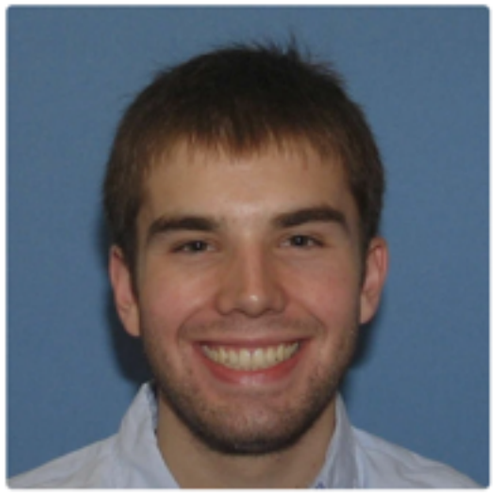
Optical Conductivity in the Cuprates: from Mottness to Scale Invariance

Thanks to: NSF, EFRC (DOE)

Brandon Langley



Garrett Vanacore

Kridsangaphong Limtragool

Optical spectra of $La_{2-x}Sr_xCuO_4$: Effect of carrier doping on the electronic structure of the CuO_2 plane

S. Uchida
Engineering Research Institute, University of Tokyo, Yayoi, Tokyo 113, Japan

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Department of Applied Physics, University of Tokyo, Hongo, Tokyo 113, Japan

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S. Tajima
Superconducting Research Laboratory, International Superconductivity Technology Center, Shinonome, Tokyo, Japan
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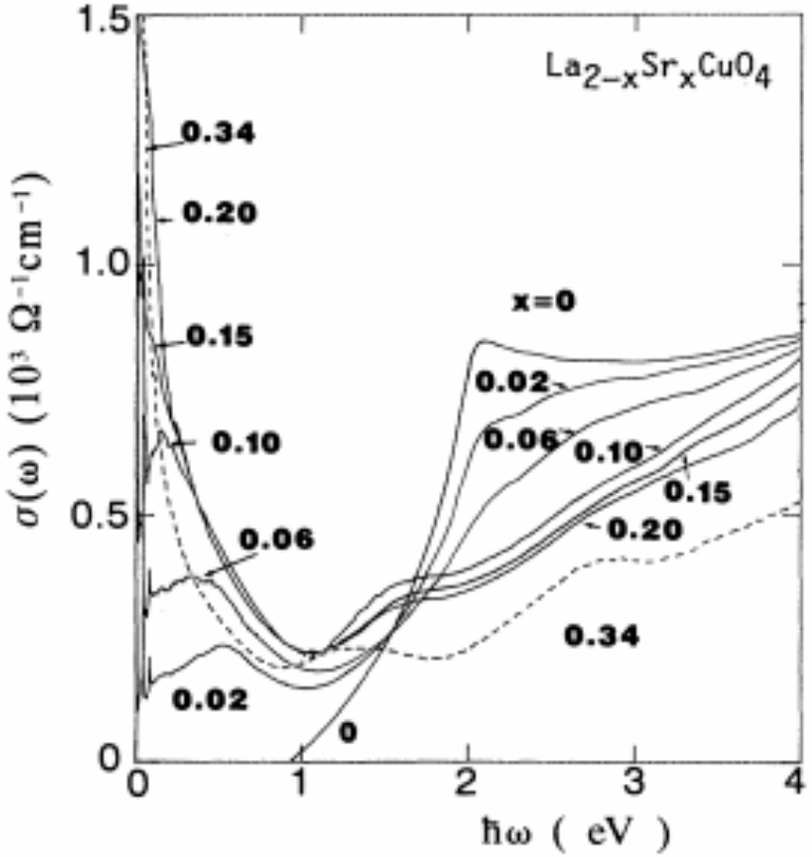

