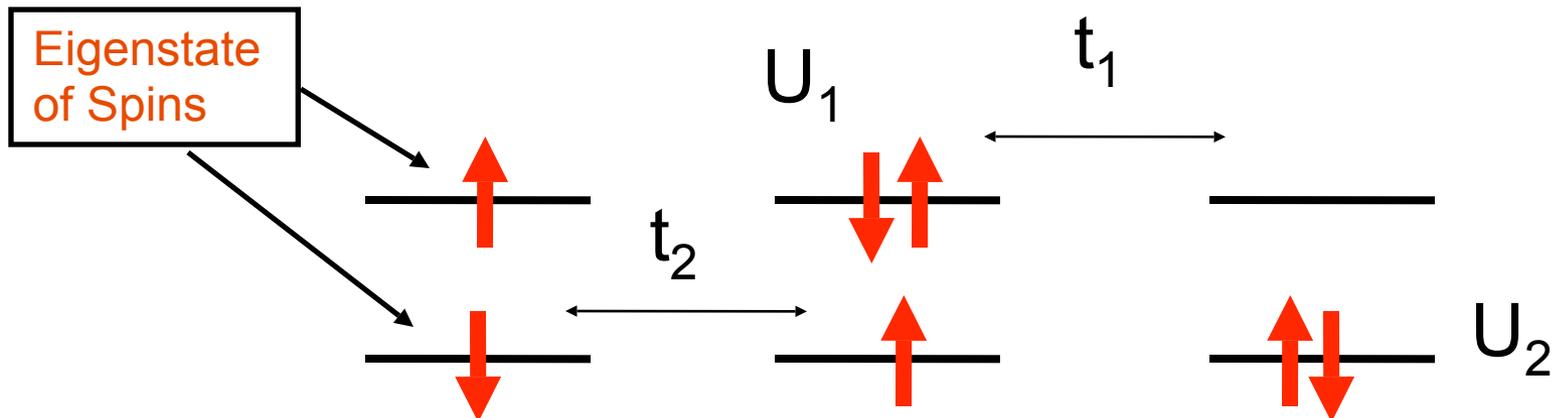
Only the Γ_{15} p-levels of Fe hybridize with the As 4p levels. The Γ_{12} levels remain non bonding.

Each of the hybridized levels is doubly degenerate, though not an eigenstate of S_z . The only hybridized level which is an eigenstate of S_z is E_0 . The lowest-energy configuration of the spins in E_0 and E_2 is indicated. As a result, only the x-y component produces a non-zero moment.

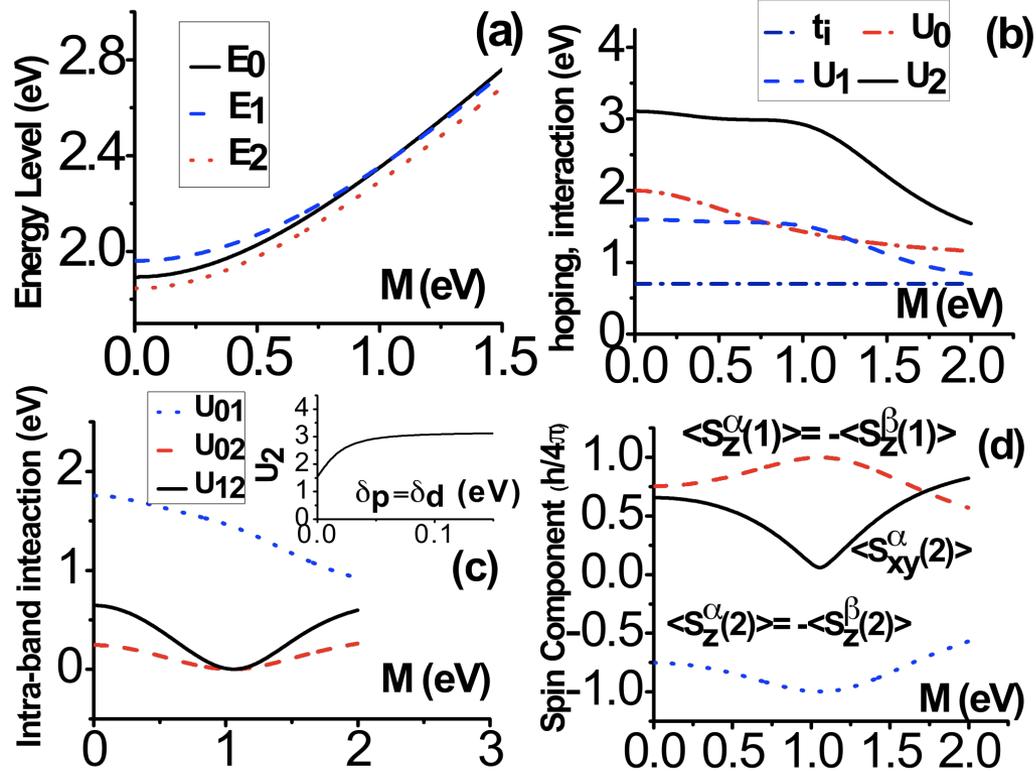
Hamiltonian in orbital basis:

$$H = \sum_{\mathbf{a}, \mathbf{i}} E_{\mathbf{a}} c_{\mathbf{a}, \mathbf{i}}^{\dagger}(\sigma) c_{\mathbf{a}, \mathbf{i}}(\sigma) - \sum_{\mathbf{a}, \langle \mathbf{i}, \mathbf{j} \rangle} t_{\mathbf{a}} c_{\mathbf{a}, \mathbf{i}}^{\dagger}(\sigma) c_{\mathbf{a}, \mathbf{j}}(\sigma) + \text{h.c.}$$

$$+ \sum_{\mathbf{a}} U_{\mathbf{a}} n_{\mathbf{i}, \mathbf{a} \uparrow} n_{\mathbf{i}, \mathbf{a} \downarrow},$$



Magnitude of parameters

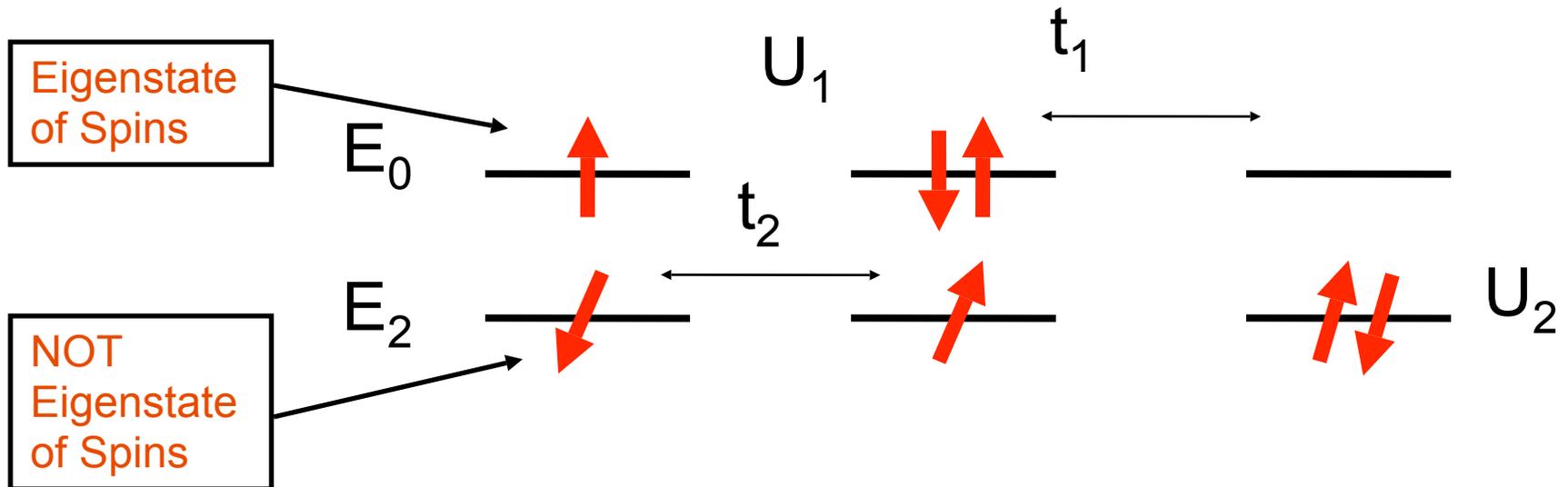


(a) Energy levels, (b) hopping matrix elements, on-site interactions, (c) intra-band interaction and (d) spin components as a function of hybridization M . Shown in the inset of (c) is the on-site interaction U_2 as a function of the crystal field (monoclinic distortion).

Hamiltonian in new basis

$$\mathbf{H} = \sum_{\mathbf{a},i} E_{\mathbf{a}} c_{\mathbf{a},i}^{\dagger}(\mu) c_{\mathbf{a},i}(\mu) - \sum_{\mathbf{a},\mathbf{b},\langle i,j \rangle} t_{\mathbf{a},\mathbf{b}}(\mu,\nu) c_{\mathbf{a},i}^{\dagger}(\mu) c_{\mathbf{b},j}(\nu) + \text{h.c.}$$

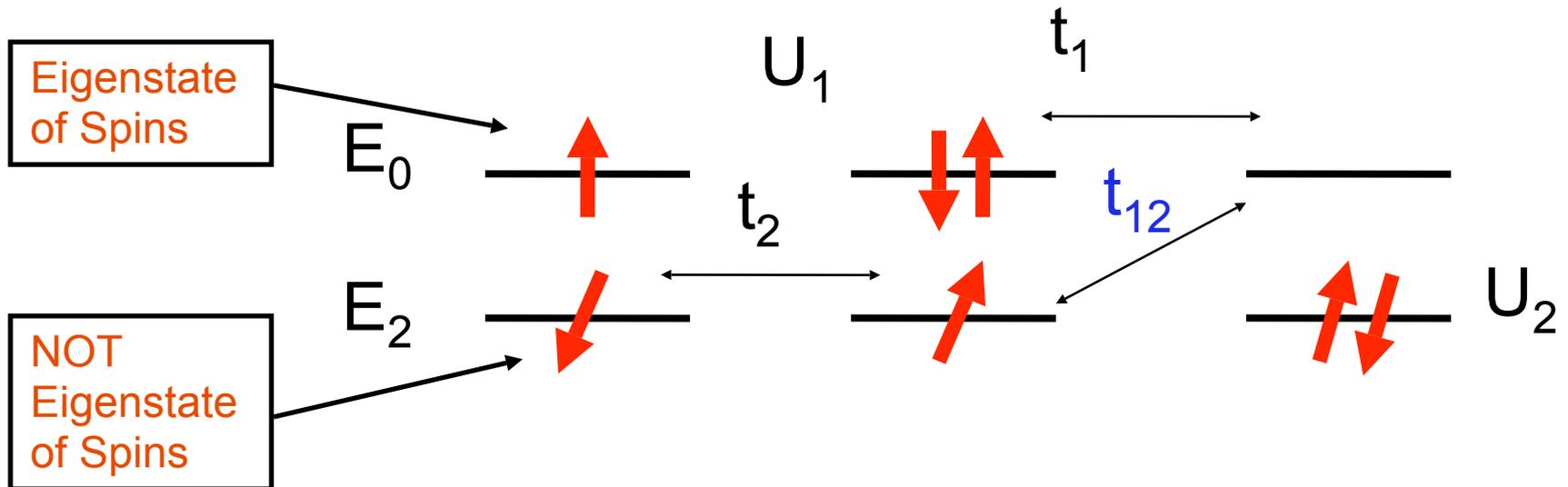
$$+ \sum_{\mathbf{a},\mathbf{b},i} U_{\mathbf{a}\mathbf{b}}(\mu\nu) n_{i\mathbf{a}}(\mu) n_{i\mathbf{b}}(\nu),$$



Hamiltonian in new basis

$$\mathbf{H} = \sum_{\mathbf{a},i} E_{\mathbf{a}} c_{\mathbf{a},i}^{\dagger}(\mu) c_{\mathbf{a},i}(\mu) - \sum_{\mathbf{a},\mathbf{b},\langle i,j \rangle} t_{\mathbf{a},\mathbf{b}}(\mu,\nu) c_{\mathbf{a},i}^{\dagger}(\mu) c_{\mathbf{b},j}(\nu) + \text{h.c.}$$

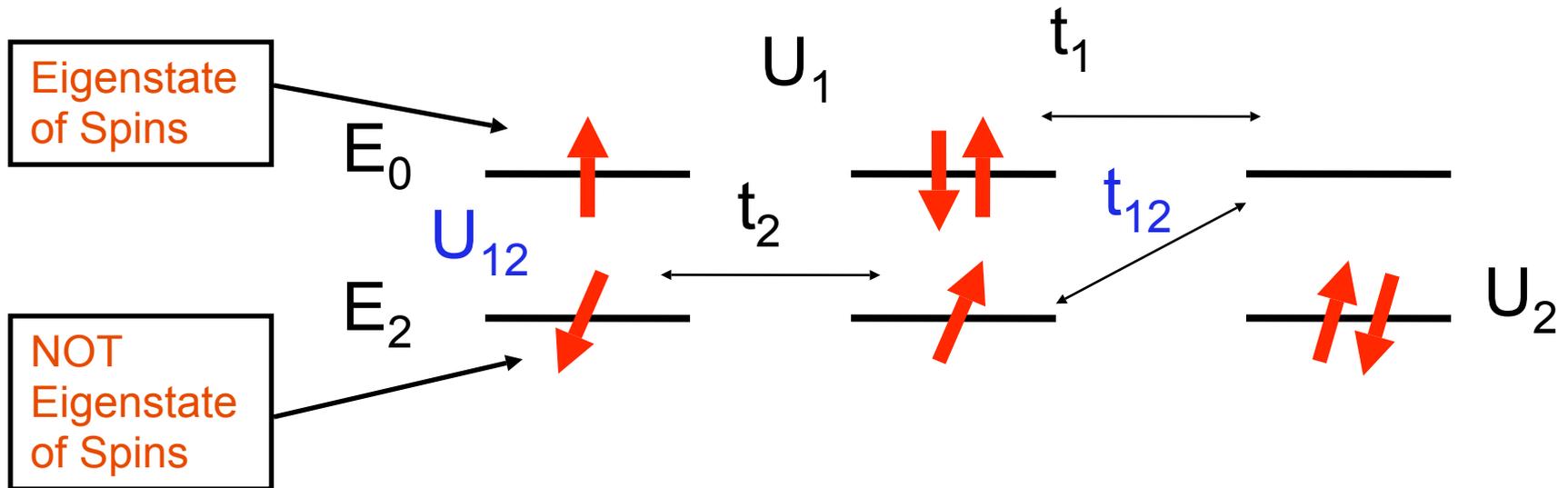
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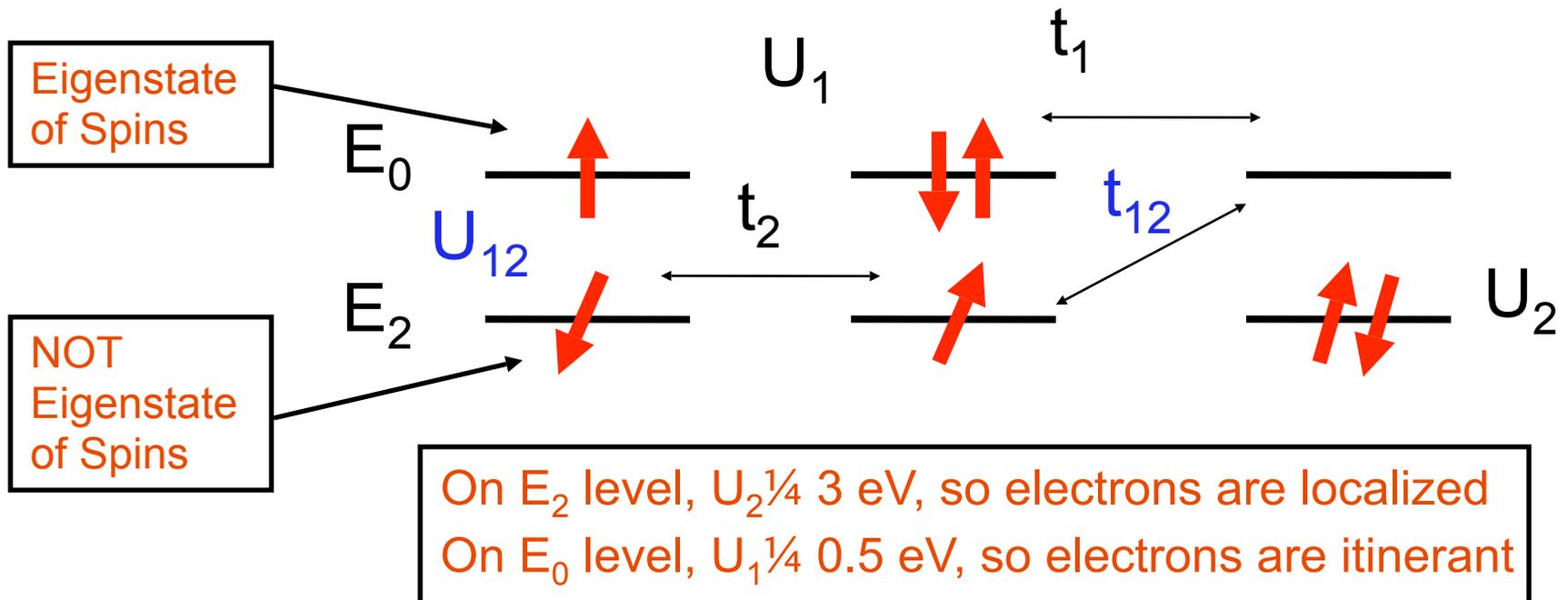
$$+ \sum_{\mathbf{a}, \mathbf{b}, \mathbf{i}} U_{\mathbf{a} \mathbf{b}}(\mu \nu) n_{\mathbf{i} \mathbf{a}}(\mu) n_{\mathbf{i} \mathbf{b}}(\nu),$$



Hamiltonian in new basis

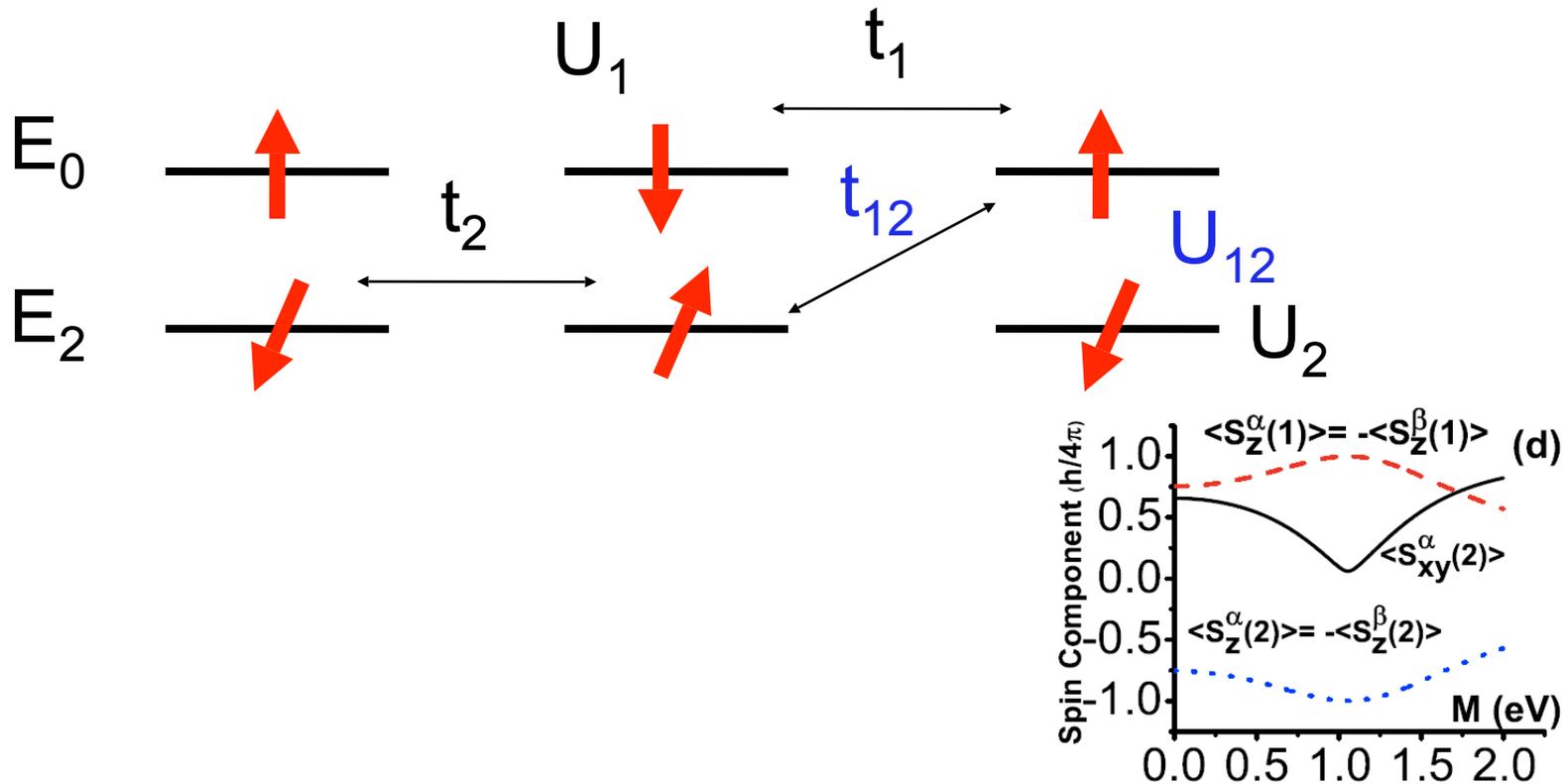
$$\mathbf{H} = \sum_{\mathbf{a}, \mathbf{i}} E_{\mathbf{a}} c_{\mathbf{a}, \mathbf{i}}^{\dagger}(\mu) c_{\mathbf{a}, \mathbf{i}}(\mu) - \sum_{\mathbf{a}, \mathbf{b}, \langle \mathbf{i}, \mathbf{j} \rangle} t_{\mathbf{a}, \mathbf{b}}(\mu, \nu) c_{\mathbf{a}, \mathbf{i}}^{\dagger}(\mu) c_{\mathbf{b}, \mathbf{j}}(\nu) + \text{h.c.}$$

$$+ \sum_{\mathbf{a}, \mathbf{b}, \mathbf{i}} U_{\mathbf{a}\mathbf{b}}(\mu\nu) n_{\mathbf{ia}}(\mu) n_{\mathbf{ib}}(\nu),$$



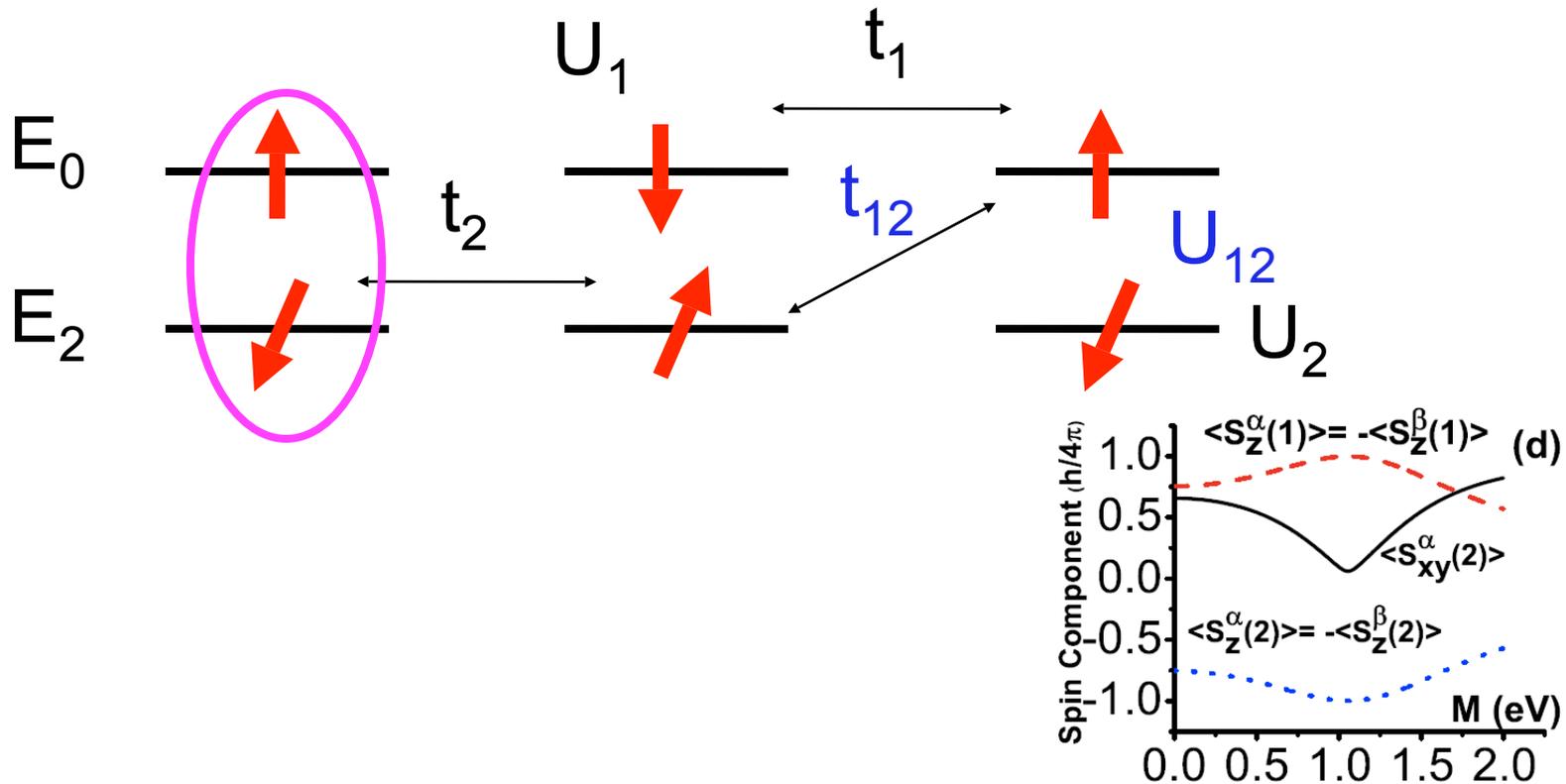
Why the magnetic moments are so small?

$\mu_z = 0.06\mu_B$ $\mu_{xy} = 0.25\mu_B$ agree with experiments



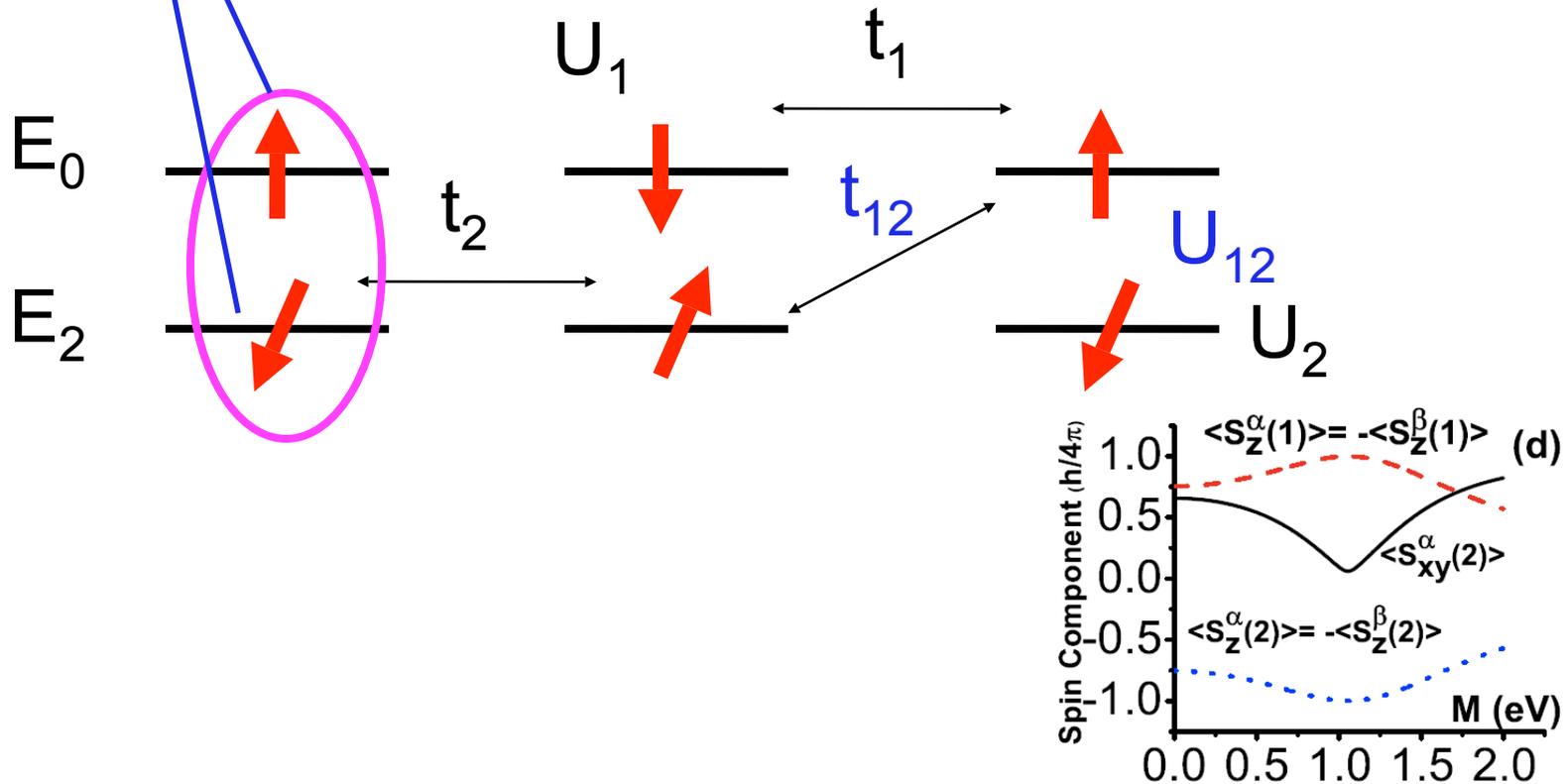
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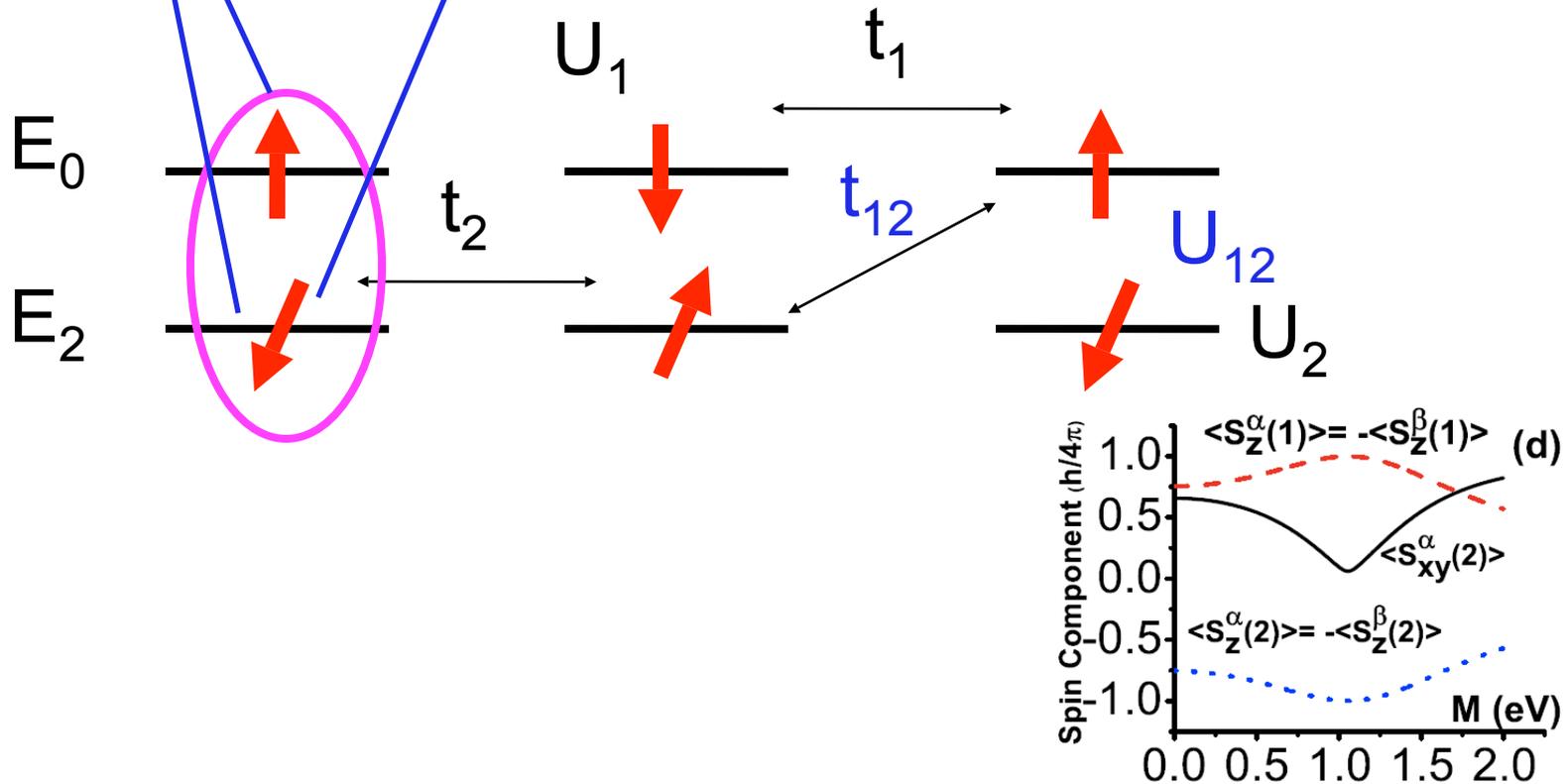
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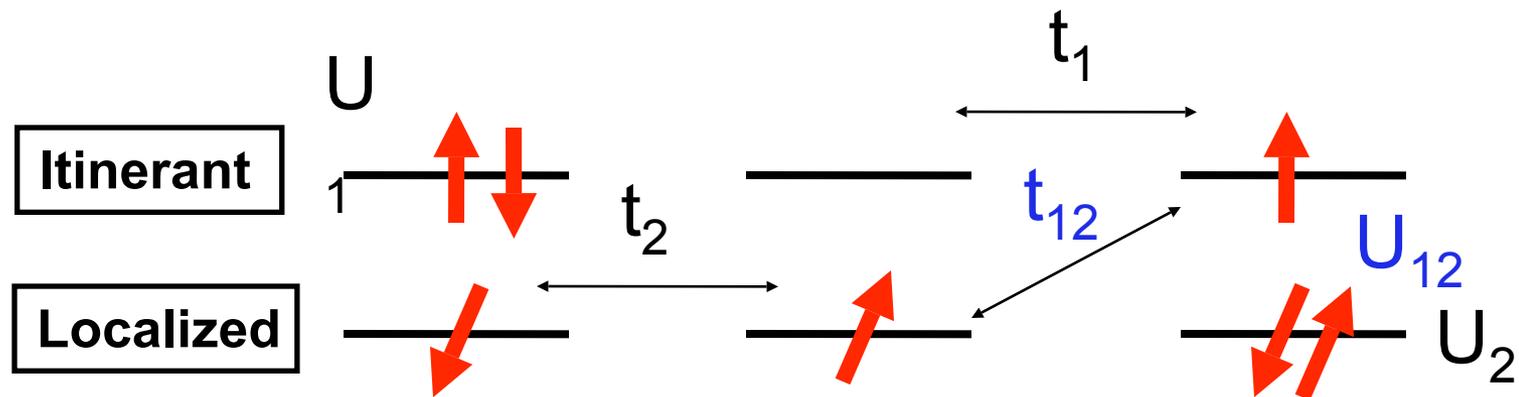
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Localized magnetic moments are screened by the itinerant electrons

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Localized magnetic moments are screened by the itinerant electrons

We also get the minimal model:
Itinerant-localized dichotomy



Itinerant-localized dichotomy

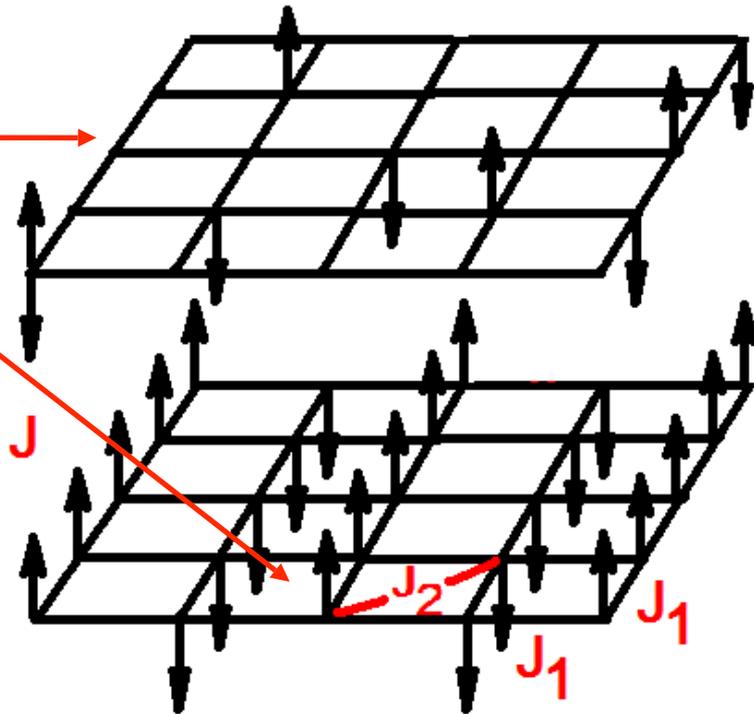
$$H = H_e + H_s + H_{sf}$$

$$H_e = \sum \epsilon_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma}^\dagger c_{\mathbf{k},\sigma}$$

$$H_s = J_1 \sum_{n.n} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{n.n.n} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$H_{sf} = J \sum c_{i,\alpha}^\dagger \sigma_{\alpha\beta} c_{i,\beta} \cdot \mathbf{S}_i$$

Kondo coupling

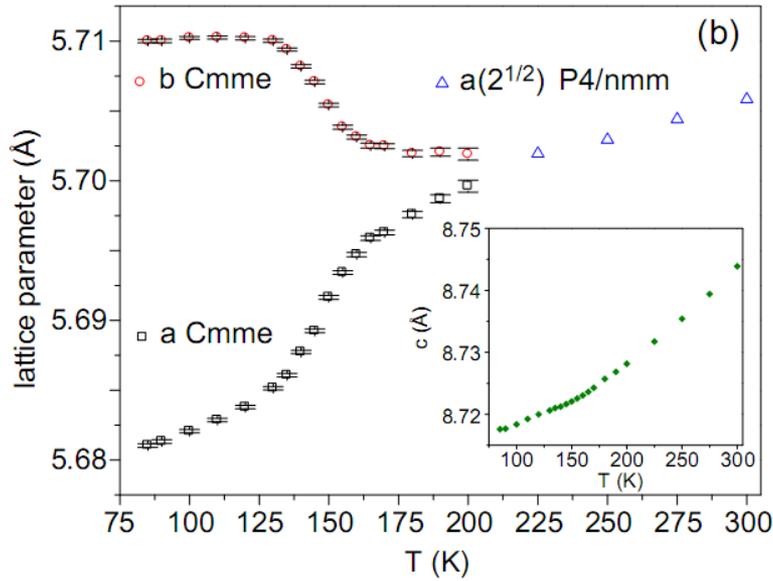


Itinerant electrons interact with each other by propagating magnons in localized level

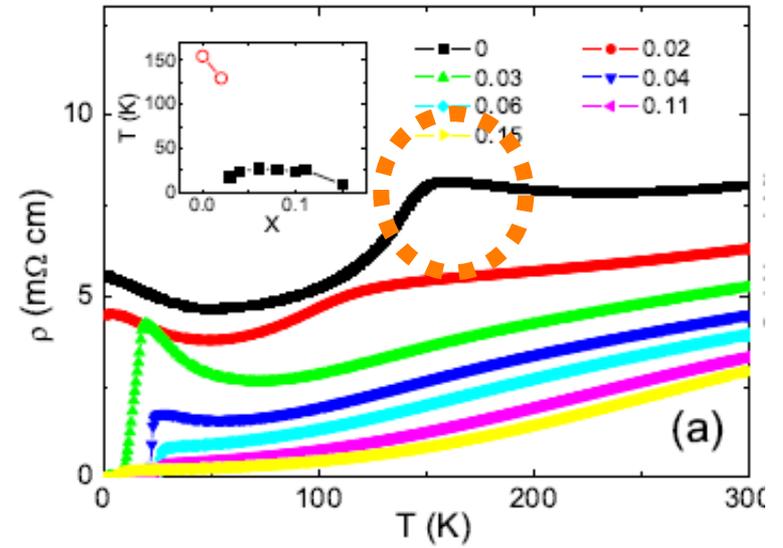
2.) What causes the structural phase transition and resistivity anomaly?

W. Lv, J. Wu, P. Phillips, PRB, 80, 224506 (2009)

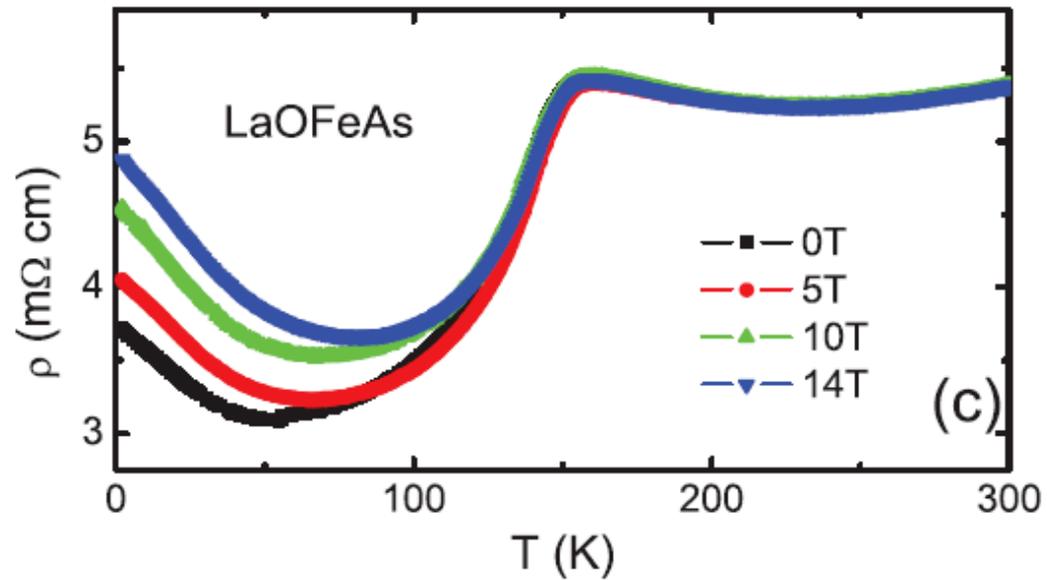
• **Structural Phase Transition**



• **Resistivity Anomaly** $T_{RA} = T_{SPT}$



SPT is insensitive to external magnetic field



What breaks
symmetry in x-y plane?

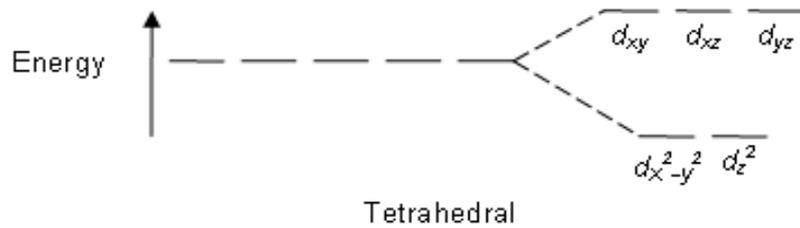
What breaks
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orbital ordering

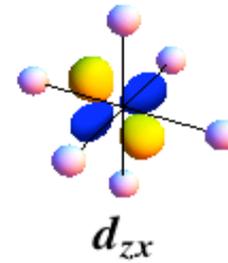
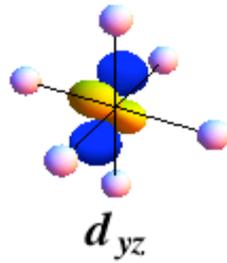
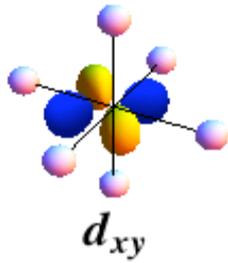
F. Krüger, *et al.* PRB (2009)
R.R.P. Singh, arXiv:0903.4408
W. Lv, *et al.* PRB (2009)
A.M. Turner, *et al.* PRB (2009)
C.-C. Lee, *et al.* PRL (2009)

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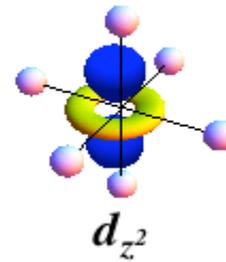
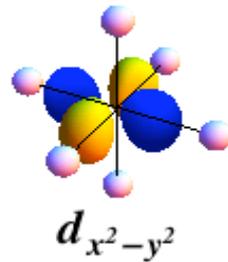
Orbital ordering (multi-bands)



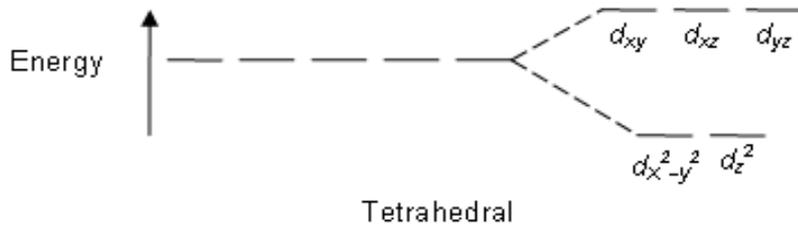
t_{2g}



e_g

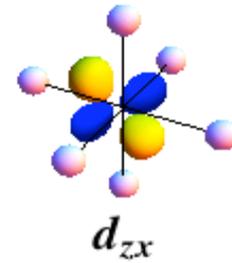
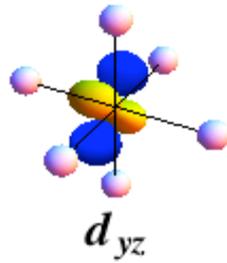


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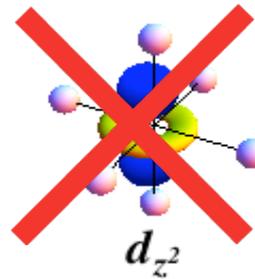
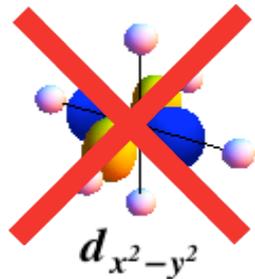


$d_{x^2-y^2}$, d_{z^2} , d_{xy} , with the rotational symmetry in xy -plane, will not contribute to SPT.

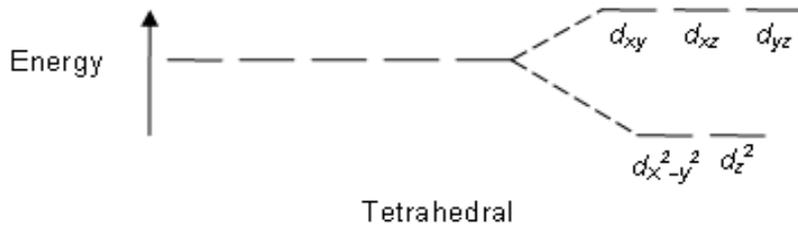
t_{2g}



e_g

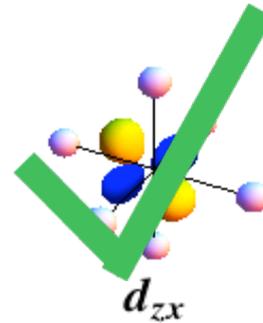
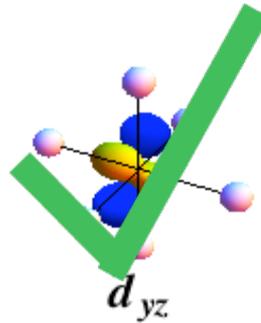
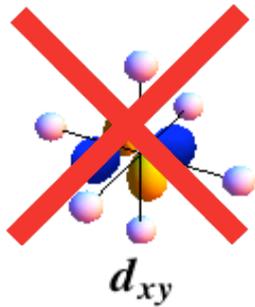


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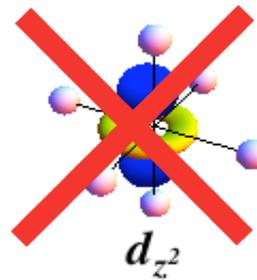
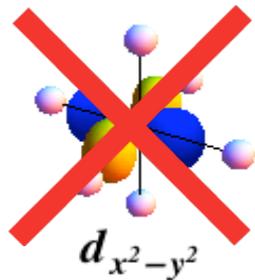


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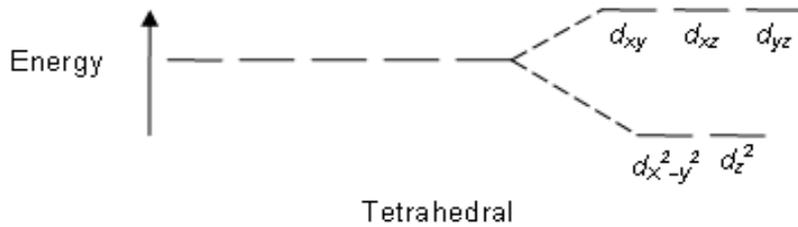
t_{2g}



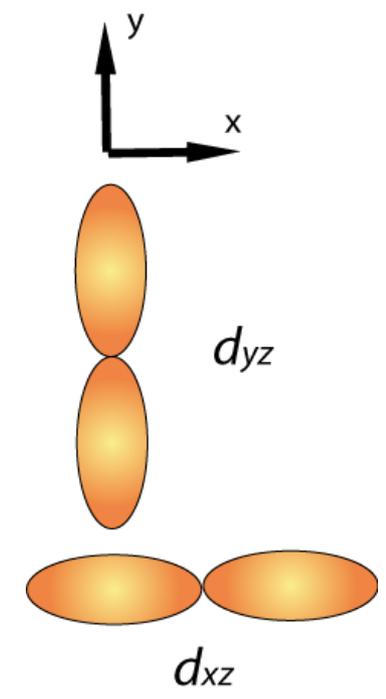
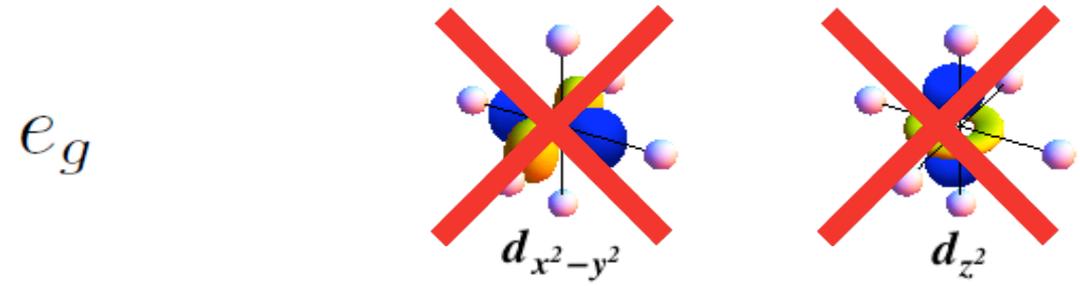
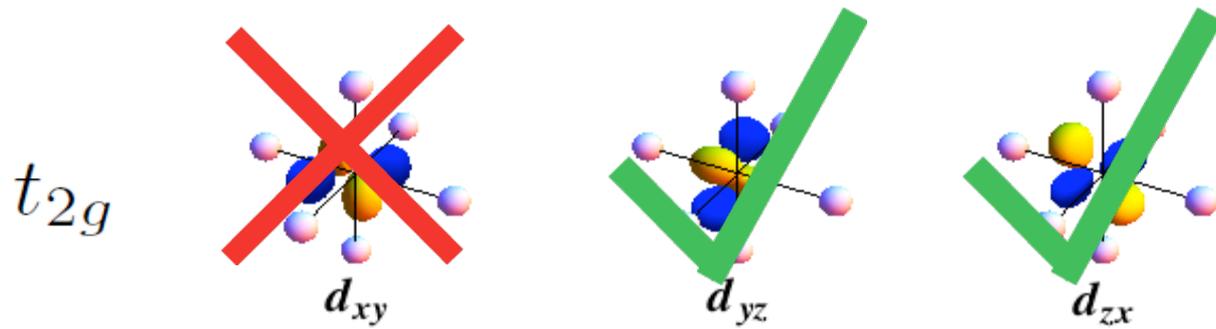
e_g



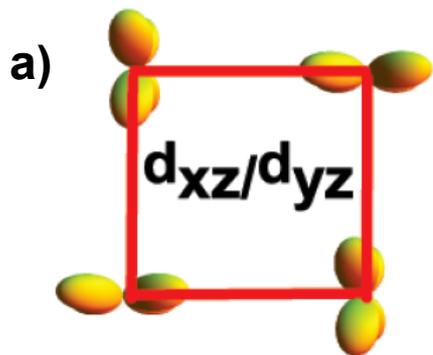
Orbital ordering (multi-bands)



$d_{x^2-y^2}$, d_{z^2} , d_{xy} , with the rotational symmetry in xy -plane, will not contribute to SPT.