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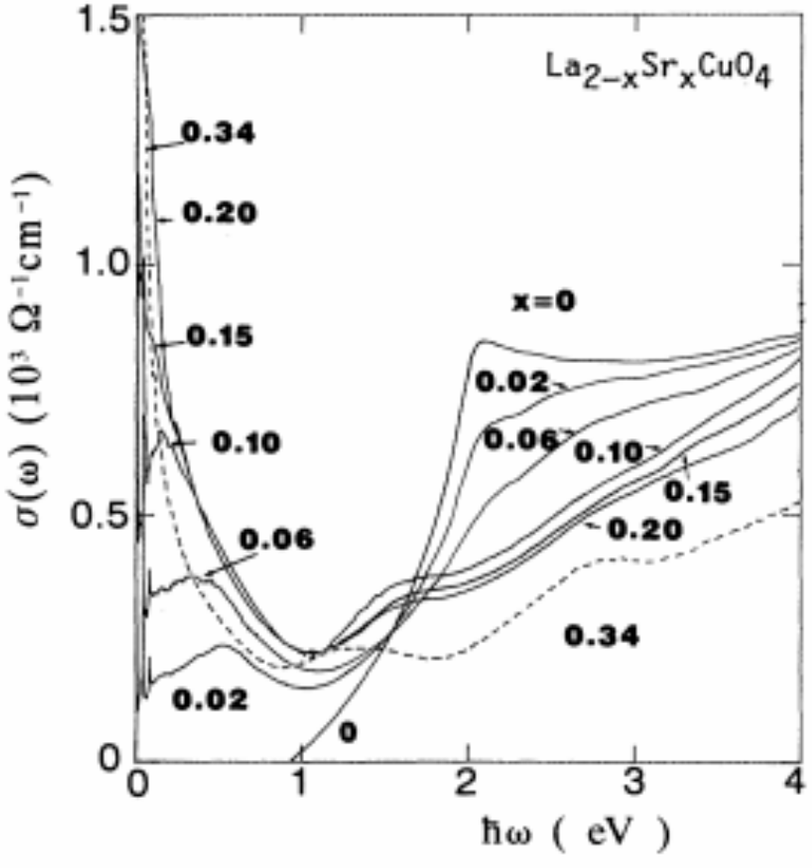

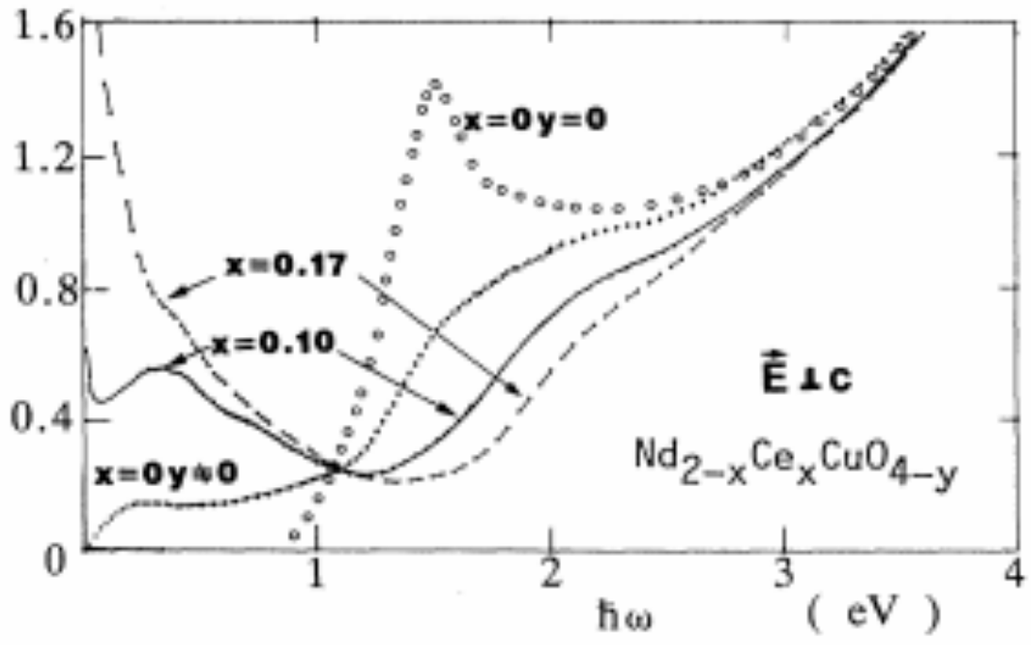

Growth of the optical conductivity in the Cu-O planes

S. L. Cooper, G. A. Thomas, J. Orenstein, D. H. Rapkine, A. J. Millis, S-W. Cheong, and A. S. Cooper
AT&T Bell Laboratories, Murray Hill, New Jersey 07974