$T > T_{SPT}$



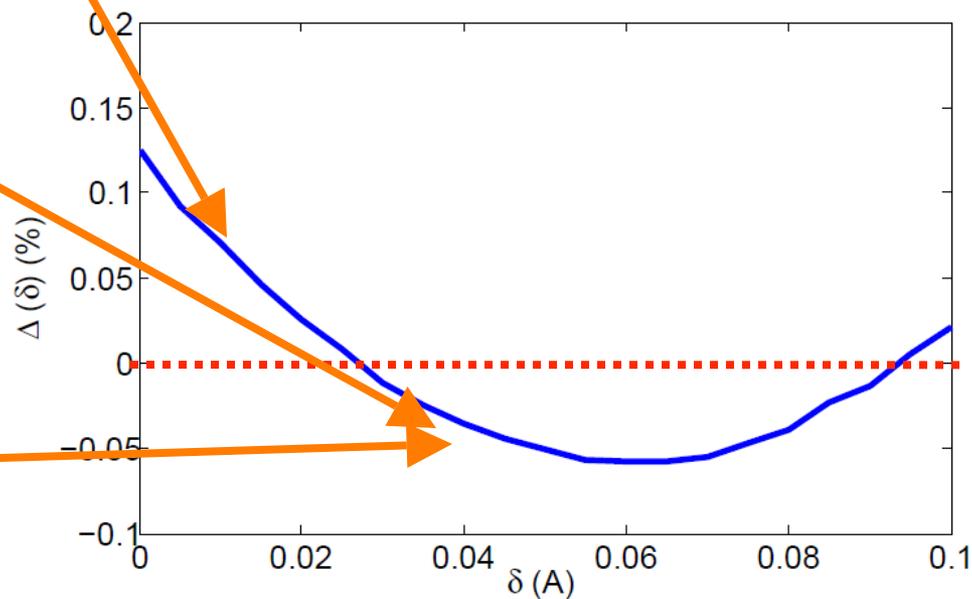
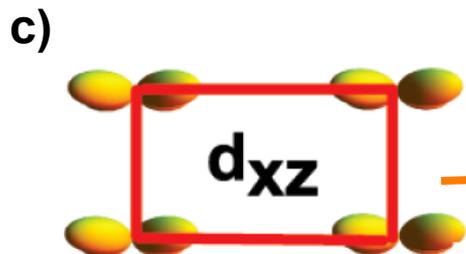
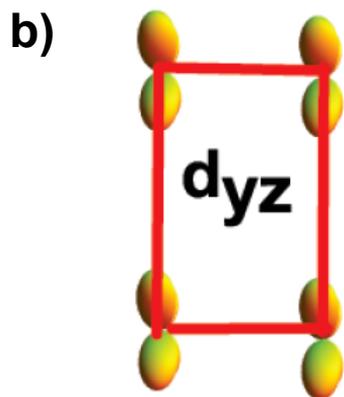
Coulomb Repulsion:

$$U = e^2 \int |\psi_{i_1 m_1}(\mathbf{r})|^2 \frac{e^{-|\mathbf{r}-\mathbf{r}'|/r_0}}{|\mathbf{r}-\mathbf{r}'|} |\psi_{i_2 m_2}(\mathbf{r}')|^2$$

Energy Difference:

$$\Delta(\delta) = \frac{U_b(\delta) - U_a}{U_a}$$

$T < T_{SPT}$



SPT in Ising Universality Class

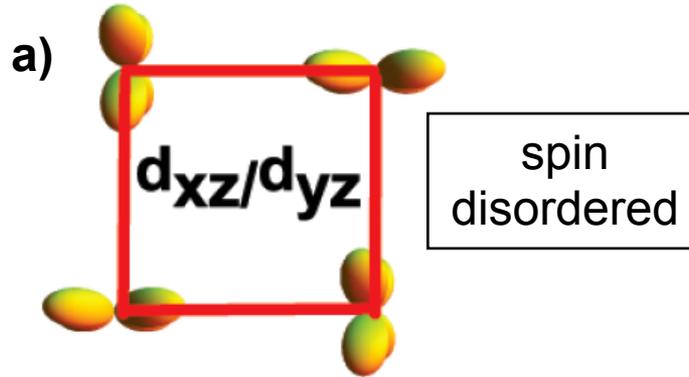
$$H_{\text{SPT}} = -J_{\text{SPT}} \sum_{\langle i,j \rangle} M_i M_j$$

$$M_i = \pm 1, i = d_{yz}, d_{xz}$$

SPT-induced Collinear AF

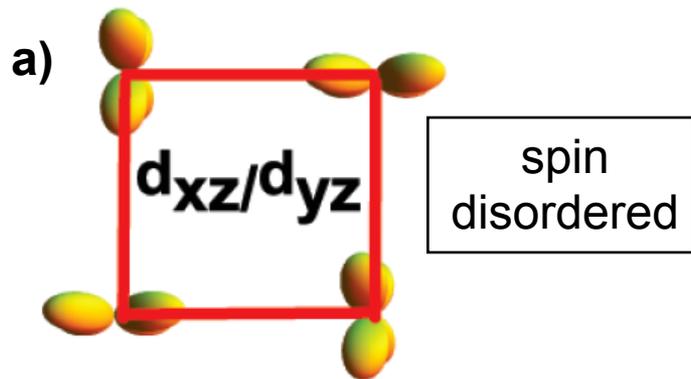
SPT-induced Collinear AF

$$T > T_{SPT}$$

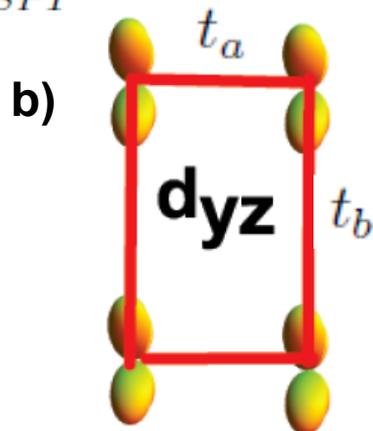


SPT-induced Collinear AF

$T > T_{SPT}$

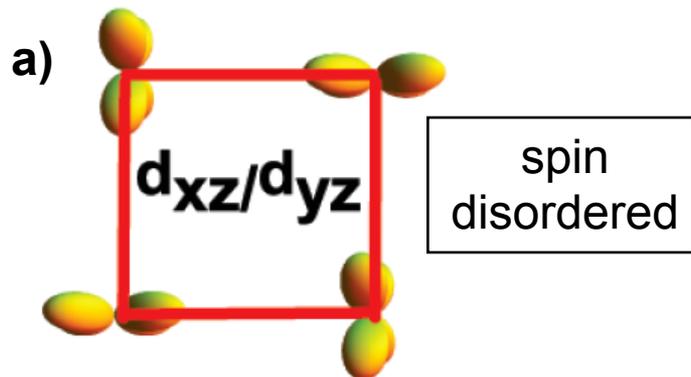


$T < T_{SPT}$

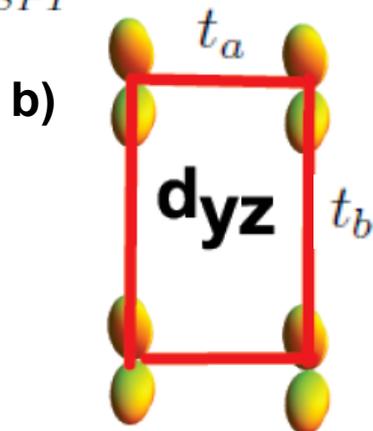


SPT-induced Collinear AF

$T > T_{SPT}$



$T < T_{SPT}$



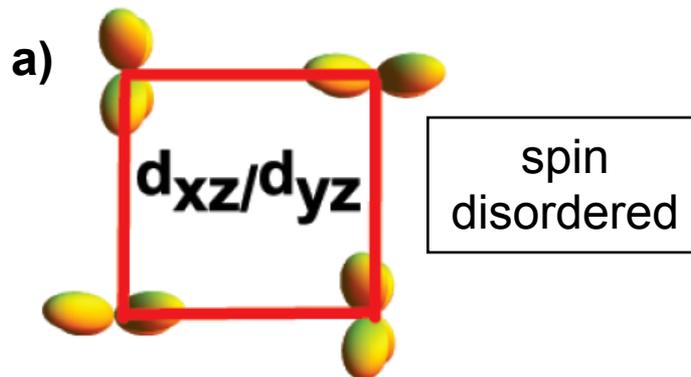
$$t_b > t_a$$

$$J \sim \frac{t^2}{U}$$

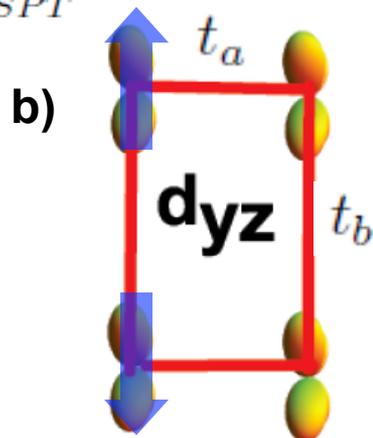
$$J_b > J_a$$

SPT-induced Collinear AF

$T > T_{SPT}$



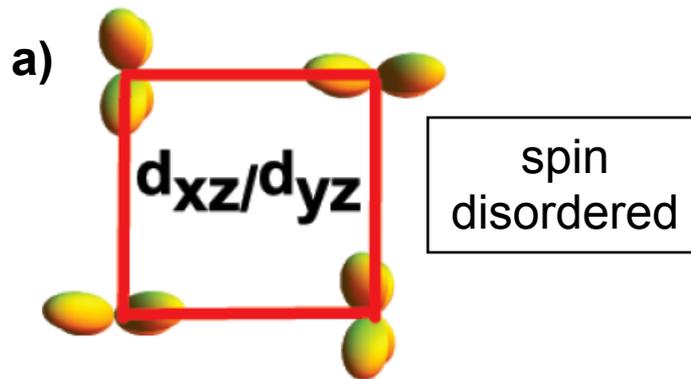
$T < T_{SPT}$



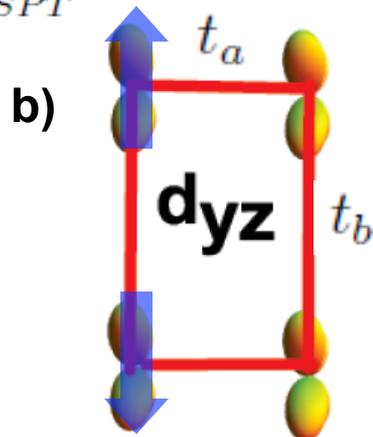
$$\begin{aligned} t_b &> t_a \\ J &\sim \frac{t^2}{U} \\ J_b &> J_a \end{aligned}$$

SPT-induced Collinear AF

$T > T_{SPT}$



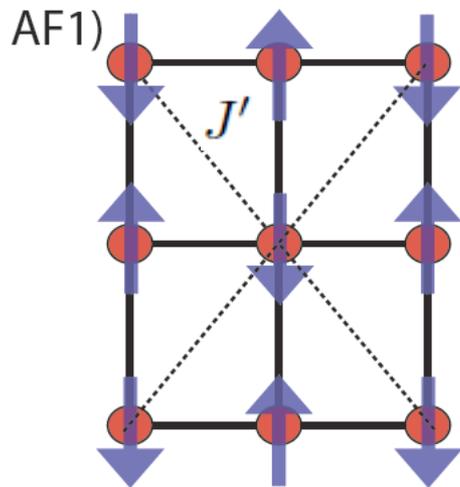
$T < T_{SPT}$



$$t_b > t_a$$

$$J \sim \frac{t^2}{U}$$

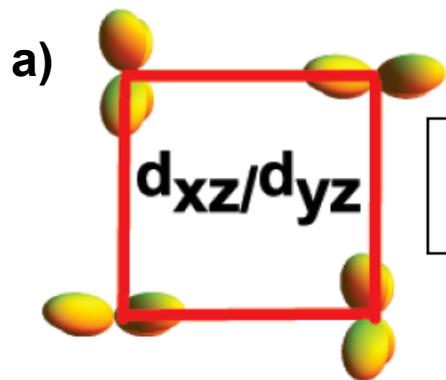
$$J_b > J_a$$



$$E_1 = -2J_a + 4J'$$

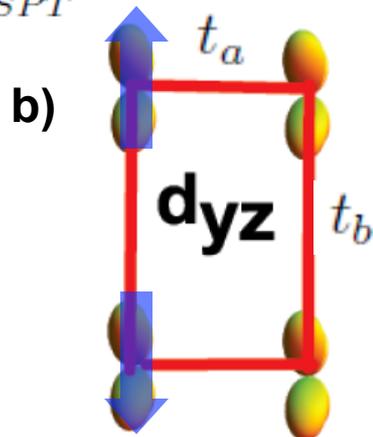
SPT-induced Collinear AF

$T > T_{SPT}$



spin disordered

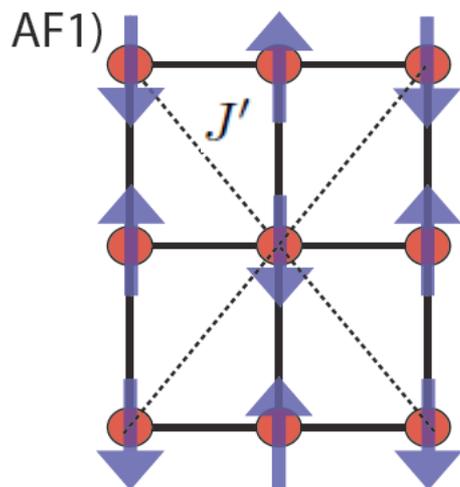
$T < T_{SPT}$



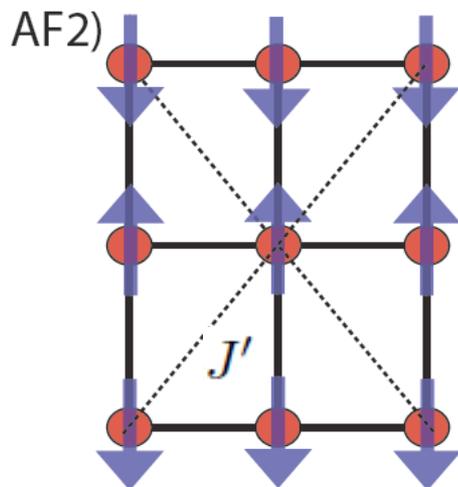
$$t_b > t_a$$

$$J \sim \frac{t^2}{U}$$

$$J_b > J_a$$



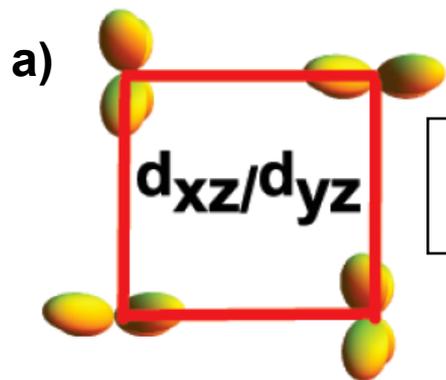
$$E_1 = -2J_a + 4J'$$



$$E_2 = +2J_a - 4J'$$

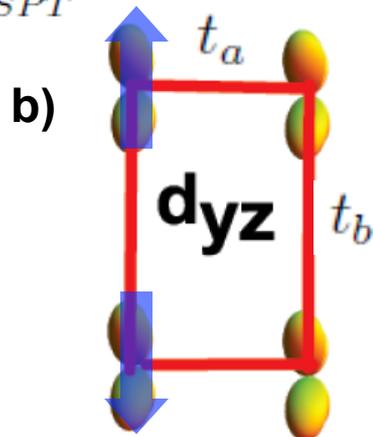
SPT-induced Collinear AF

$T > T_{SPT}$



spin
disordered

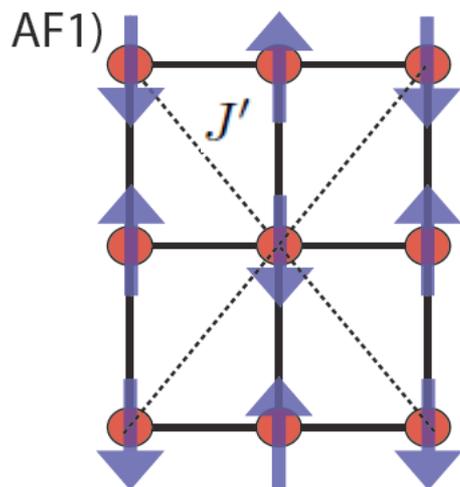
$T < T_{SPT}$



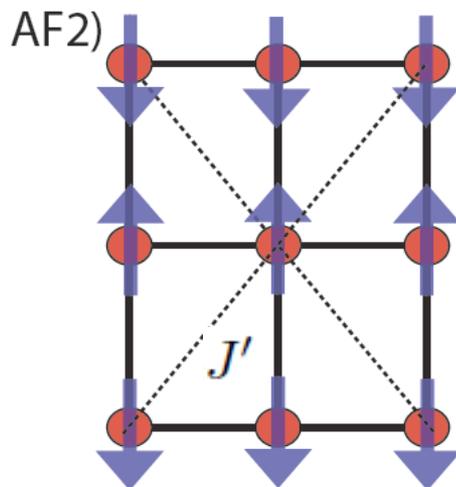
$$t_b > t_a$$

$$J \sim \frac{t^2}{U}$$

$$J_b > J_a$$



$$E_1 = -2J_a + 4J'$$

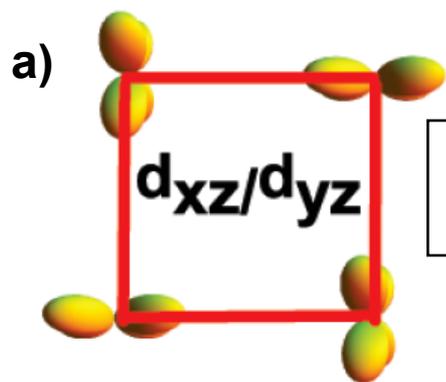


$$E_2 = +2J_a - 4J'$$

$$J' > \frac{J_a}{2}$$

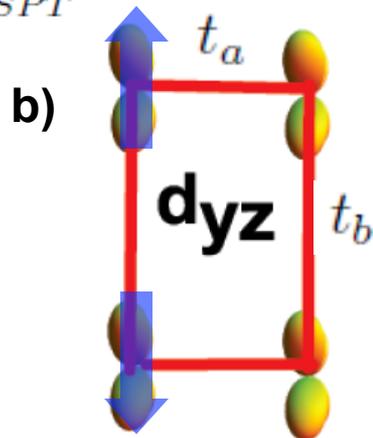
SPT-induced Collinear AF

$T > T_{SPT}$

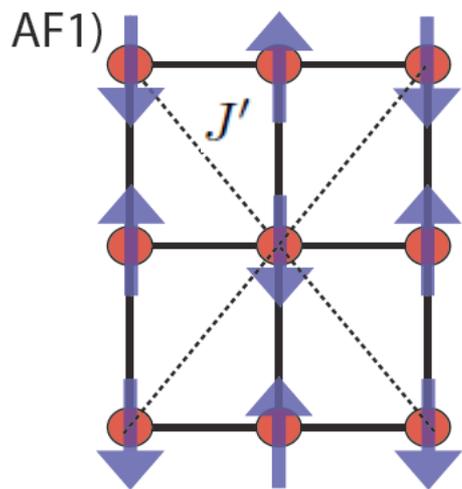


spin disordered

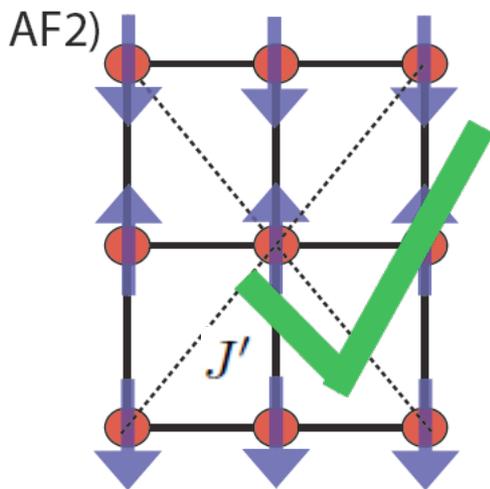
$T < T_{SPT}$



$$\begin{aligned} t_b &> t_a \\ J &\sim \frac{t^2}{U} \\ J_b &> J_a \end{aligned}$$



$$E_1 = -2J_a + 4J'$$



$$E_2 = +2J_a - 4J'$$

$$J' > \frac{J_a}{2}$$

SPT-induced magnetism

$$\begin{aligned} H_{\text{SO}} = & J_{\text{SPT}} \sum_{\langle i,j \rangle} M_i M_j + \sum_{\langle\langle i,j \rangle\rangle} J_2 (M_i, M_j) \mathbf{S}_i \cdot \mathbf{S}_j \\ & + \sum_i J_{1x} (M_i, M_{i+\hat{x}}) \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{x}} \\ & + \sum_i J_{1y} (M_i, M_{i+\hat{y}}) \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{y}} \end{aligned}$$

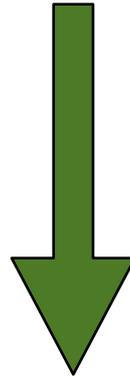
$$J_{1x} (M_i, M_j) = \delta_{M_i, M_j} (J_{1b} \delta_{M_i, 1} + J_{1a} \delta_{M_i, -1})$$

$$J_{1y} (M_i, M_j) = \delta_{M_i, M_j} (J_{1a} \delta_{M_i, 1} + J_{1b} \delta_{M_i, -1})$$

$$J_2 (M_i, M_j) = \delta_{M_i, M_j} J_2$$

122 Fe-Fe is shorter: $J_{\{1b\}}$ is enhanced

orbital ordering



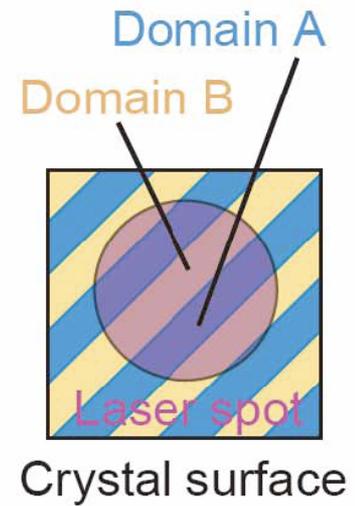
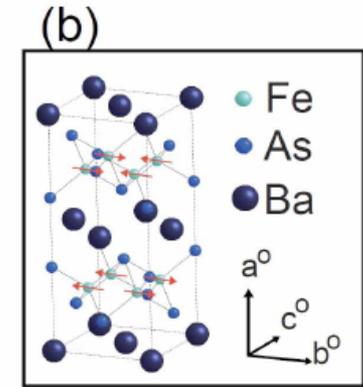
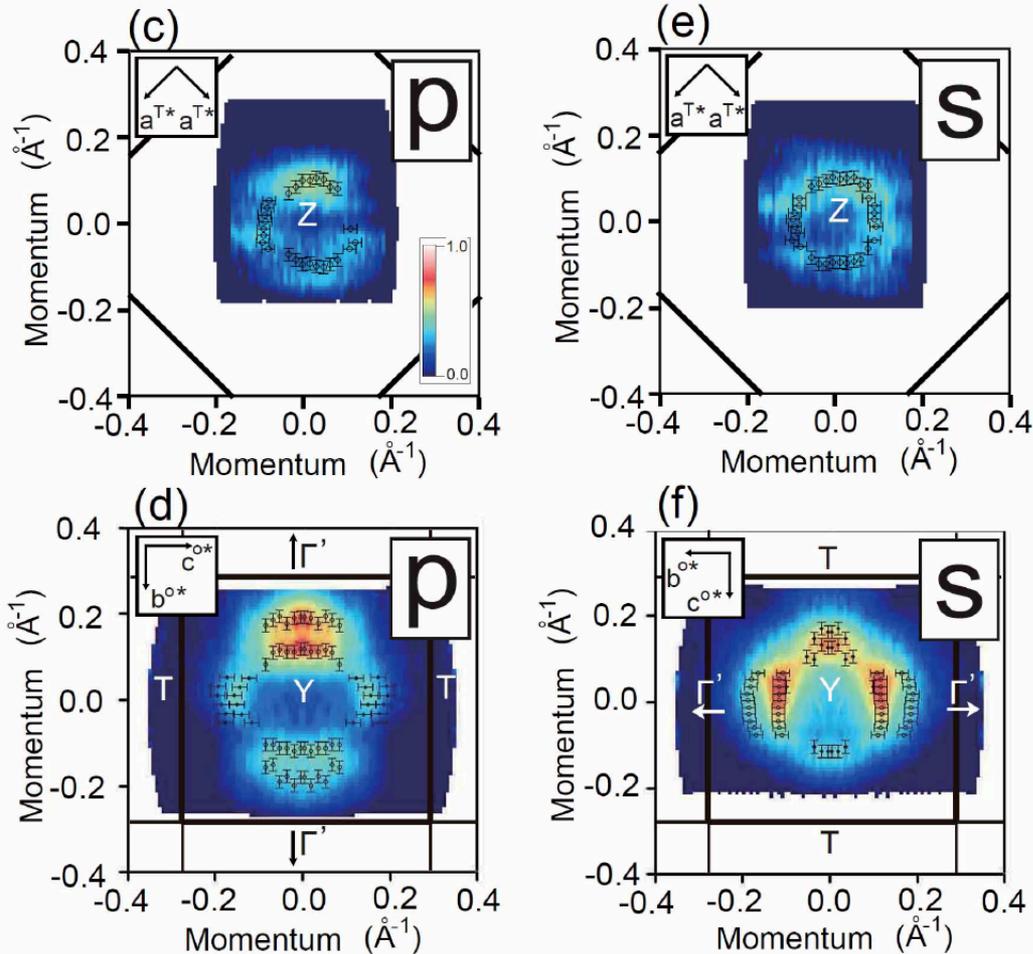
transport anisotropies