Z. Fisk
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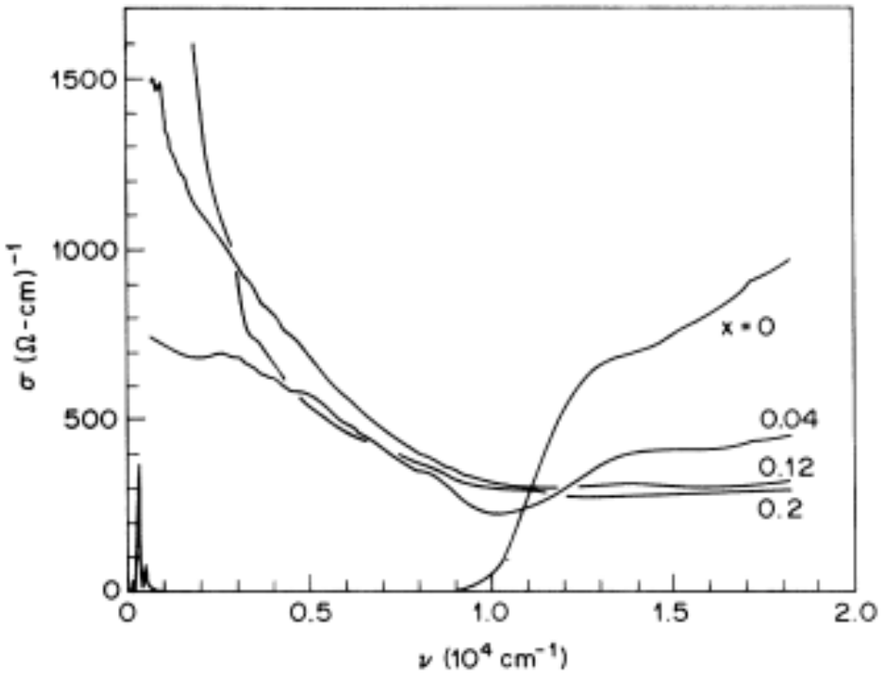
(Received 7 March 1990)



$$N_{\mathrm{eff}}(\Omega) = \frac{2mV_{\mathrm{cell}}}{\pi e^2} \int_0^{\Omega} \sigma(\omega) d\omega$$

optical gap

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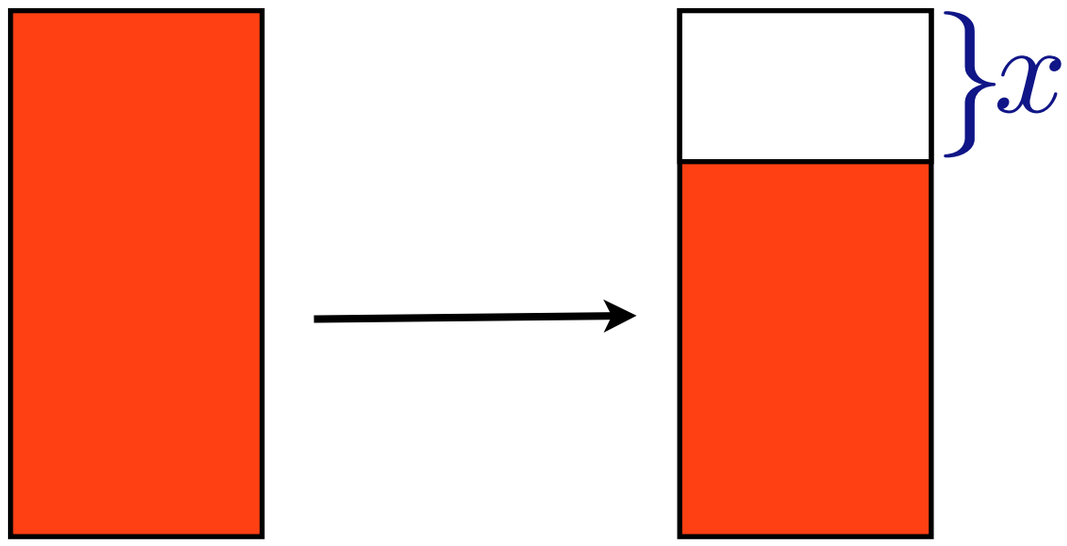