Orbital-polarized Fermi surface in antiferromagnetic state of

Shimojima, et al.
arXiv:0904.1632

BaFe₂As₂

• Polarized ARPES



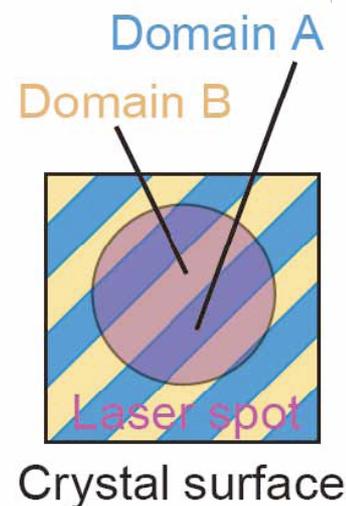
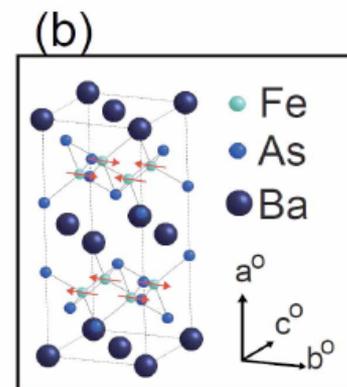
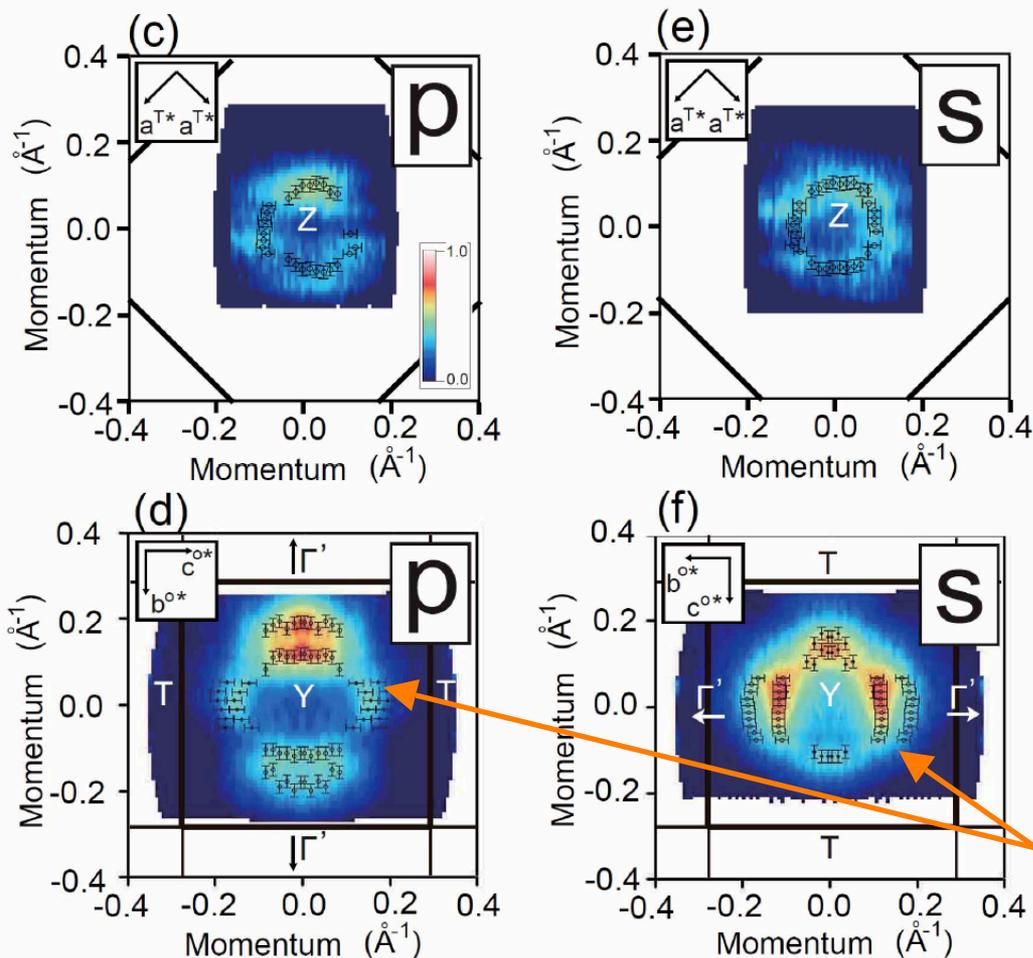
Orbital-polarized Fermi surface in antiferromagnetic state of

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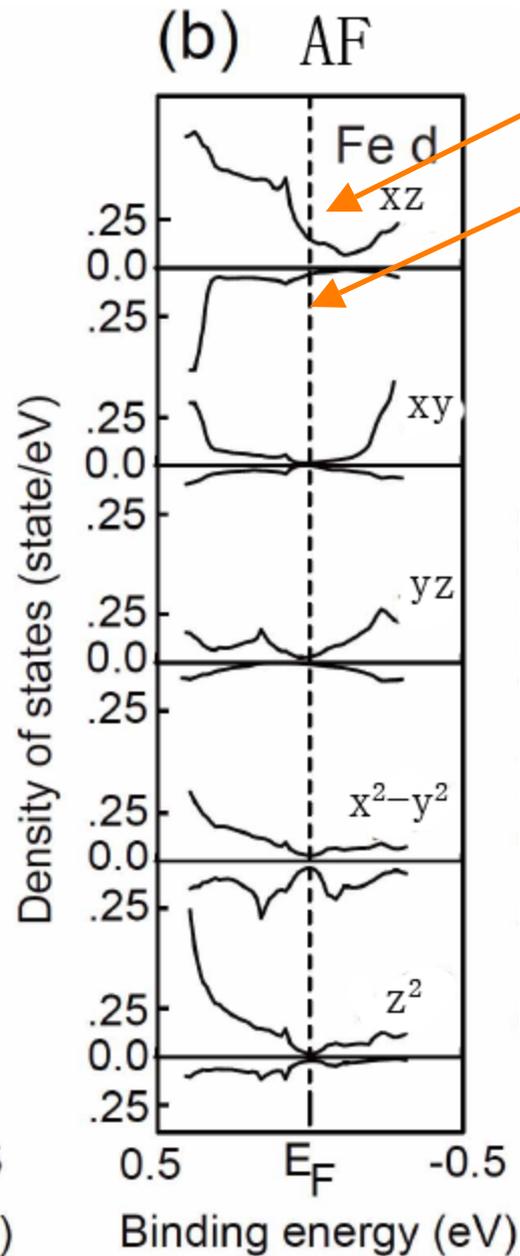
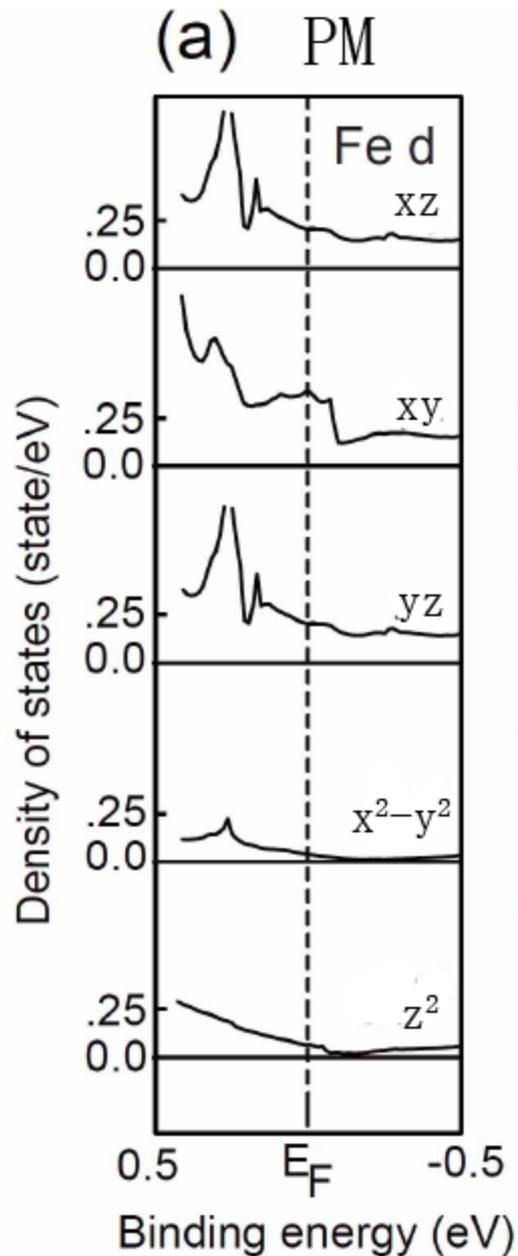
BaFe₂As₂

• Polarized ARPES



d_{xz} in domain A

d_{xz} in domain B



(a) PM state:

1. d_{xz} , d_{yz} degenerate
2. Fermi surface is composed by multiple orbitals

(b) AF state:

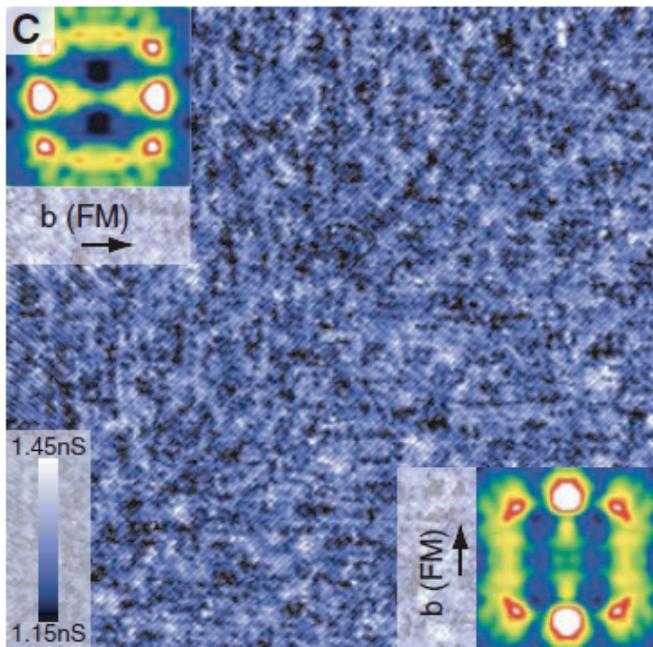
1. Localized moment is formed by d_{xz}
2. Fermi surface is orbital-polarized

OO-induced anisotropies

OO-induced anisotropies

T.-M. Chuang, *et al.*
Science (2010)

FM easy axis!

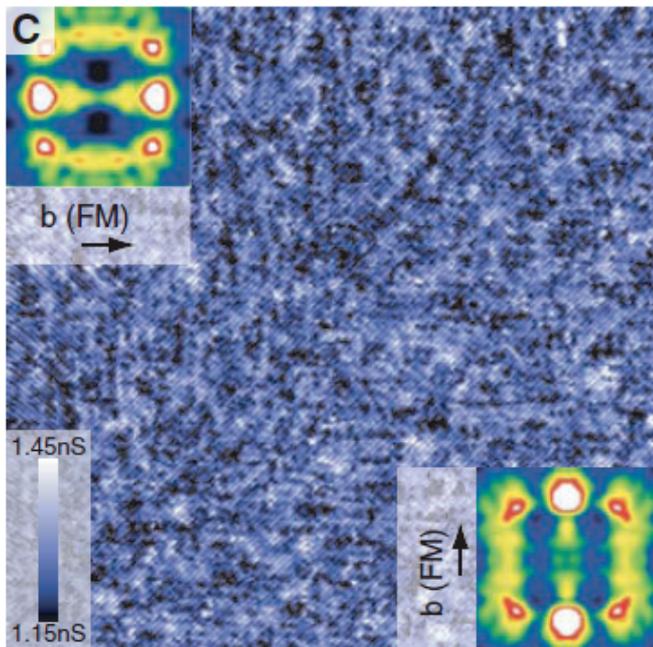


OO-induced anisotropies

T.-M. Chuang, *et al.*
Science (2010)

• *STM*

FM easy axis!



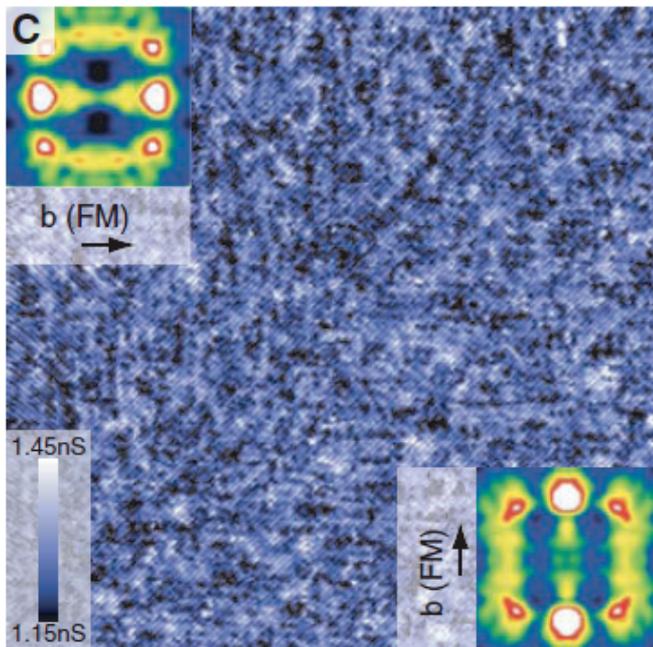
OO-induced anisotropies

AF is easy axis!

T.-M. Chuang, *et al.*
Science (2010)

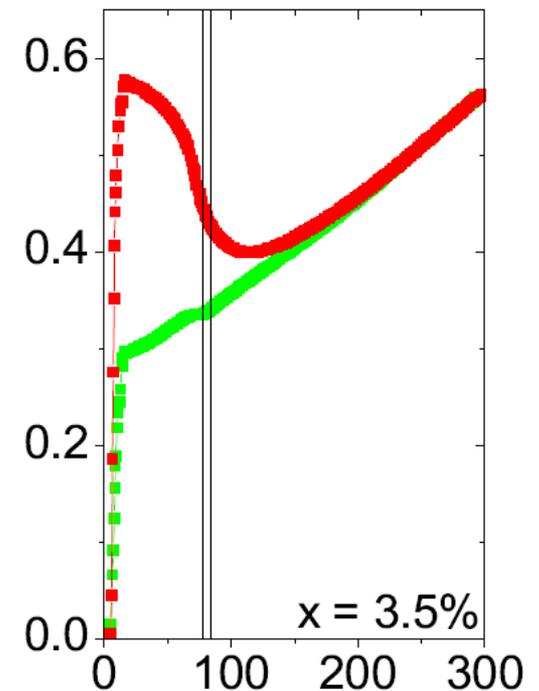
• *STM*

FM easy axis!



• *Resistivity*

J.-H. Chu, *et al.*
arXiv:1002.3364



Tetragonal : Even occupancy of two degenerate orbitals d_{xz} and d_{yz}

Jahn-Teller Effect (orbital ordering)

Orthorhombic : Break the degeneracy of two orbitals d_{xz} and d_{yz} , occupy only one orbital

A gap open between d_{xz} and d_{yz}

Resistivity Anomaly

$$T > T_{SPT}$$

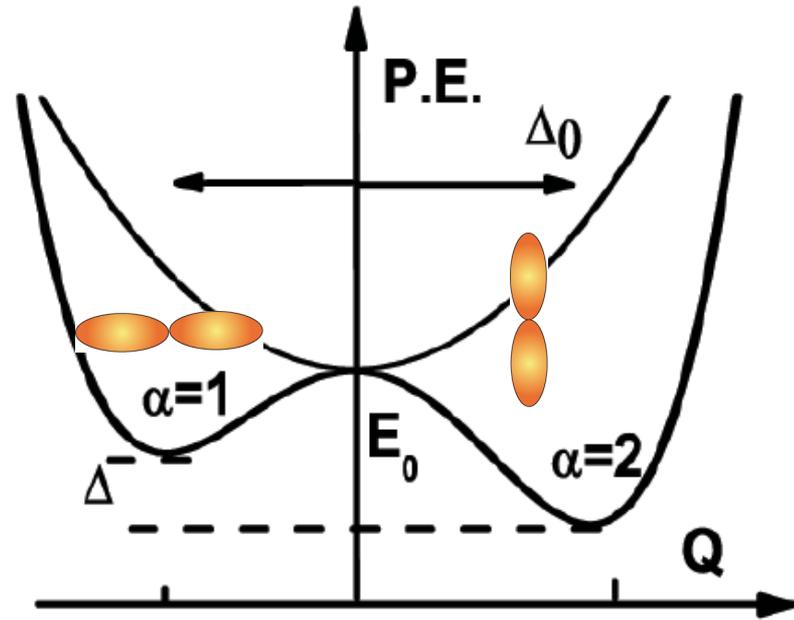
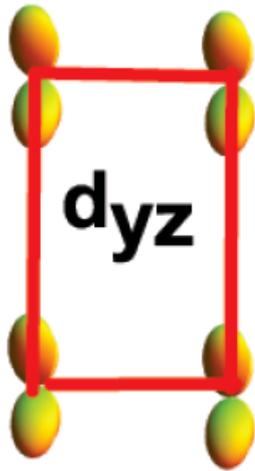
d_{xz}

d_{yz}



two-level system:
Kondo-like problem

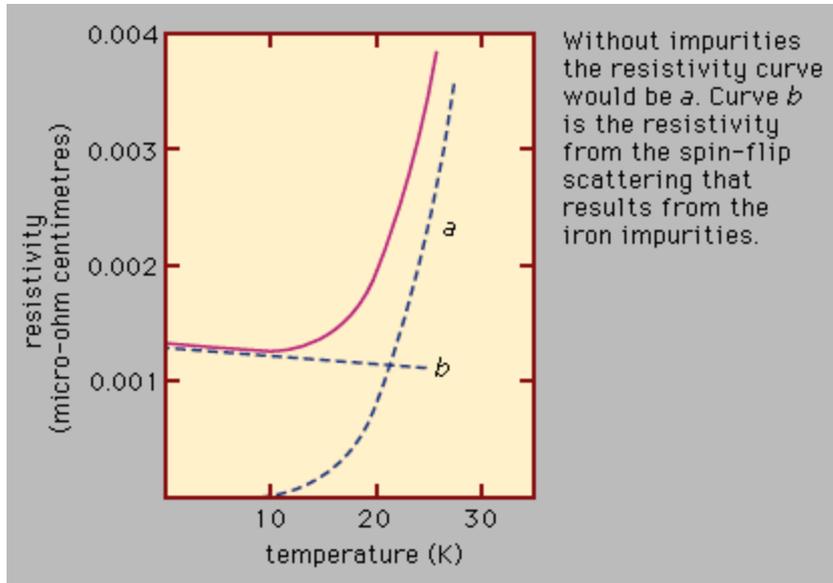
A Two-level System



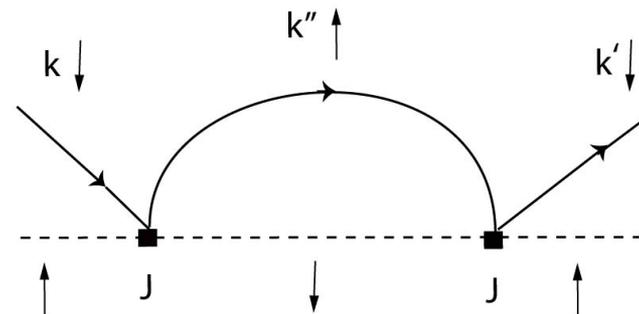
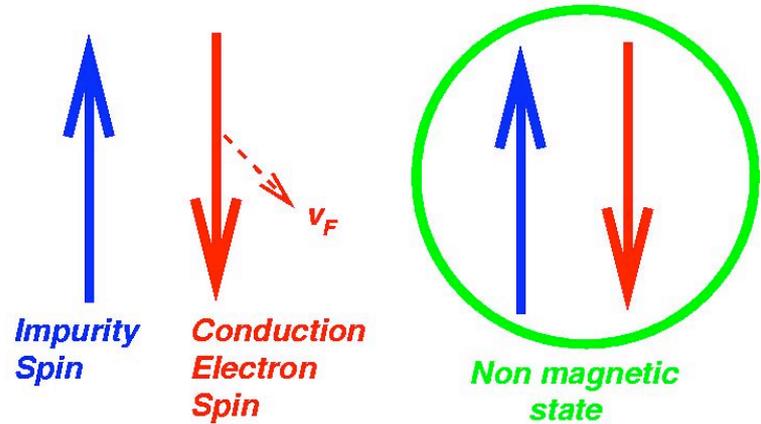
$$H_{\text{TLS}} = \lambda_{ps} \sum_{\alpha} a_{\alpha}^{\dagger} a_{\alpha} + \frac{1}{2} \Delta \sum_{\alpha\beta} a_{\alpha}^{\dagger} \sigma_{\alpha\beta}^z a_{\beta} + \frac{1}{2} \Delta_0 \sum_{\alpha\beta} a_{\alpha}^{\dagger} \sigma_{\alpha\beta}^x a_{\beta}$$

$$E = \sqrt{\Delta_0^2 + \Delta^2}$$

Kondo effect

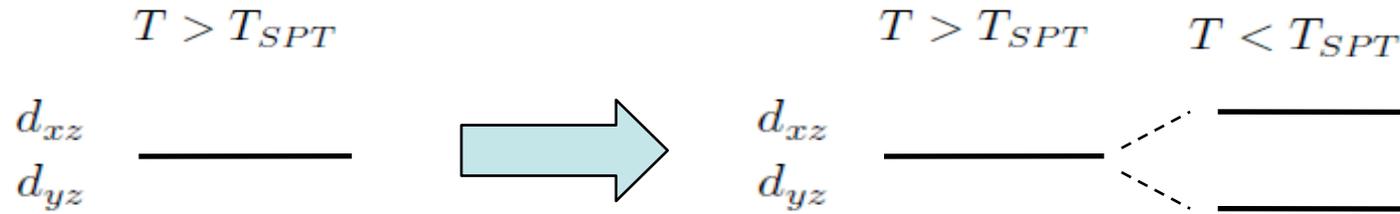


High T - weak coupling Low T - strong coupling

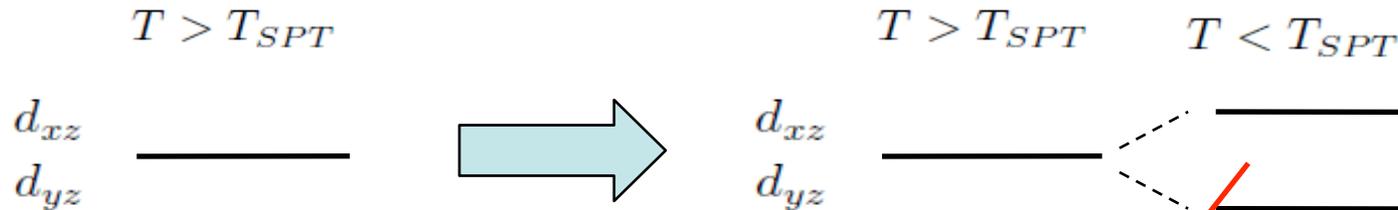


Resistivity anomaly

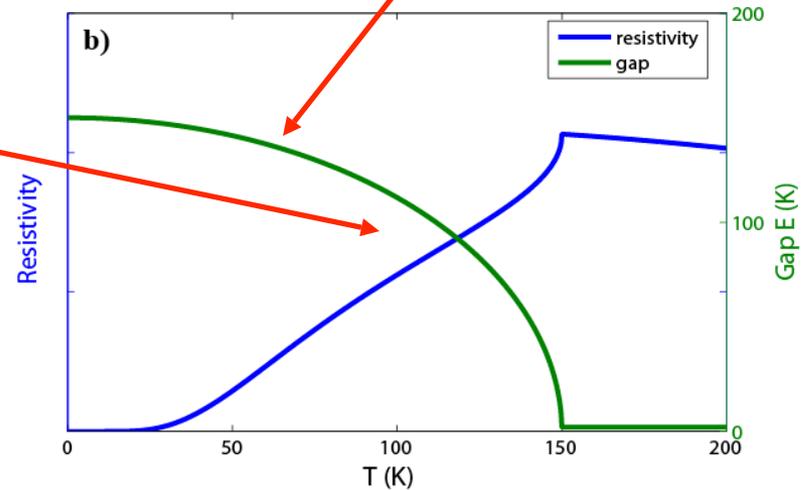
Resistivity anomaly



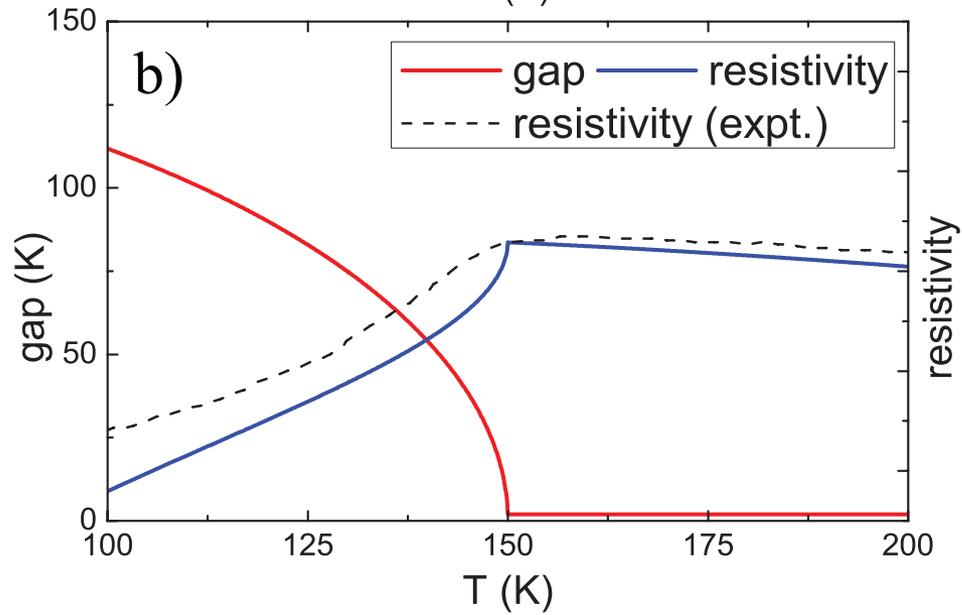
Resistivity anomaly



Opening of the gap makes scattering difficult; Resistivity decreases



Resistivity Anomaly



Conclusion I:

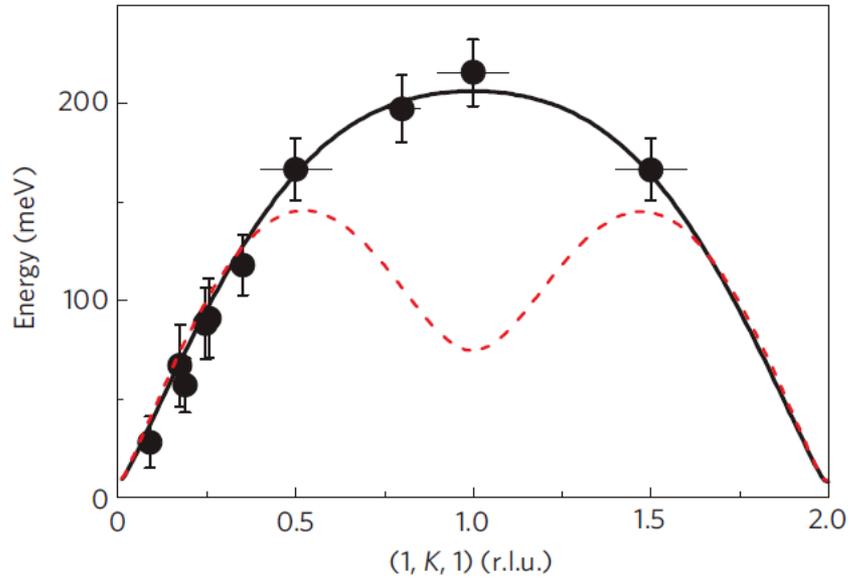
Orbital-ordering

```
graph TD; A[Orbital-ordering] --> B[Structural Phase Transition]; B --> C[Resistivity anomaly];
```

Structural Phase Transition

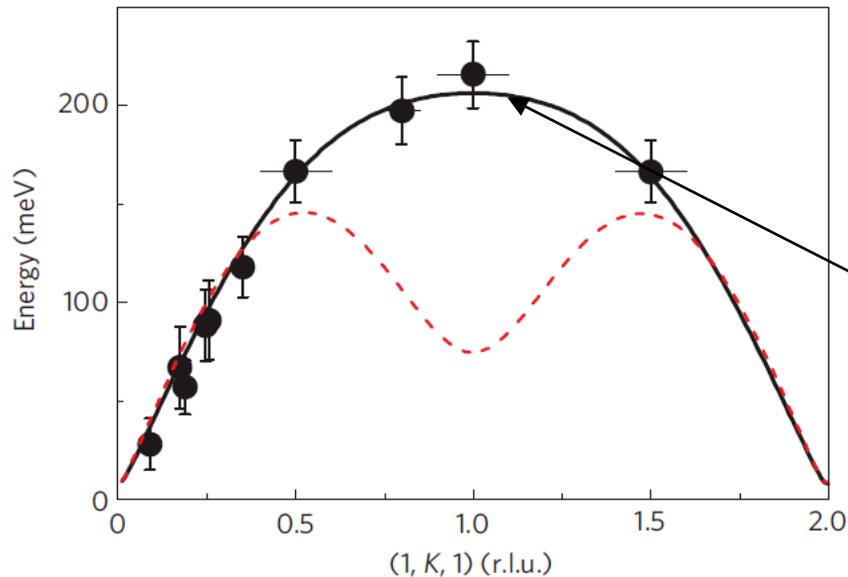
Resistivity anomaly

spin-wave spectrum



origin of ferromagnetic coupling?

spin-wave spectrum



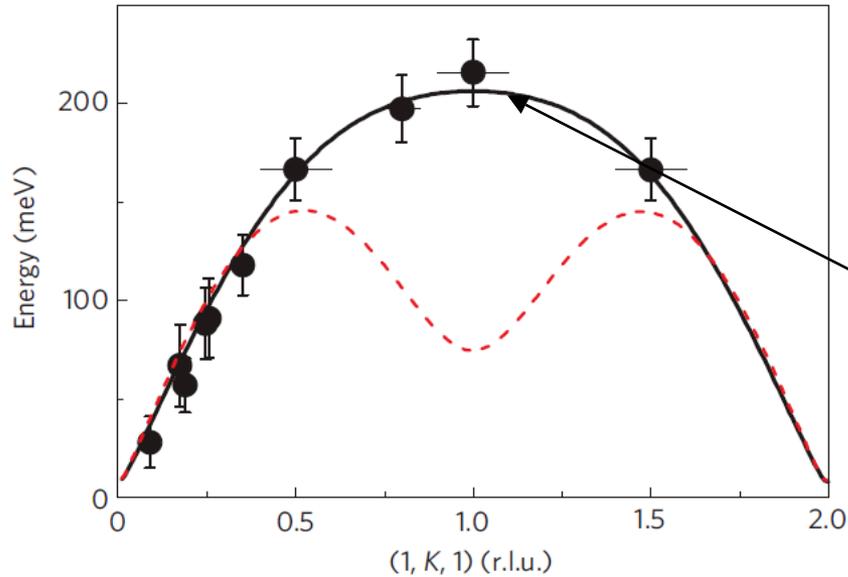
$$SJ_1^a = 49.9 \pm 9.9 \text{ meV}$$

$$SJ_1^b = -5.7 \pm 4.5 \text{ meV}$$

$$SJ_2 = 18.9 \pm 3.4 \text{ meV}$$

origin of ferromagnetic coupling?

spin-wave spectrum



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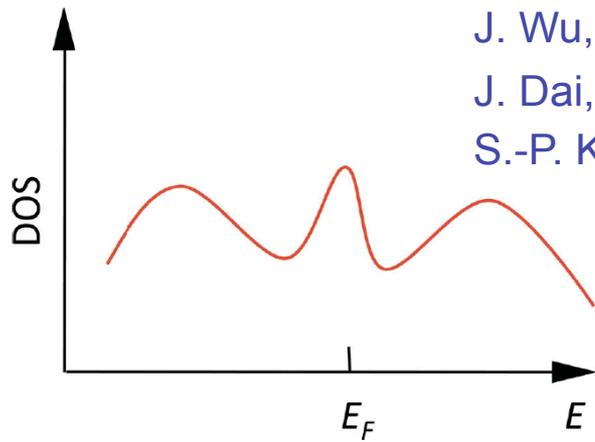
origin of ferromagnetic coupling?

- *A local-itinerant model*

J. Wu, *et al.* PRL (2008)

J. Dai, *et al.* PNAS (2009)

S.-P. Kou, *et al.* EPL (2009)

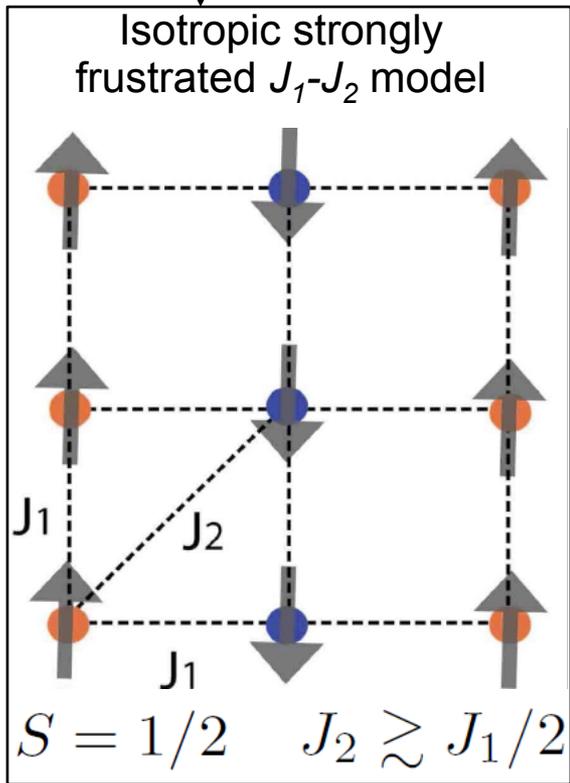
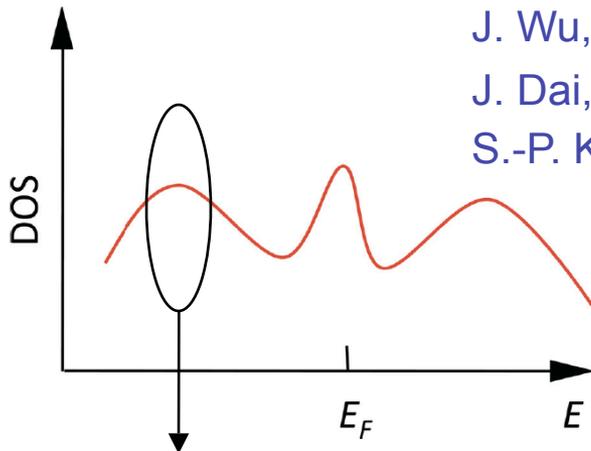


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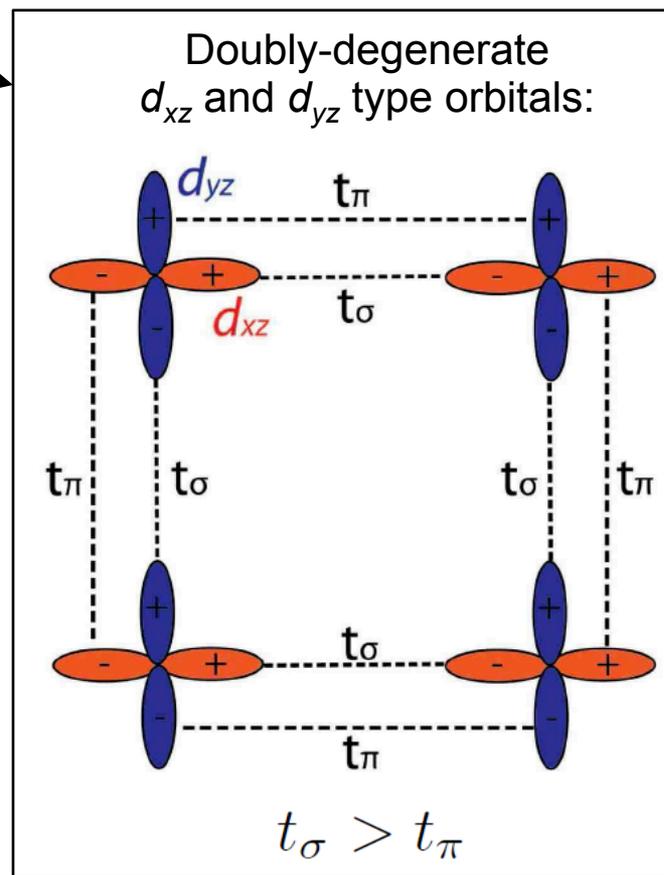
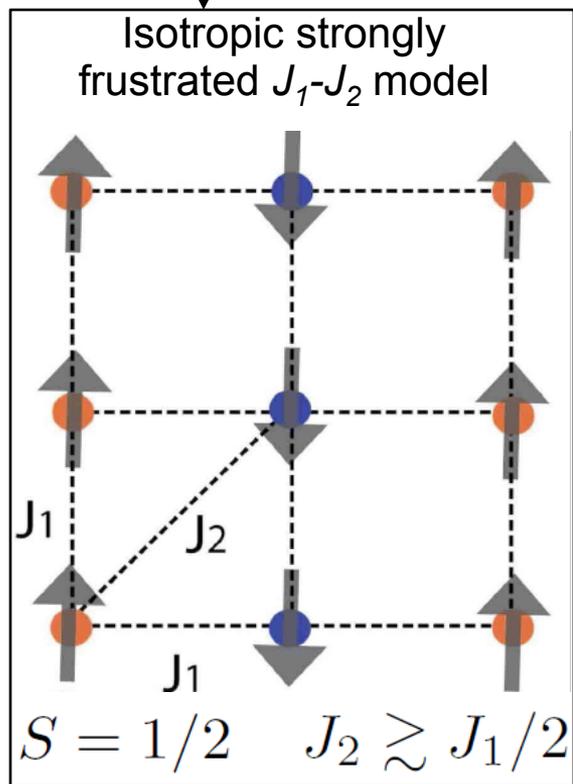
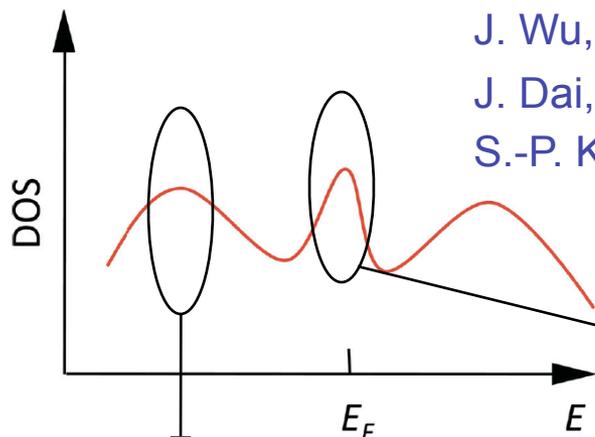


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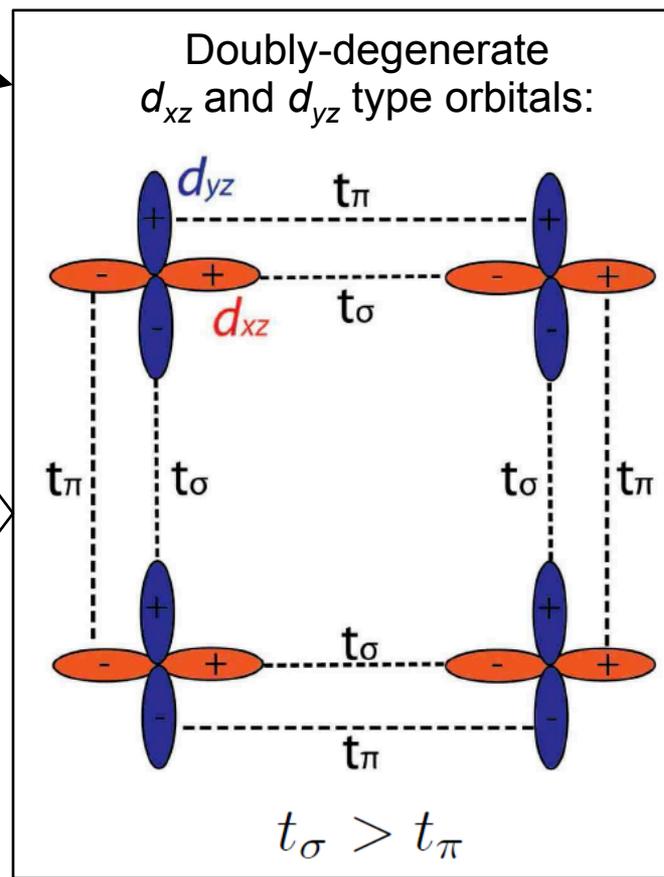
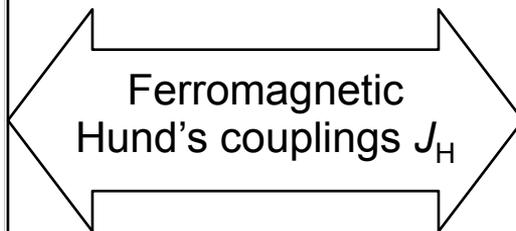
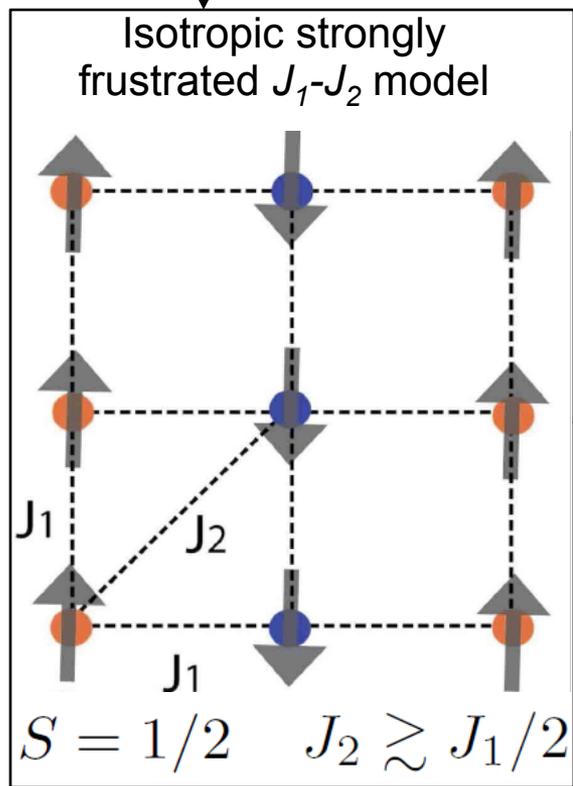
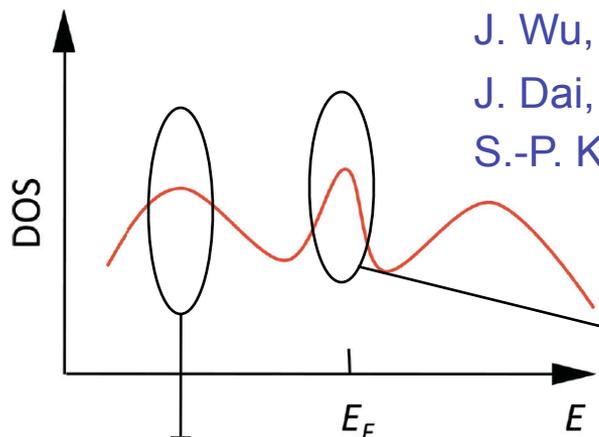


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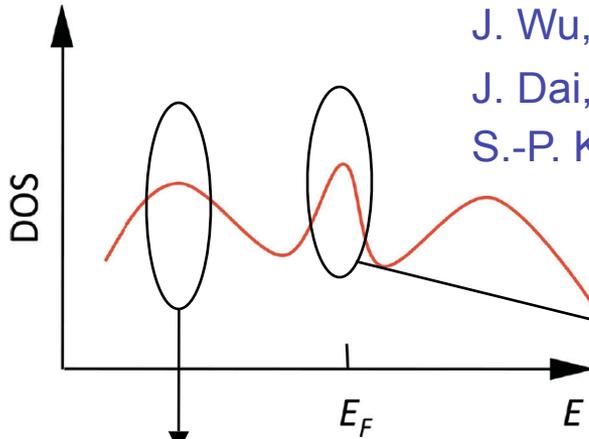
J. Dai, *et al.* PNAS (2009)

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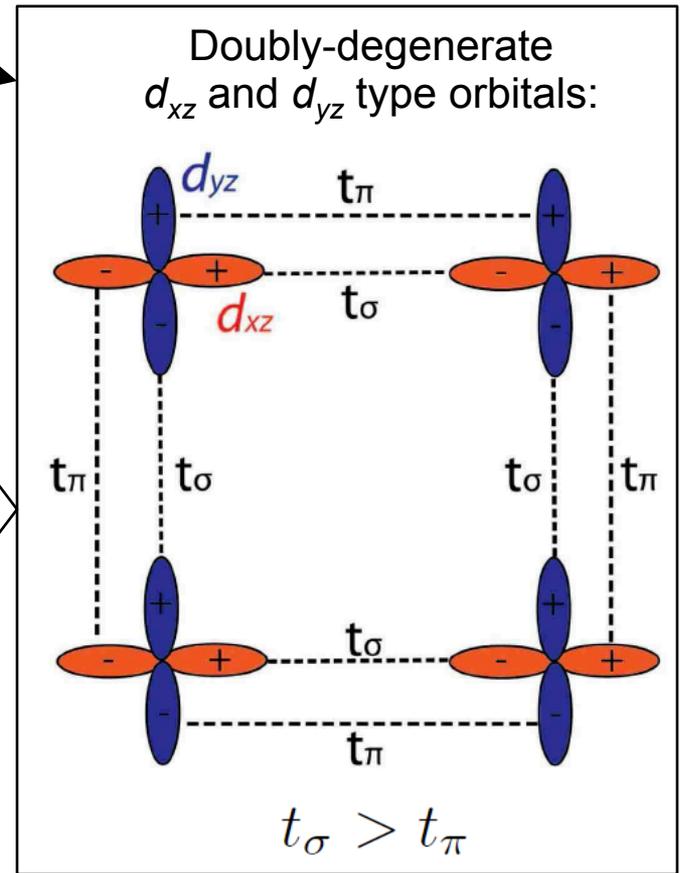
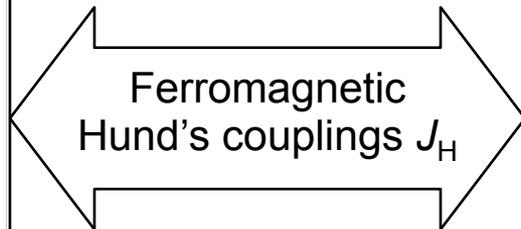
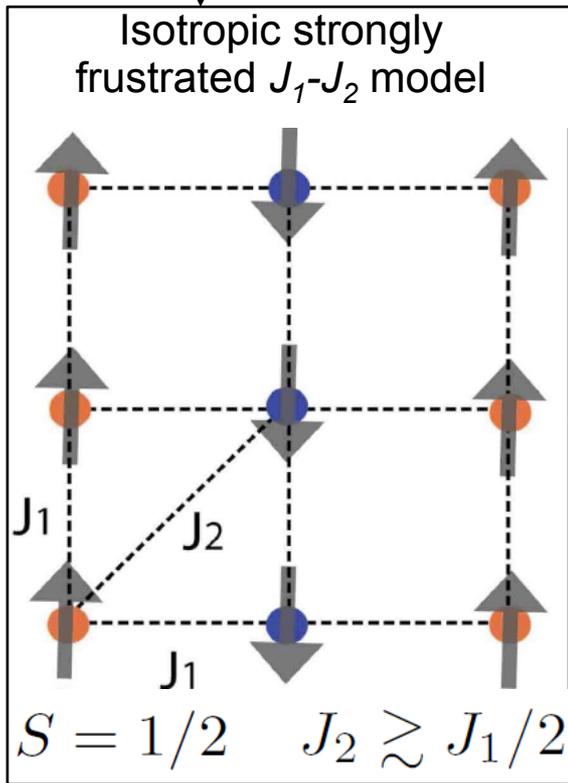
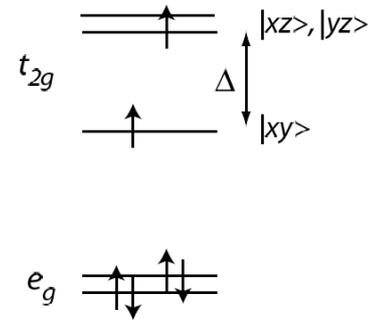


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 J. Dai, et al. PNAS (2009)
 S.-P. Kou, et al. EPL (2009)



consistent with the local multiplet structure:



Double-Exchange Model

$$\mathcal{H}_{\text{loc}} = \frac{J_1}{S^2} \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \frac{J_2}{S^2} \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

unlike manganites

$$\nu J_H \ll \infty$$

Double-Exchange Model

$$\mathcal{H}_{\text{loc}} = \frac{J_1}{S^2} \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \frac{J_2}{S^2} \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$\mathcal{H}_{\text{it}} = - \sum_{ij, \alpha\beta, \nu} t_{ij}^{\alpha\beta} c_{i\alpha\nu}^\dagger c_{j\beta\nu} + \frac{V}{2} \sum_{i, \alpha \neq \beta, \nu\nu'} \hat{n}_{i\alpha\nu} \hat{n}_{i\beta\nu'}$$

$$\mathcal{H}_{\text{H}}^{(0)} = -\frac{J_{\text{H}}}{2} \sum_{k, \alpha, \nu} \nu \tilde{c}_{k\alpha\nu}^\dagger \tilde{c}_{k\alpha\nu}$$

unlike manganites

$$\nu J_{\text{H}} \ll \infty$$

• *local moments:*

$$\mathcal{H}_{\text{loc}} = \frac{J_1}{S^2} \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \frac{J_2}{S^2} \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

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sublattice rotations



Holstein-Primakoff bosons

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sublattice rotations

Holstein-Primakoff bosons

$$\mathcal{H}_{\text{loc}}^{\text{SW}} = \sum_q \left[A_0(q) \left(a_q^\dagger a_q + a_{-q} a_{-q}^\dagger \right) + B_0(q) \left(a_q^\dagger a_{-q}^\dagger + a_{-q} a_q \right) \right]$$

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sublattice rotations

mean-field decoupling

$$\begin{aligned} \mathcal{H}_e &= \mathcal{H}_{\text{it}}^{\text{mf}} + \mathcal{H}_{\text{H}}^{(0)} \\ &= \sum_{k, \alpha, \nu} \left[\left(\varepsilon_1^\alpha(k) - \nu \frac{J_{\text{H}}}{2} + V \rho_{\bar{\alpha}} \right) \tilde{c}_{k\alpha\nu}^\dagger \tilde{c}_{k\alpha\nu} + \varepsilon_2^\alpha(k) \tilde{c}_{k\alpha\nu}^\dagger \tilde{c}_{k\alpha\bar{\nu}} \right] \end{aligned}$$

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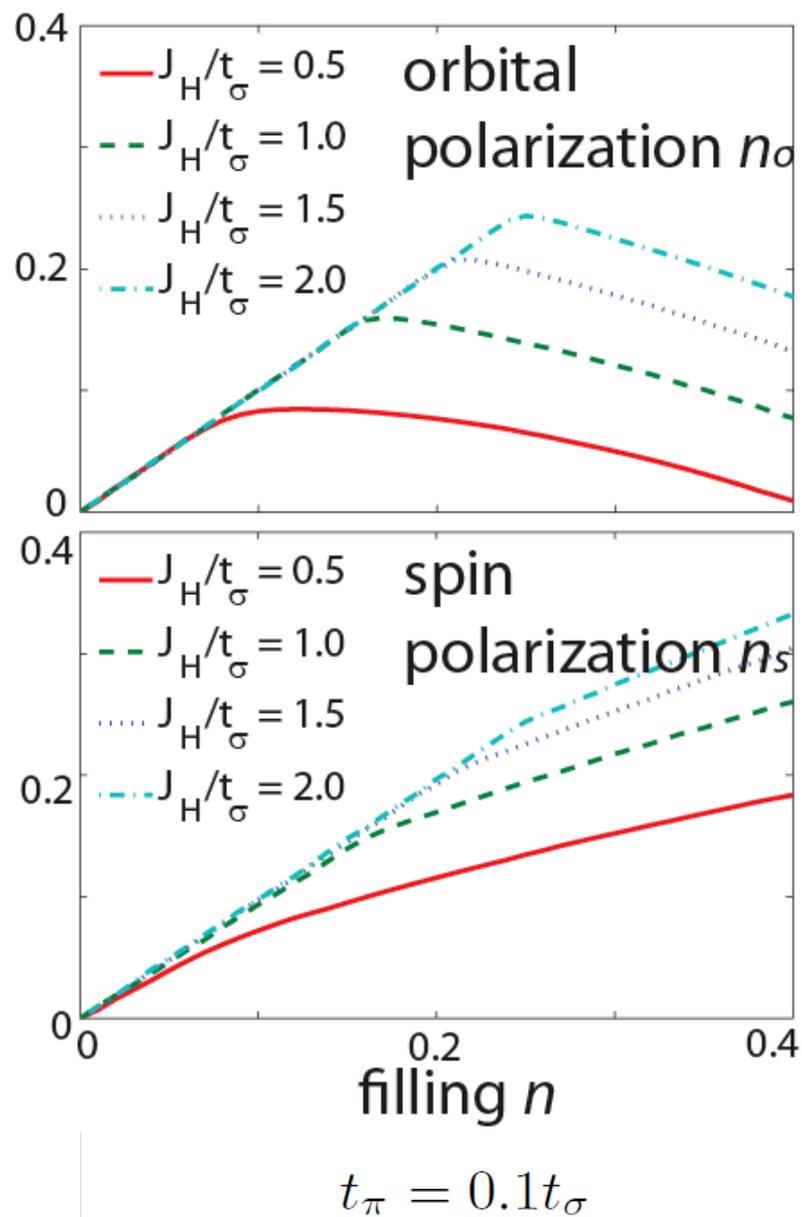
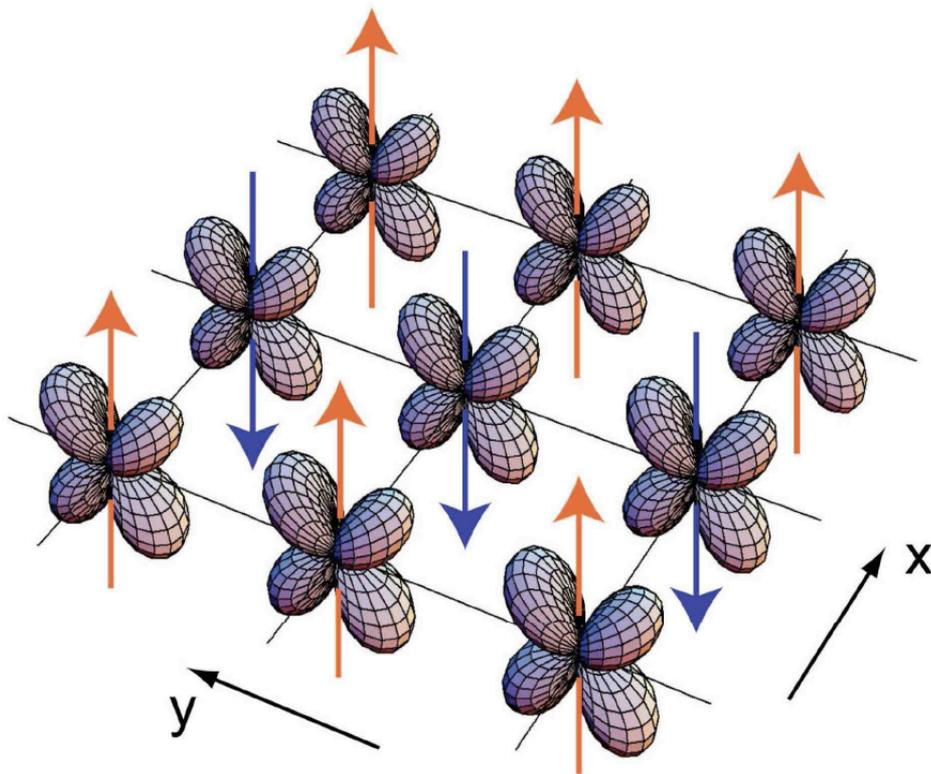
$$\mathcal{H}_{\text{H}}^{(0)} = -\frac{J_{\text{H}}}{2} \sum_{k, \alpha, \nu} \nu \tilde{c}_{k\alpha\nu}^\dagger \tilde{c}_{k\alpha\nu}$$

$$= \sum_{k, \alpha, \nu} \left[\left(\varepsilon_1^\alpha(k) - \nu \frac{J_{\text{H}}}{2} + V \rho_{\bar{\alpha}} \right) \tilde{c}_{k\alpha\nu}^\dagger \tilde{c}_{k\alpha\nu} + \varepsilon_2^\alpha(k) \tilde{c}_{k\alpha\nu}^\dagger \tilde{c}_{k\alpha\bar{\nu}} \right]$$

- Orbital ordering

$$n_o = \sum_{\nu} (\rho_{yz,\nu} - \rho_{xz,\nu})$$

$$n_s = \sum_{\alpha} (\rho_{\alpha,\uparrow} - \rho_{\alpha,\downarrow})$$



- *Spin-wave dispersion with both super- and double-exchange*

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Eliminating terms linear in a_q , we require: $[\Delta, \mathcal{H}_e] + \mathcal{H}_H^{(1)} = 0$

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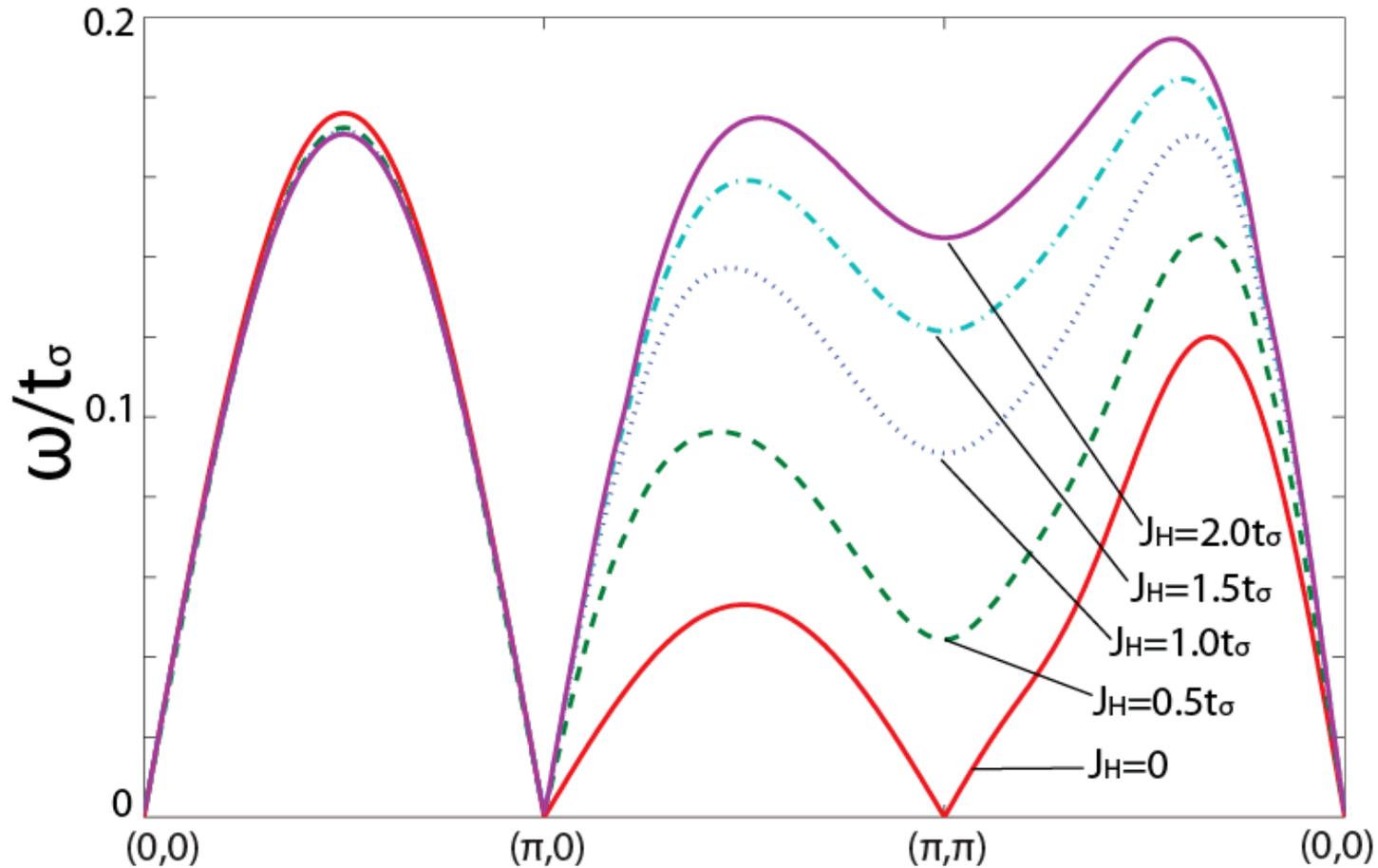
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Spin-wave dispersions: $\omega(q) = \sqrt{A^2(q) - B^2(q)}$

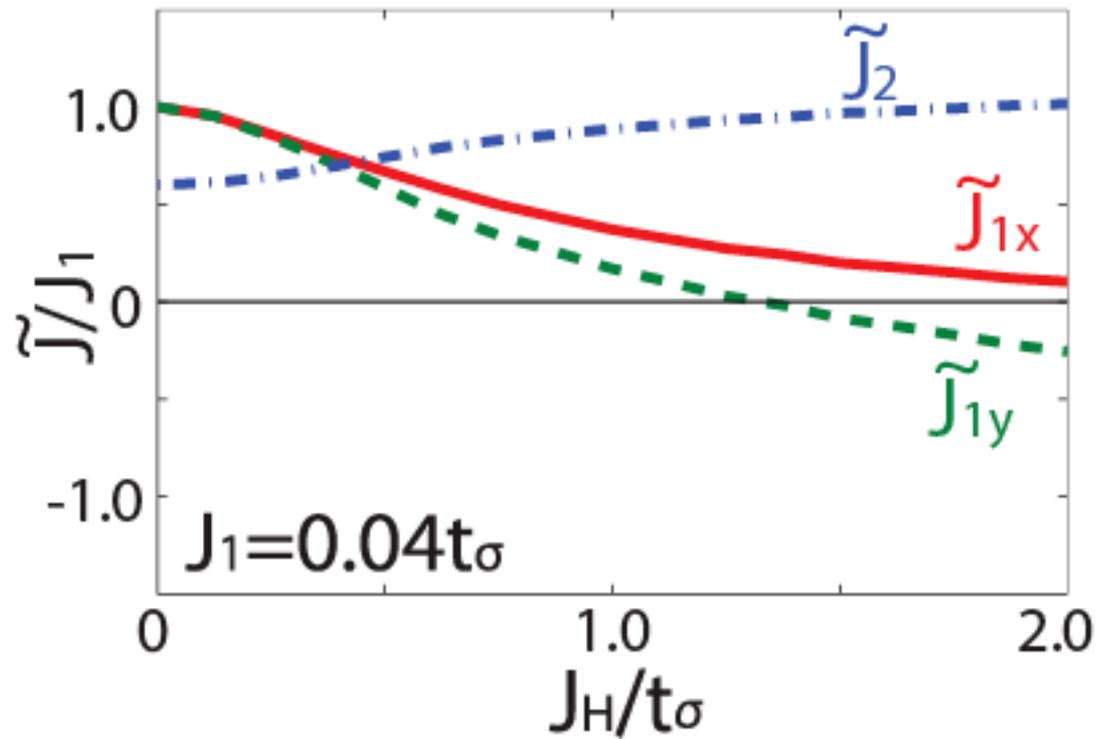
- *Spin-wave dispersion with both super- and double-exchange*



$$J_1 = 0.04t_\sigma, J_2 = 0.6J_1$$

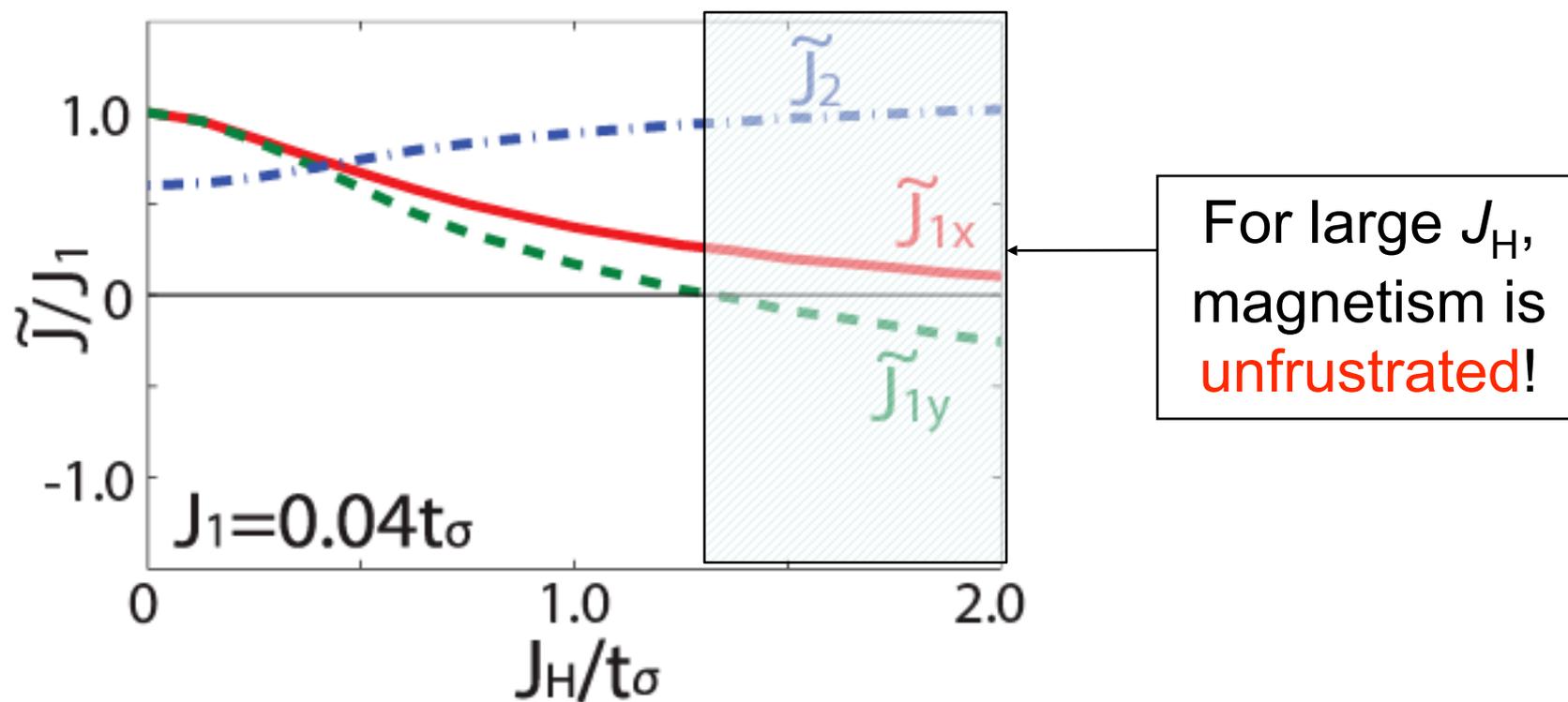
$$n = 0.1, t_\pi = 0.1t_\sigma$$

- *Fitting to an anisotropic Heisenberg model:*



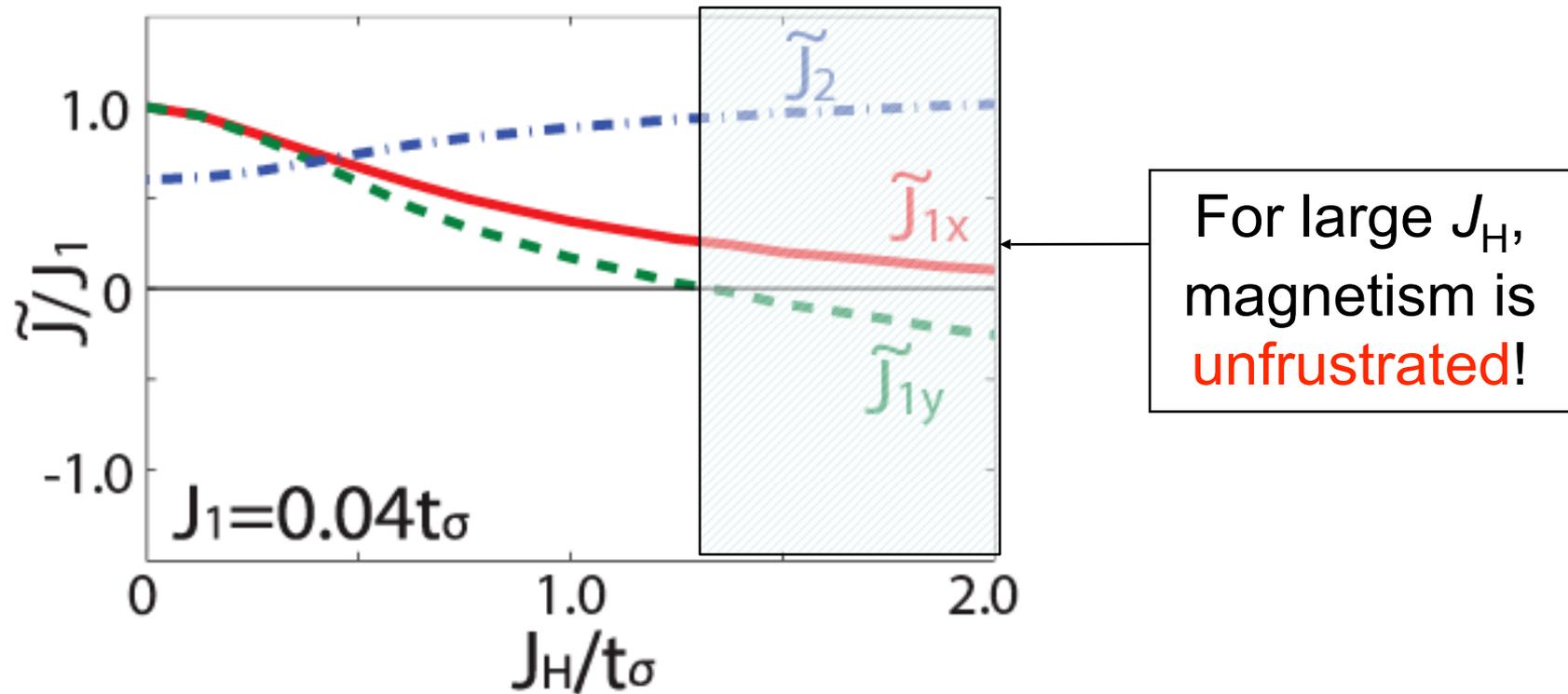
emergent ferro-orbital order

- *Fitting to an anisotropic Heisenberg model:*



emergent ferro-orbital order

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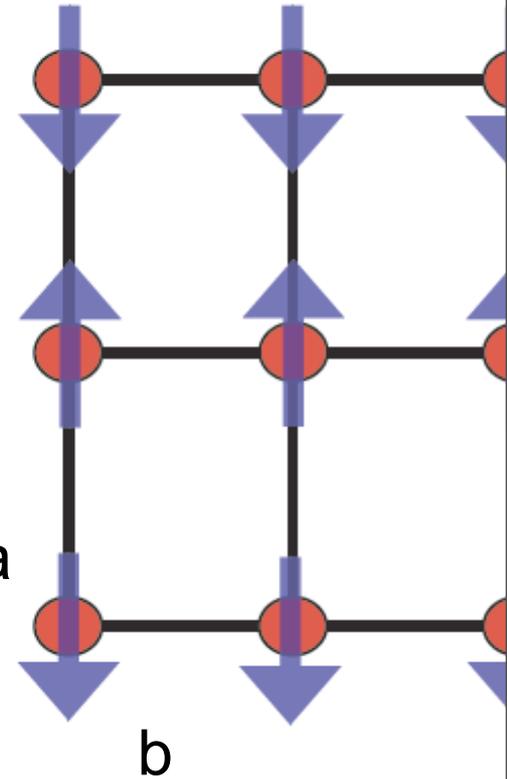
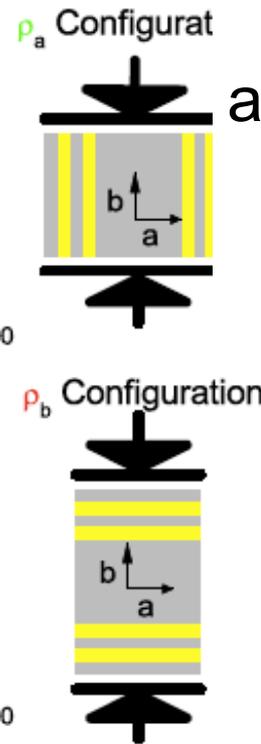
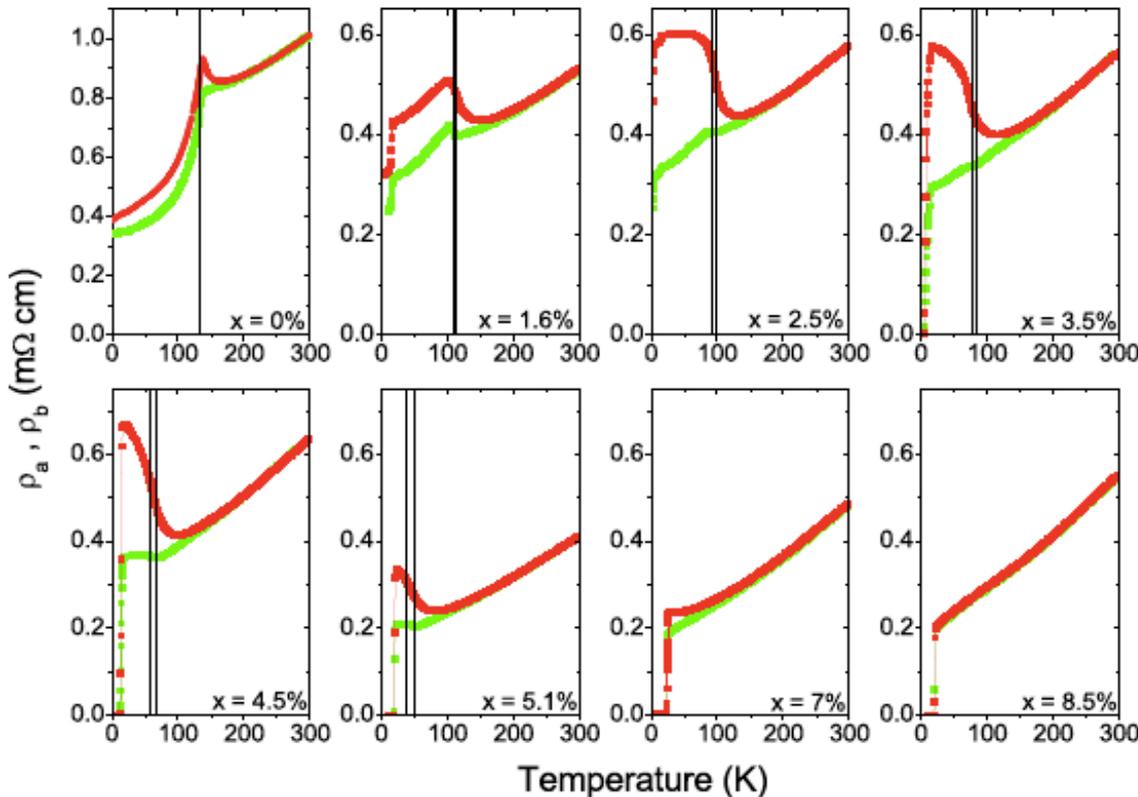
- *Comparison with experiments:*

$$J_H \sim t_\sigma \sim 1 \text{ eV} \quad \tilde{J} \sim 0.01 t_\sigma \sim 10 \text{ meV}$$

emergent ferro-orbital order

Remaining issues

1.) Expt. AF direction is the easy axis

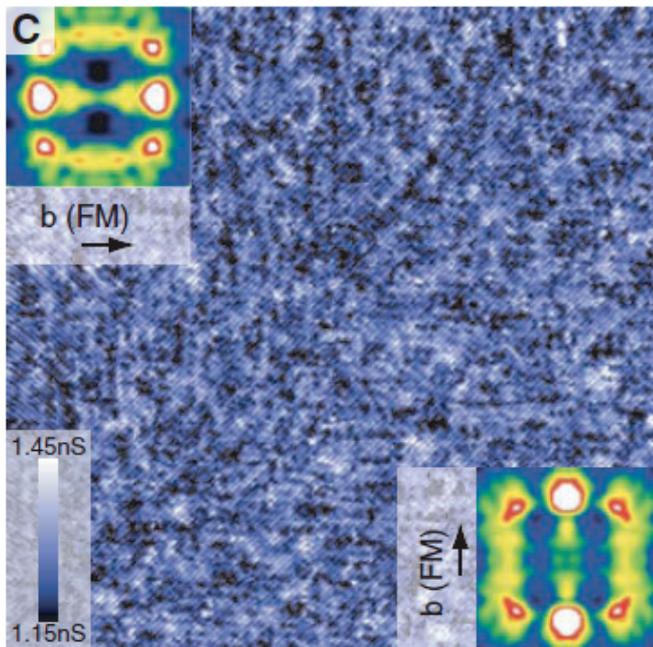


OO-induced anisotropies

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T.-M. Chuang, *et al.*
Science (2010)

FM easy axis!