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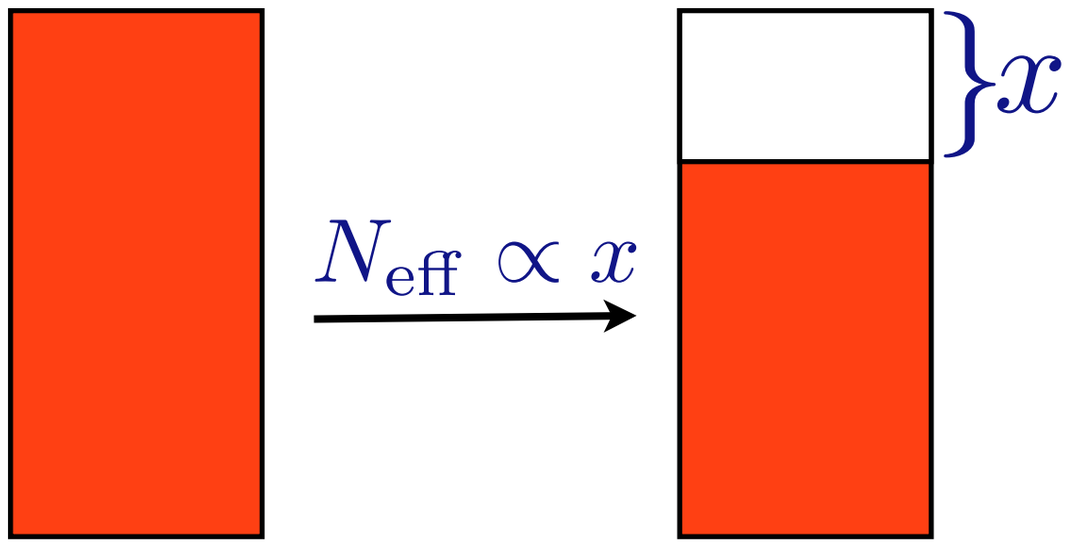
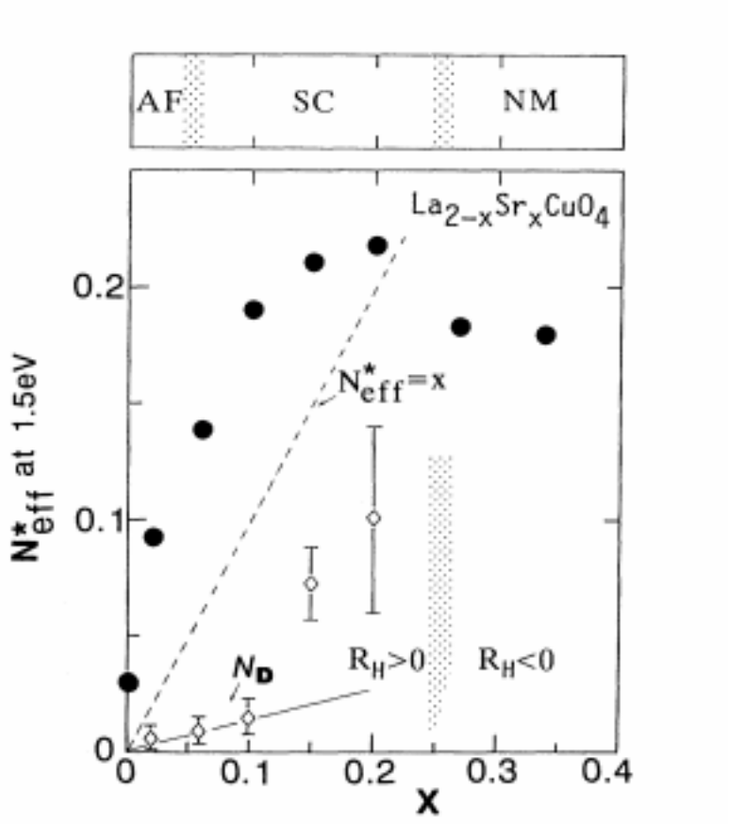
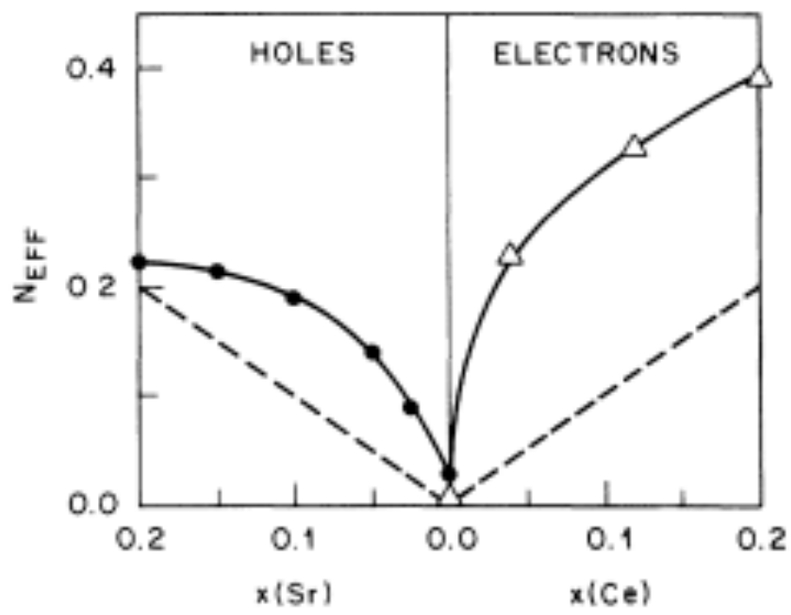