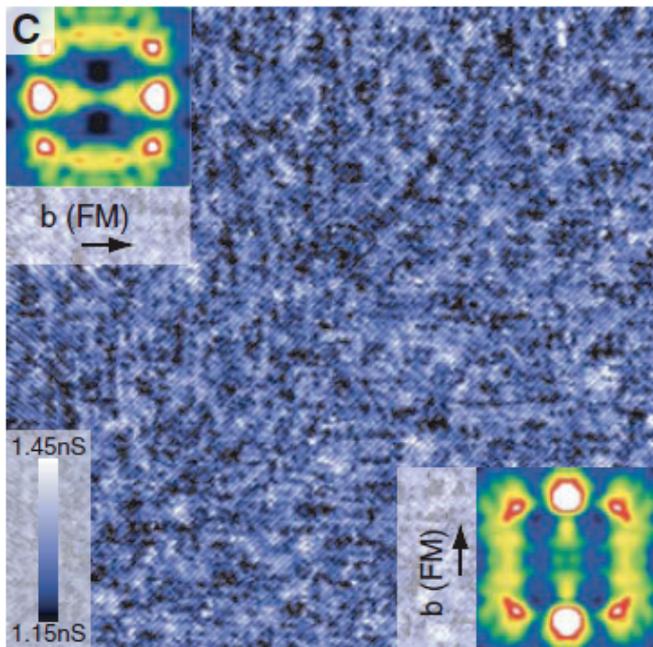


OO-induced anisotropies

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Science (2010)

• *STM*

FM easy axis!



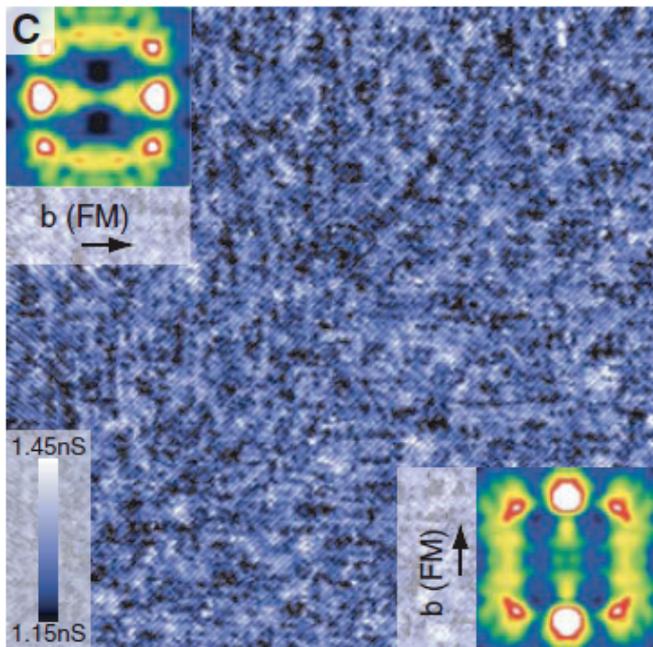
OO-induced anisotropies

AF is easy axis!

T.-M. Chuang, *et al.*
Science (2010)

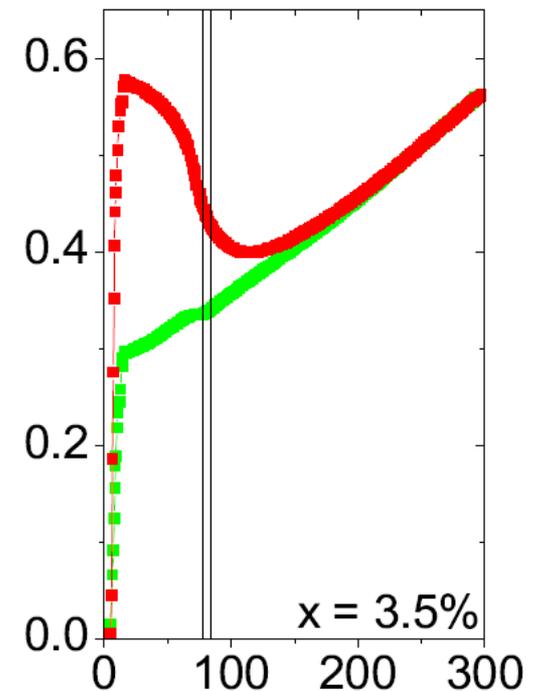
• *STM*

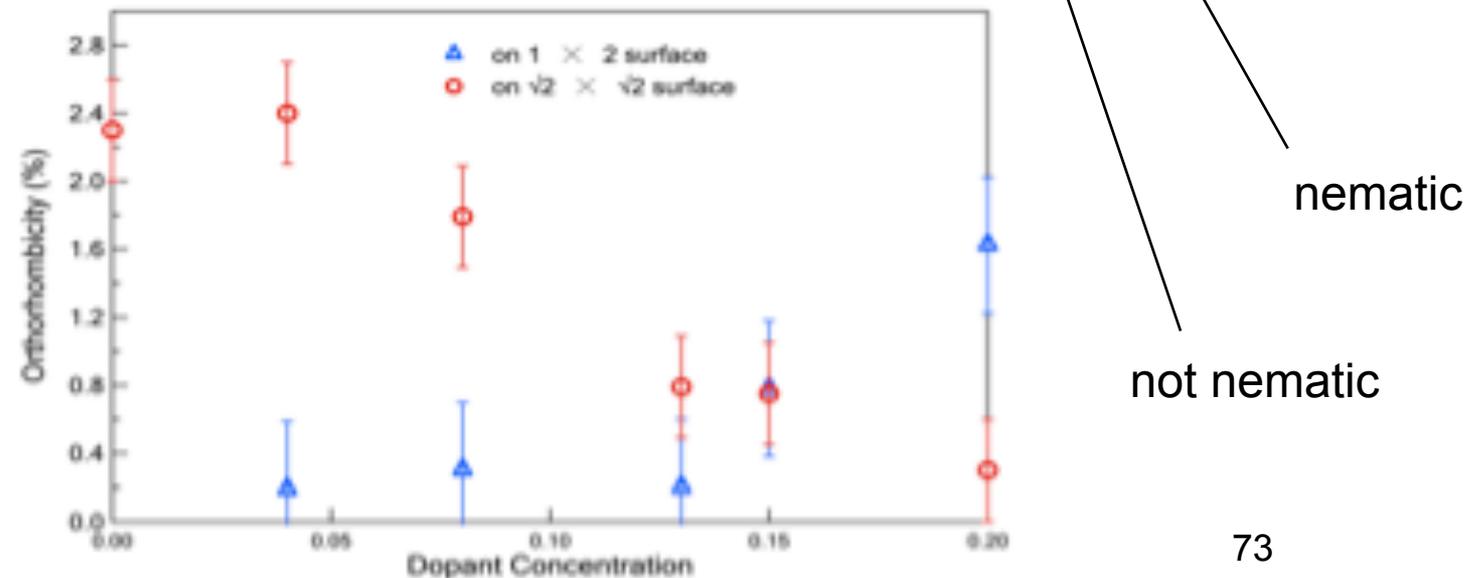
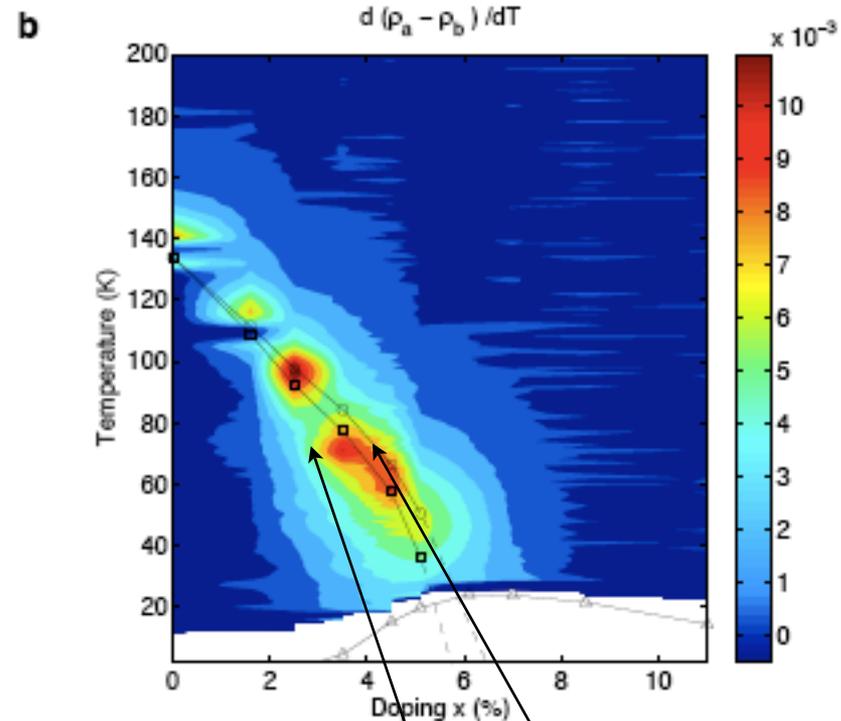
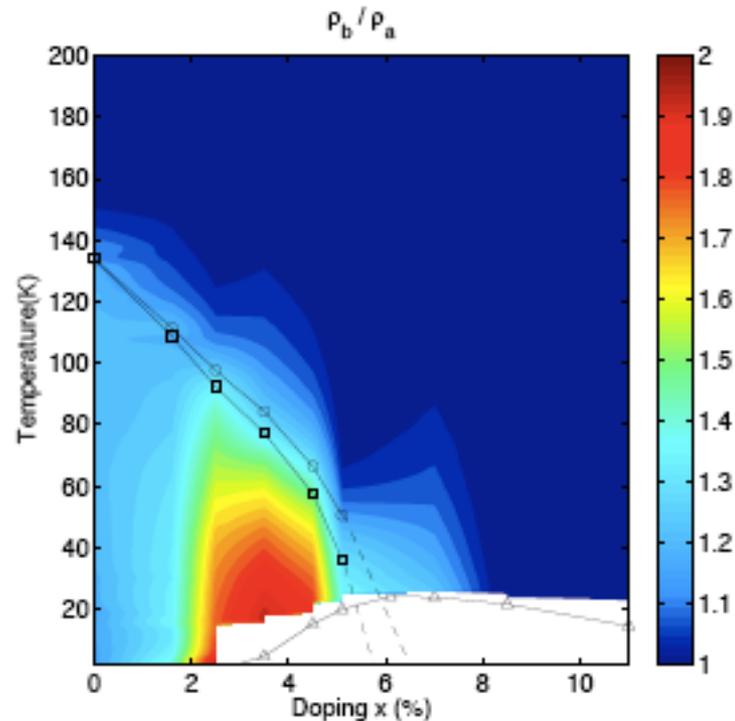
FM easy axis!



• *Resistivity*

J.-H. Chu, *et al.*
arXiv:1002.3364



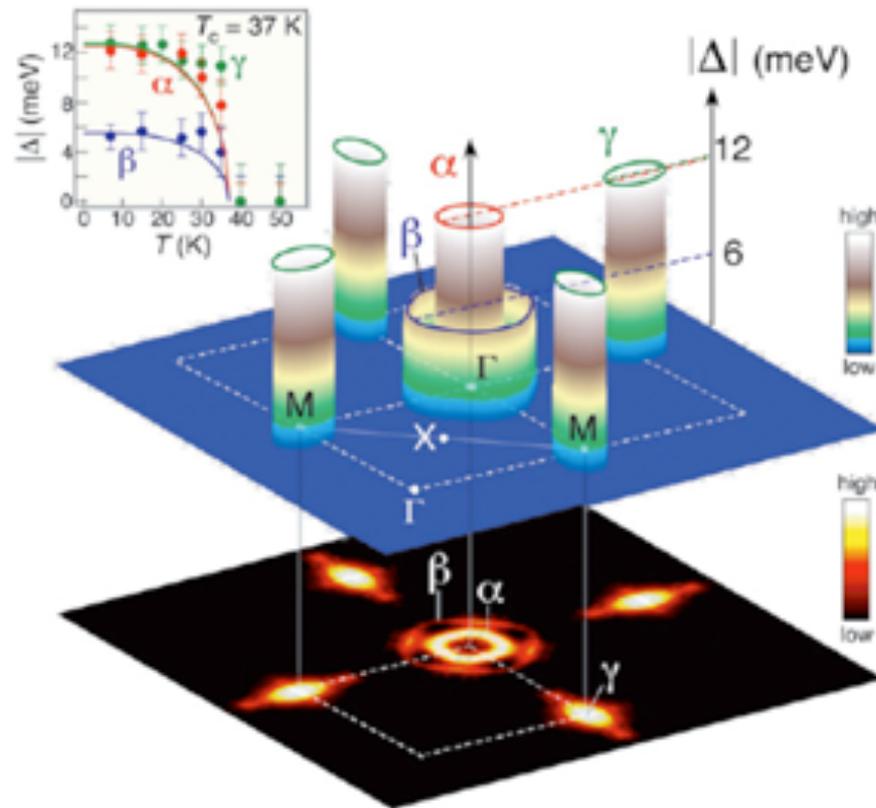


Pairing Mechanism?

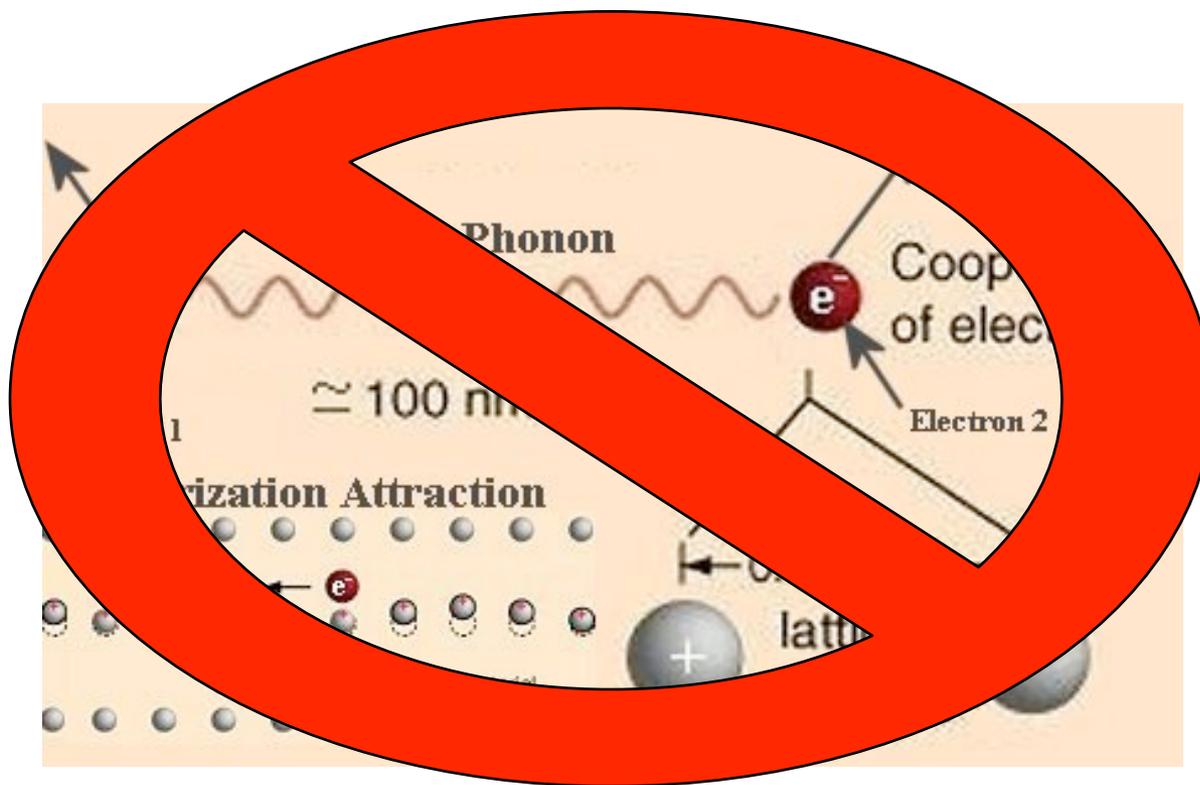
What is the pairing mechanism?

J. Wu, P. Phillips, arXiv:0901.3538

Fermi surface sheets



Pairing Mechanism?

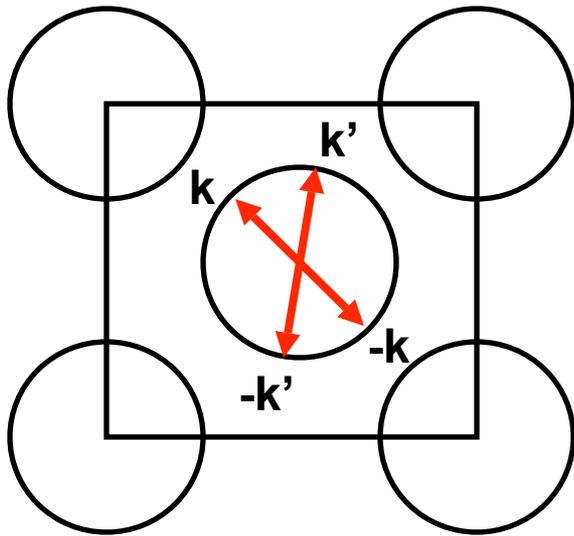


magnons not phonons

Magnon-induced interaction

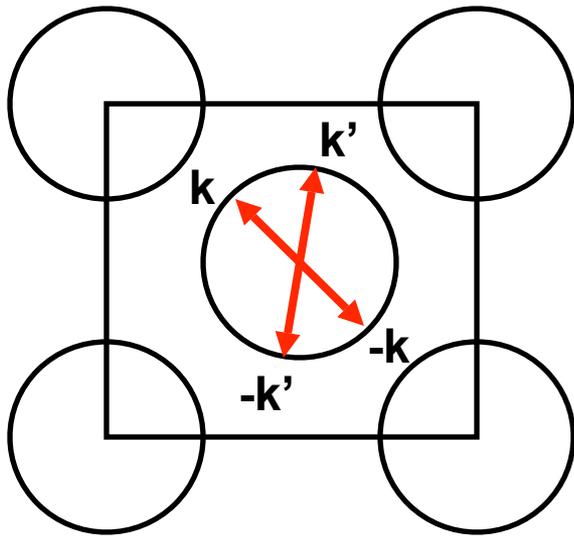
Magnon-induced interaction

Intra-band interaction

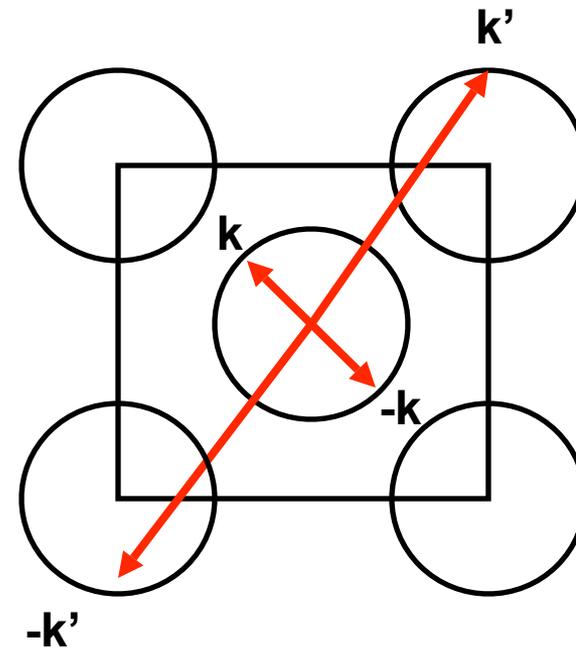


Magnon-induced interaction

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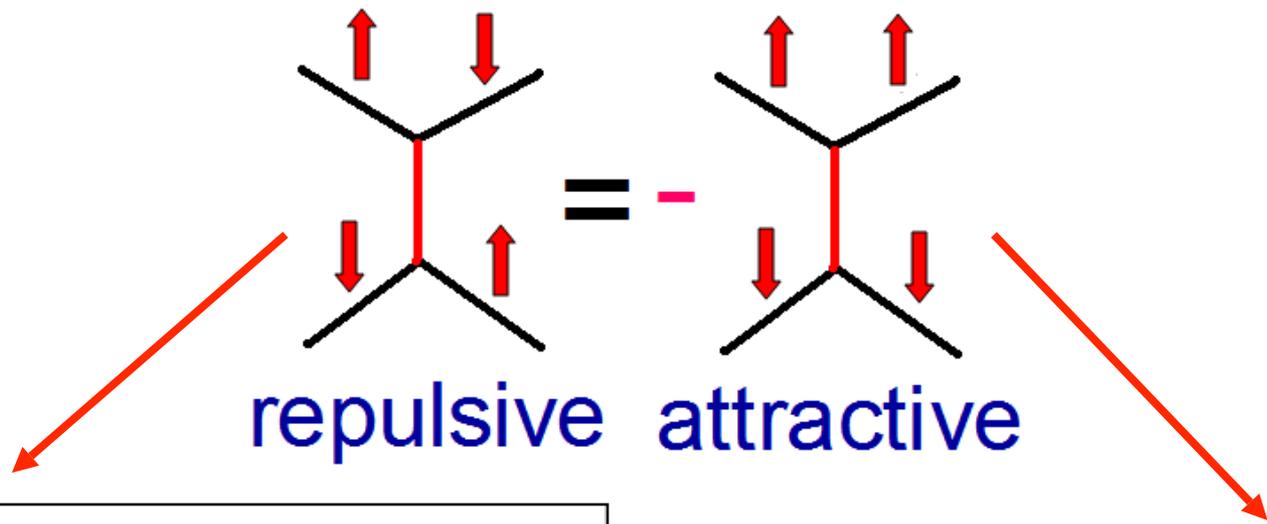


Inter-band interaction



Properties of magnon-mediated interaction

1) Both repulsive (due to spin-flip);



$$\begin{aligned}
 & V(\mathbf{q}) a_{\mathbf{k}_1+\mathbf{q},\uparrow}^\dagger a_{\mathbf{k}_2-\mathbf{q},\downarrow}^\dagger a_{\mathbf{k}_1,\downarrow} a_{\mathbf{k}_2,\uparrow} \\
 = - & V(\mathbf{q}) a_{\mathbf{k}_1+\mathbf{q},\uparrow}^\dagger a_{\mathbf{k}_2-\mathbf{q},\downarrow}^\dagger a_{\mathbf{k}_2,\uparrow} a_{\mathbf{k}_1,\downarrow}
 \end{aligned}$$

$$V(\mathbf{q}) a_{\mathbf{k}_1+\mathbf{q},\uparrow}^\dagger a_{\mathbf{k}_2-\mathbf{q},\downarrow}^\dagger a_{\mathbf{k}_1,\uparrow} a_{\mathbf{k}_2,\downarrow}$$

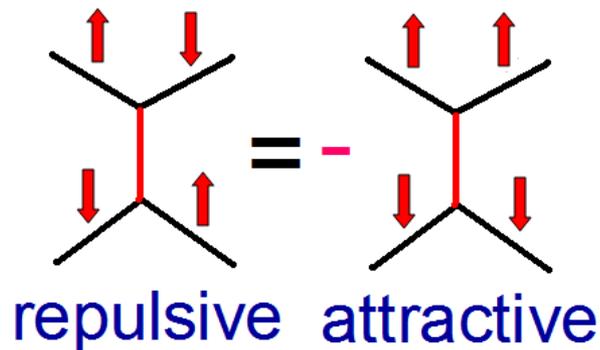
Interpretation I: pairing from repulsive interaction

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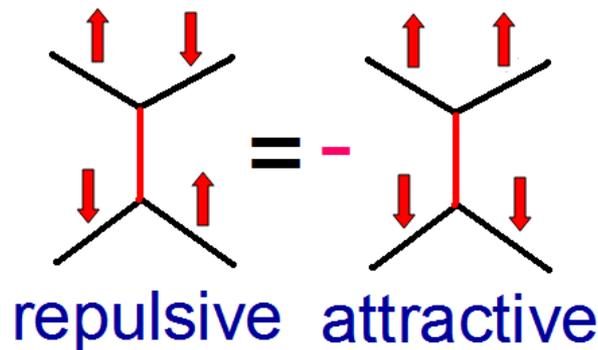
Propagating one magnon



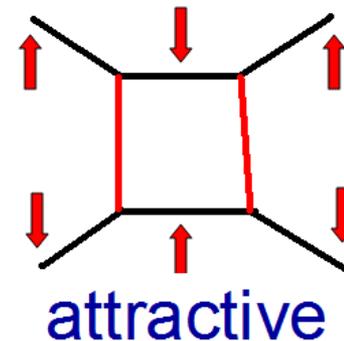
Interpretation I: pairing from repulsive interaction

1) 2 inter-band scattering bring attractive interaction for each band;

Propagating one magnon



Propagating two magnons



J. Appel and A. W. Overhauser
Physica B 199 & 200, 310(1994)

Properties of magnon-mediated interaction

2) Inter-band \gg intra-band

$$V_{ii}(\mathbf{k}, \mathbf{k}') = \frac{(J^2) |f_{\nu, \nu'}(\mathbf{k}, \mathbf{k}')|^2}{qF(\alpha)}$$

$$V_{eh}(\mathbf{k}, \mathbf{k}') = \frac{(J^2) |f_{\nu, \nu'}(\mathbf{k}, \mathbf{k}')|^2}{qG(\alpha)}$$

Intra-band

transferred momentum

Inter-band

q

$Q+q$

Why S_{\pm} -pairing symmetry?

- From gap equation:

$$\lambda \begin{pmatrix} \Delta_{\mathbf{k}}^h \\ \Delta_{\mathbf{k}}^e \end{pmatrix} = - \sum_{\mathbf{k}'} \left\langle \begin{pmatrix} V_{hh} & V_{eh} \\ V_{eh} & V_{ee} \end{pmatrix} (\mathbf{k}, \mathbf{k}') \begin{pmatrix} \Delta_{\mathbf{k}'}^h \\ \Delta_{\mathbf{k}'}^e \end{pmatrix} \right\rangle$$

- **Eigenvalue:** $\lambda_e = V_{ee}N_e(E_F)$ $\lambda_h = V_{hh}N_h(E_F)$
 $\lambda_{eh} = V_{eh} \sqrt{N_e(E_F)N_h(E_F)}$

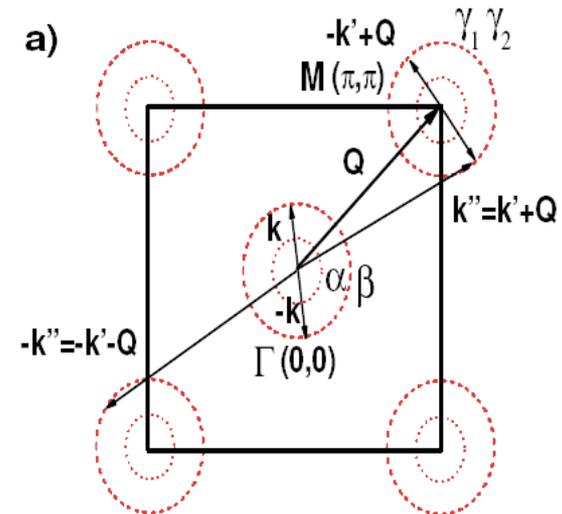
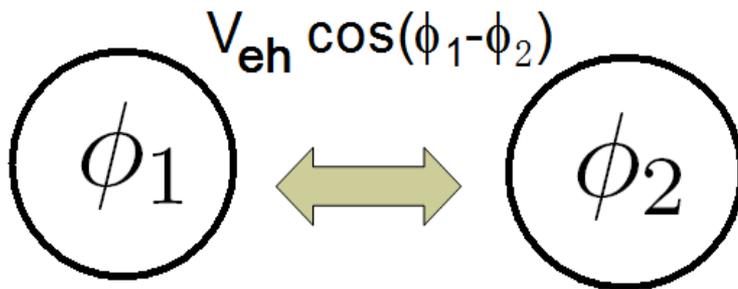
$$\lambda = -(\lambda_h + \lambda_e)/2 + \sqrt{(\lambda_h - \lambda_e)^2/4 + \lambda_{eh}^2}$$

- **Order parameters:**

$$\begin{pmatrix} \Delta_{\mathbf{k}}^h \\ \Delta_{\mathbf{k}}^e \end{pmatrix}^T \propto (\lambda_{eh}, -(\lambda + \lambda_h))^T$$

Interpretation II: Invert sign

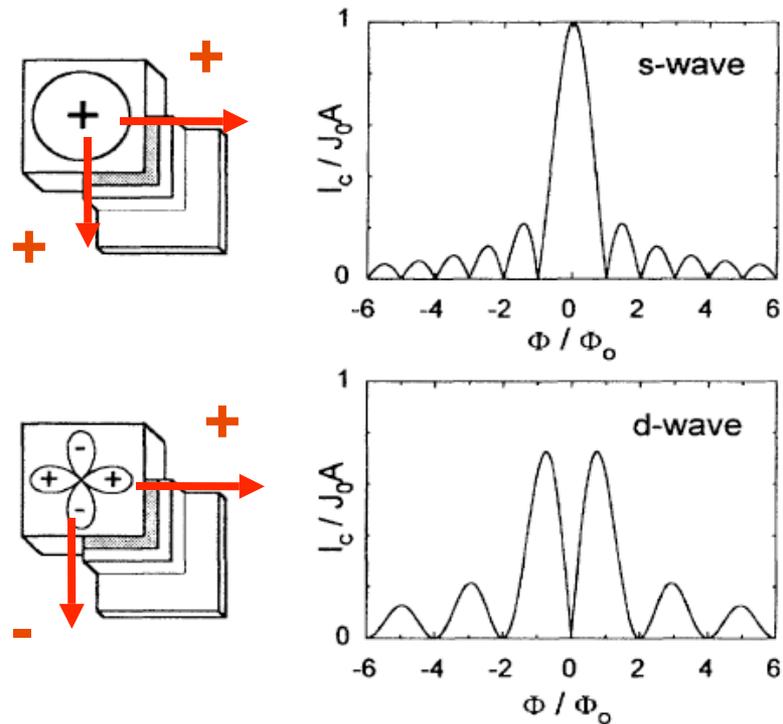
- 2) phase-shift for different bands
depends on the sign of inter-band hopping



How to detect S_{\pm} -pairing symmetry ?

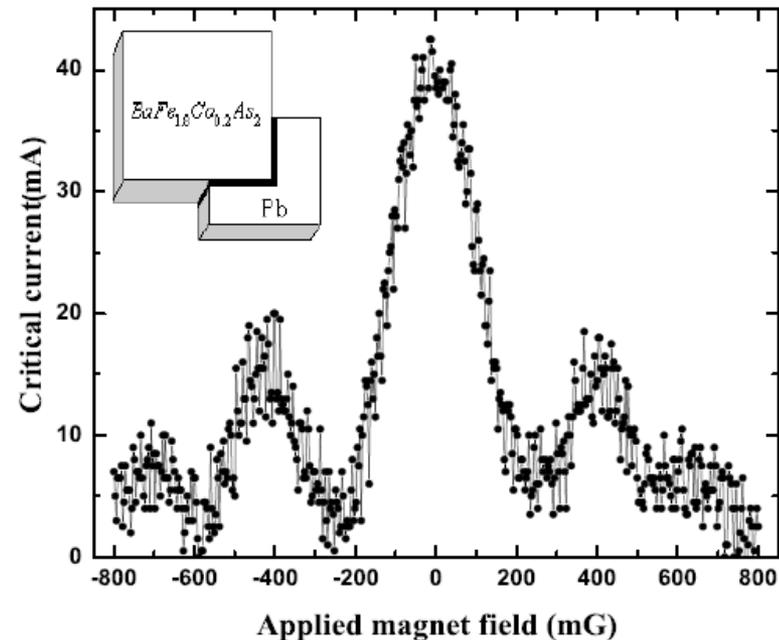
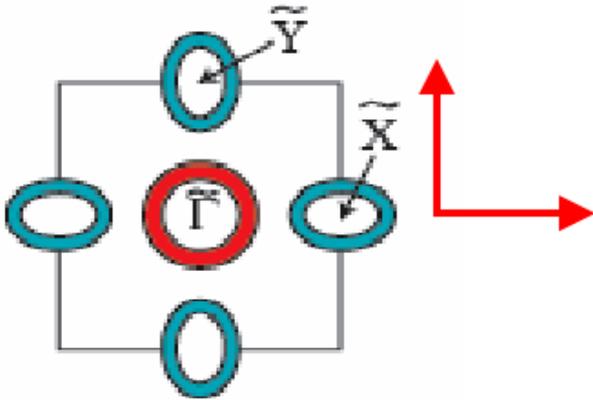
J. Wu, P. Phillips,
Phys. Rev. B 79, 092502 (2009)

Phase sensitive measurement



D.J. Van Harlingen, PRL, 1995

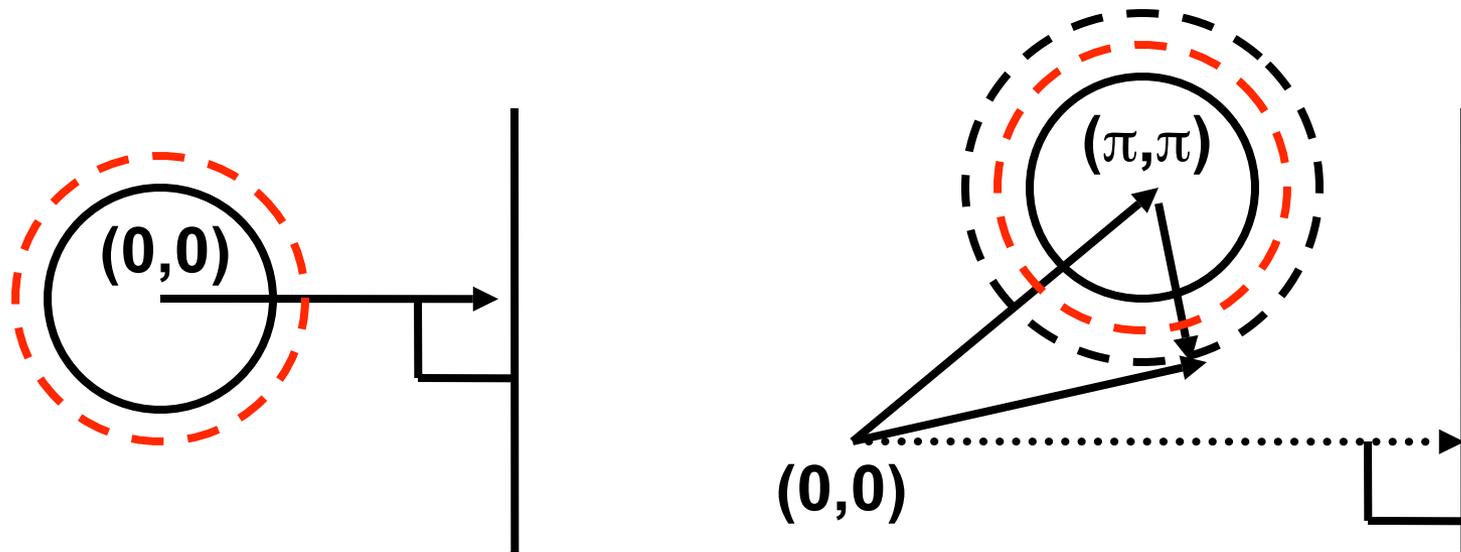
Ordinary corner Josephson junction cannot tell the inverse sign (still $A_{1,0}$ symmetry)



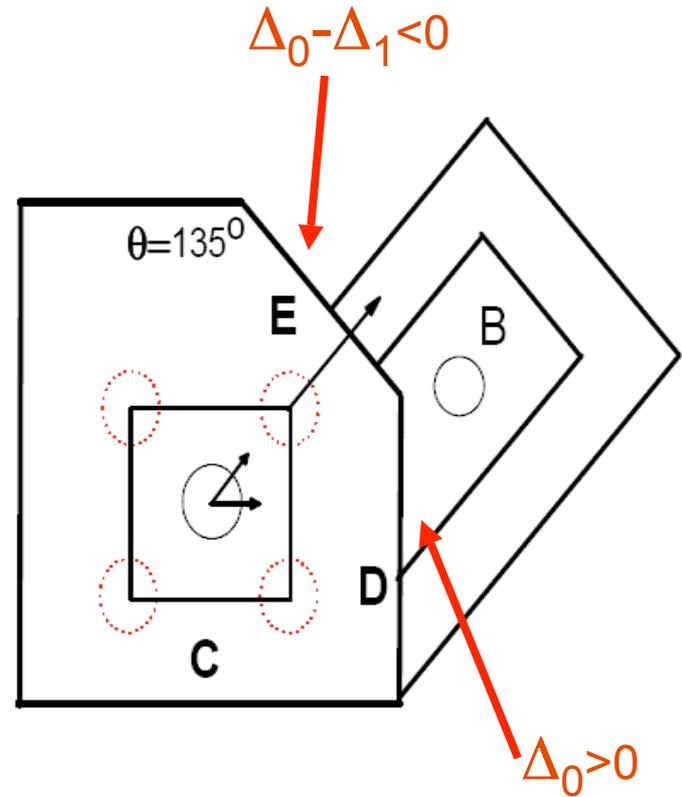
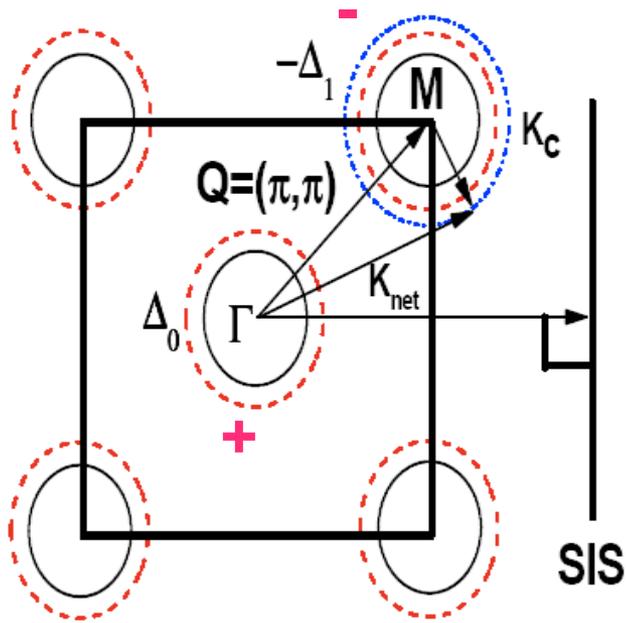
arXiv: 0812.3295

Principle of SIS junction

Superconductor-Insulator-Superconductor(SIS) Junction can only measure order parameters perpendicular to the surface of the junction.



Modified corner Josephson junction



How to test?

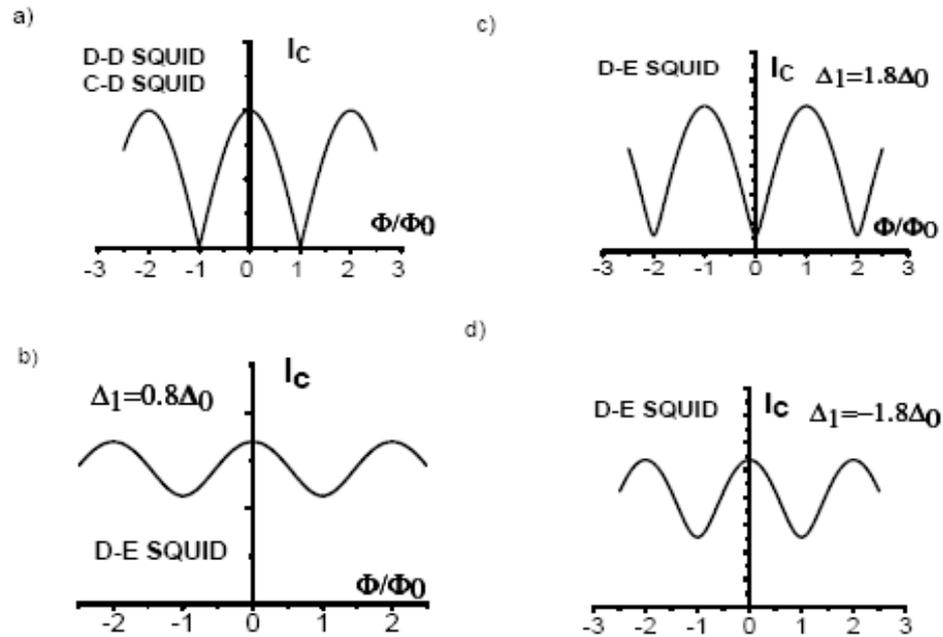


FIG. 2: All possible interference pattern for different x values where x is defined as $\Delta_1 = -x\Delta_0$. a) For junction connecting D-D or C-D faces; b) c) d) are all junction connecting D-E faces. b) $x \in [0, 1]$; c) $x \in [1, \infty)$; d) $x \in (-\infty, 0]$.

How to test?

Ordinary
90° JJ

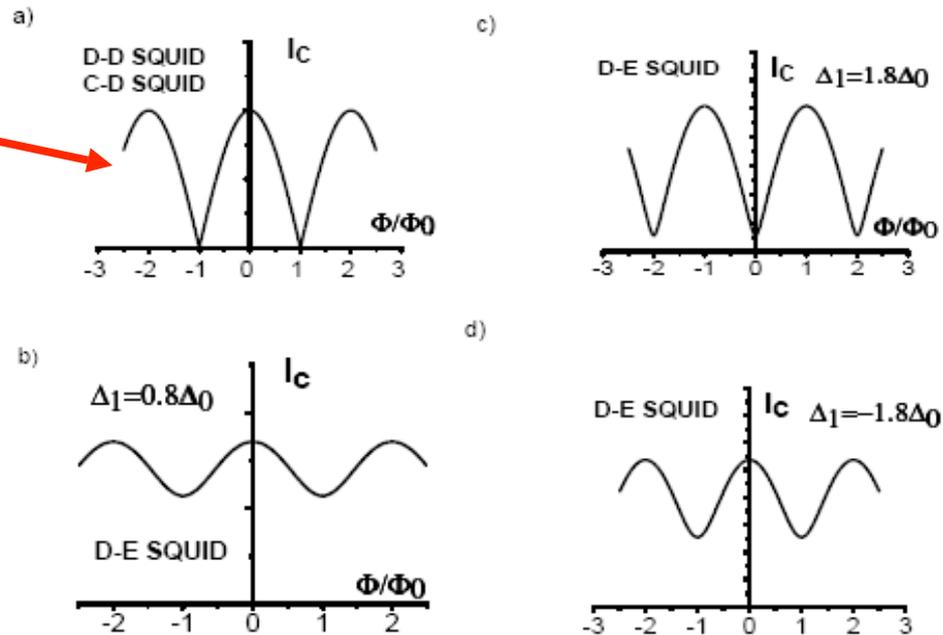
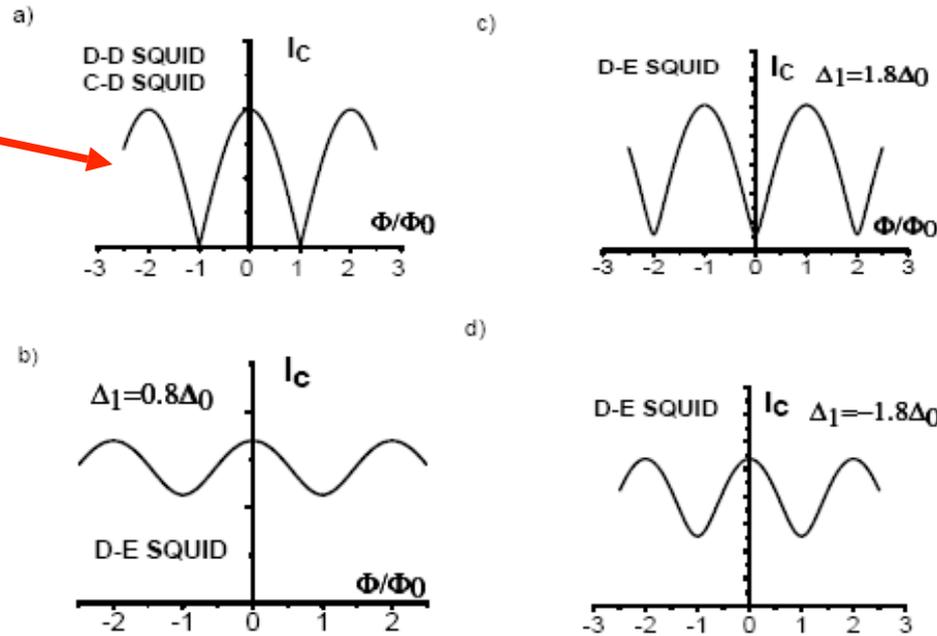


FIG. 2: All possible interference pattern for different x values where x is defined as $\Delta_1 = -x\Delta_0$. a) For junction connecting D-D or C-D faces; b) c) d) are all junction connecting D-E faces. b) $x \in [0, 1]$; c) $x \in [1, \infty)$; d) $x \in (-\infty, 0]$.

How to test?

Ordinary
90° JJ



modified
135° JJ

FIG. 2: All possible interference pattern for different x values where x is defined as $\Delta_1 = -x\Delta_0$. a) For junction connecting D-D or C-D faces; b) c) d) are all junction connecting D-E faces. b) $x \in [0, 1]$; c) $x \in [1, \infty)$; d) $x \in (-\infty, 0]$.

Summary

Key properties for Iron-pnictides:

(1) Fe-As trilayers

(2) Multi-bands

(3) Intermediate
coupling

Summary

Key properties for Iron-pnictides:

(1) Fe-As trilayers

Orbital Ordering

(2) Multi-bands

(3) Intermediate
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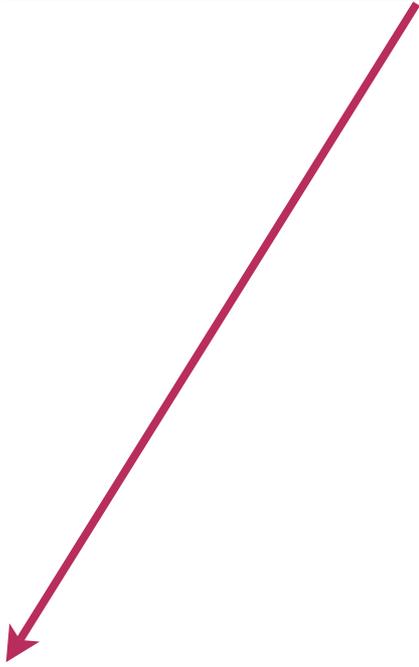
(3) Intermediate
coupling

Orbital Ordering

Itinerant-localized
dichotomy

What are the pnictides Really?

What are the pnictides Really?



Itinerant
electrons

What are the pnictides Really?

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graph TD; A[What are the pnictides Really?] --> B[Itinerant electrons]; A --> C[Localized electrons];
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Itinerant
electrons

Localized
electrons

What are the pnictides Really?

Arsenic
and Old
Lace

Itinerant
electrons

Localized
electrons

What are the pnictides Really?

Arsenic
and Old
Lace

Itinerant
electrons



"I'm not a Brewster, I'm a son of a sea cook!"^[1]

Localized
electrons

What are the pnictides Really?

Arsenic
and Old
Lace

Itinerant
electrons

Localized
electrons

What are the pnictides Really?

Arsenic
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Lace

Itinerant
electrons

Localized
electrons

?

Are the pnictides important?

Cuprates,
single-orbital
Mott-system,
low-moment,
high T_c

manganites
multi-orbital
high-moment
no T_c

pnictides
multi-orbital
low-moment
moderate
 T_c

Are the pnictides important?

Cuprates,
single-orbital
Mott-system,
low-moment,
high T_c

pnictides
multi-orbital
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moderate
 T_c

Are the pnictides important?

Cuprates,
single-orbital
Mott-system,
low-moment,
high T_c

Multi-orbital
Mott system,
low-moment
??? T_c
EFRC Center

pnictides
multi-orbital
low-moment
moderate
 T_c