Uchida, et al.



Cooper, et al.



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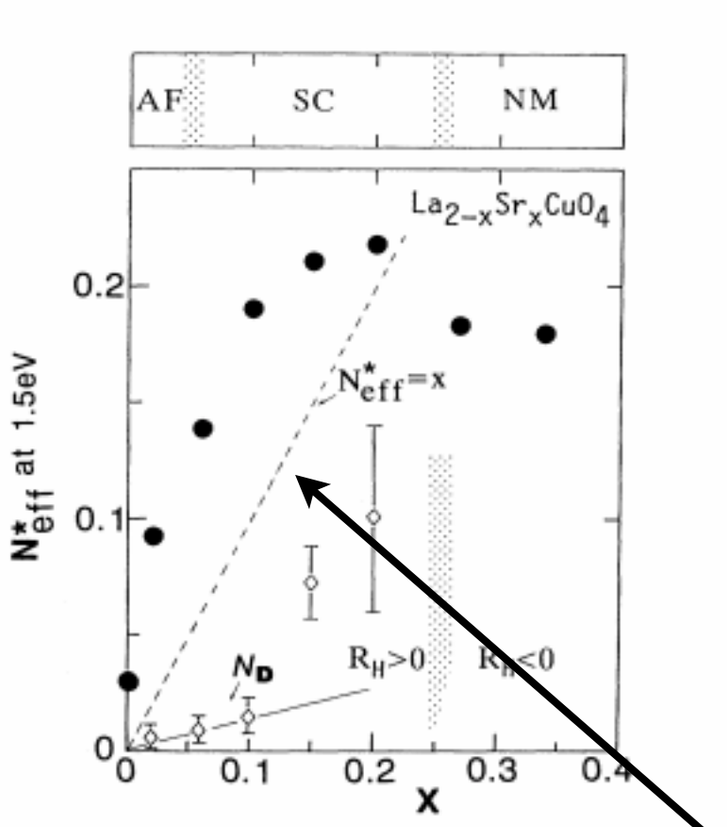

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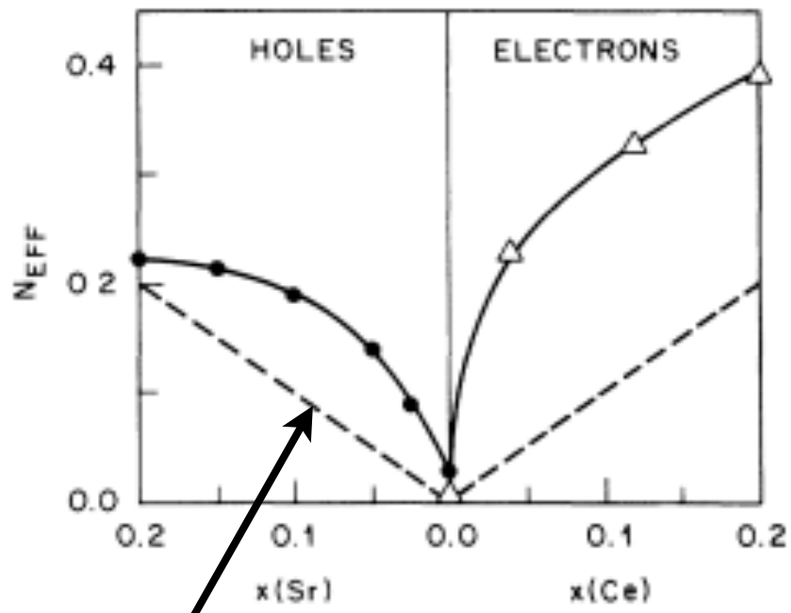
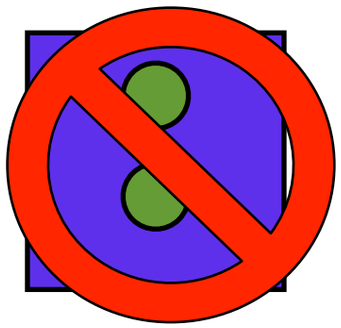


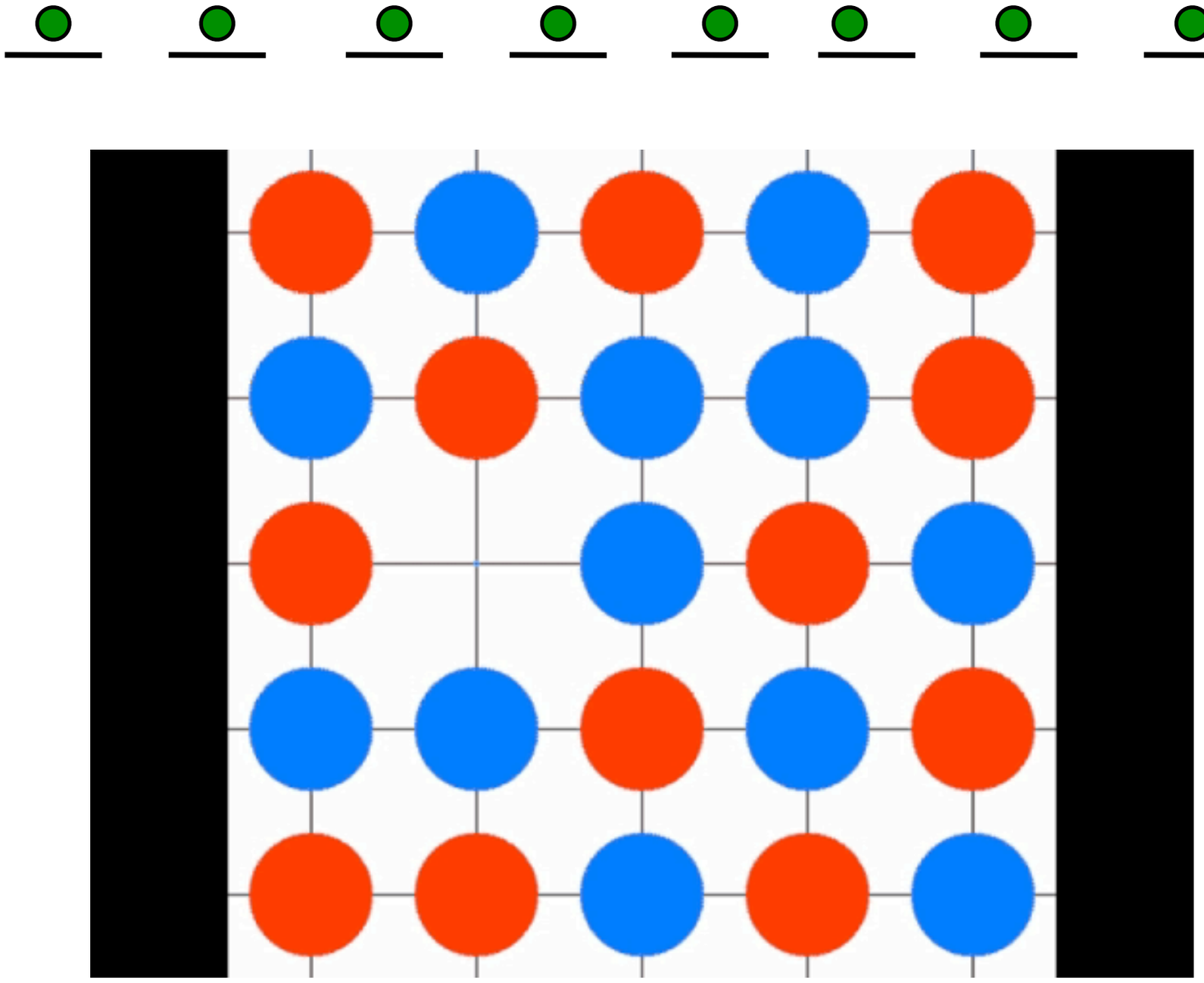
x

excess carriers?

Mottness

classical
(atomic) limit





classical
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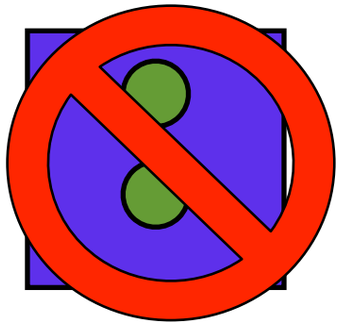

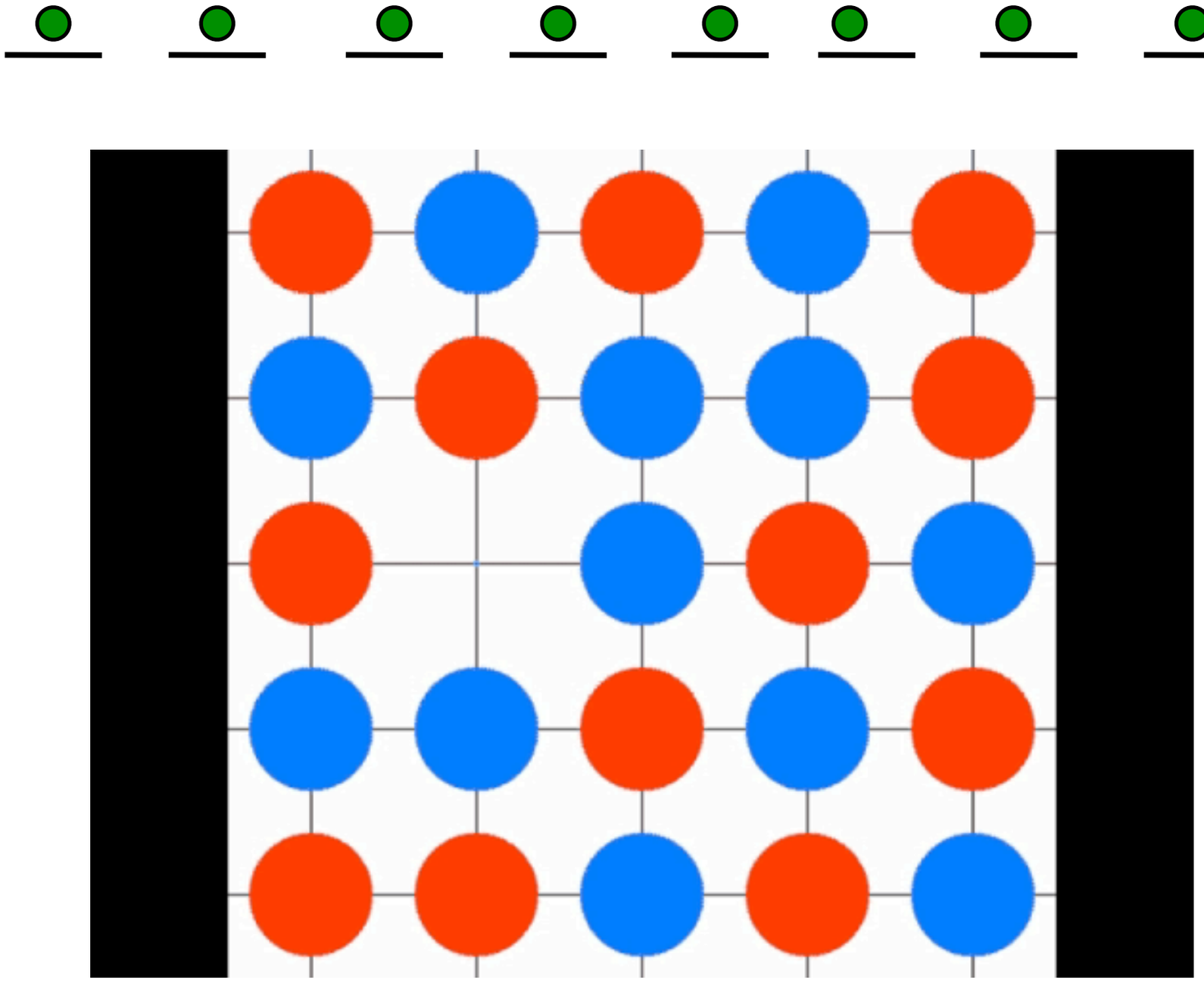

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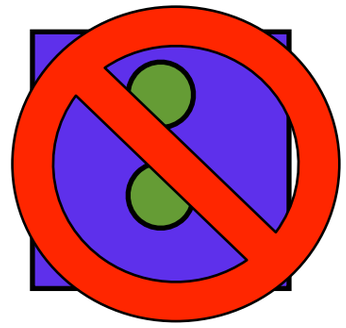
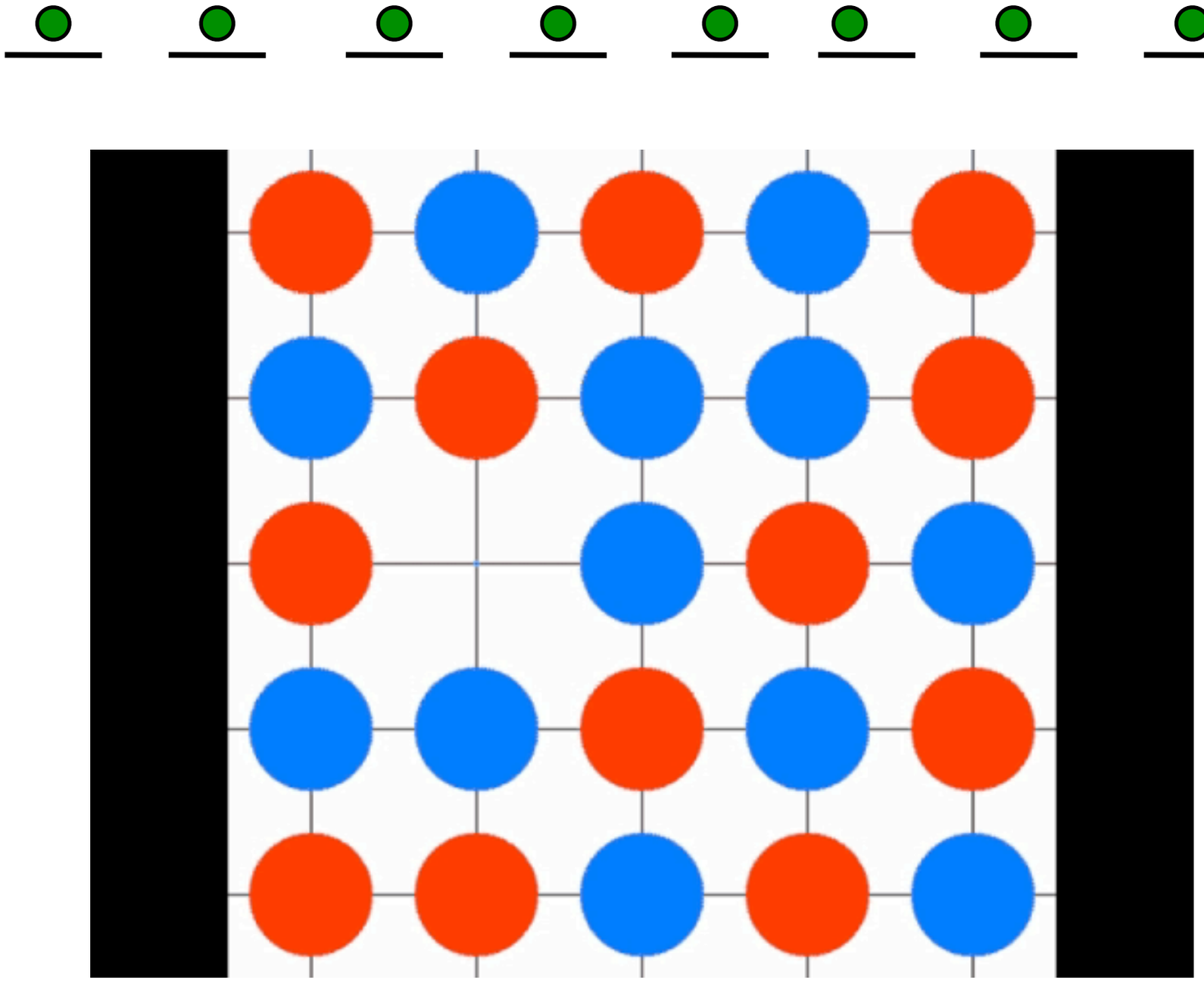


number of empty sites=x



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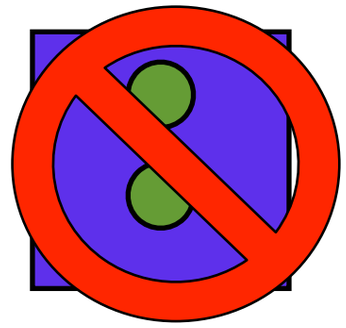
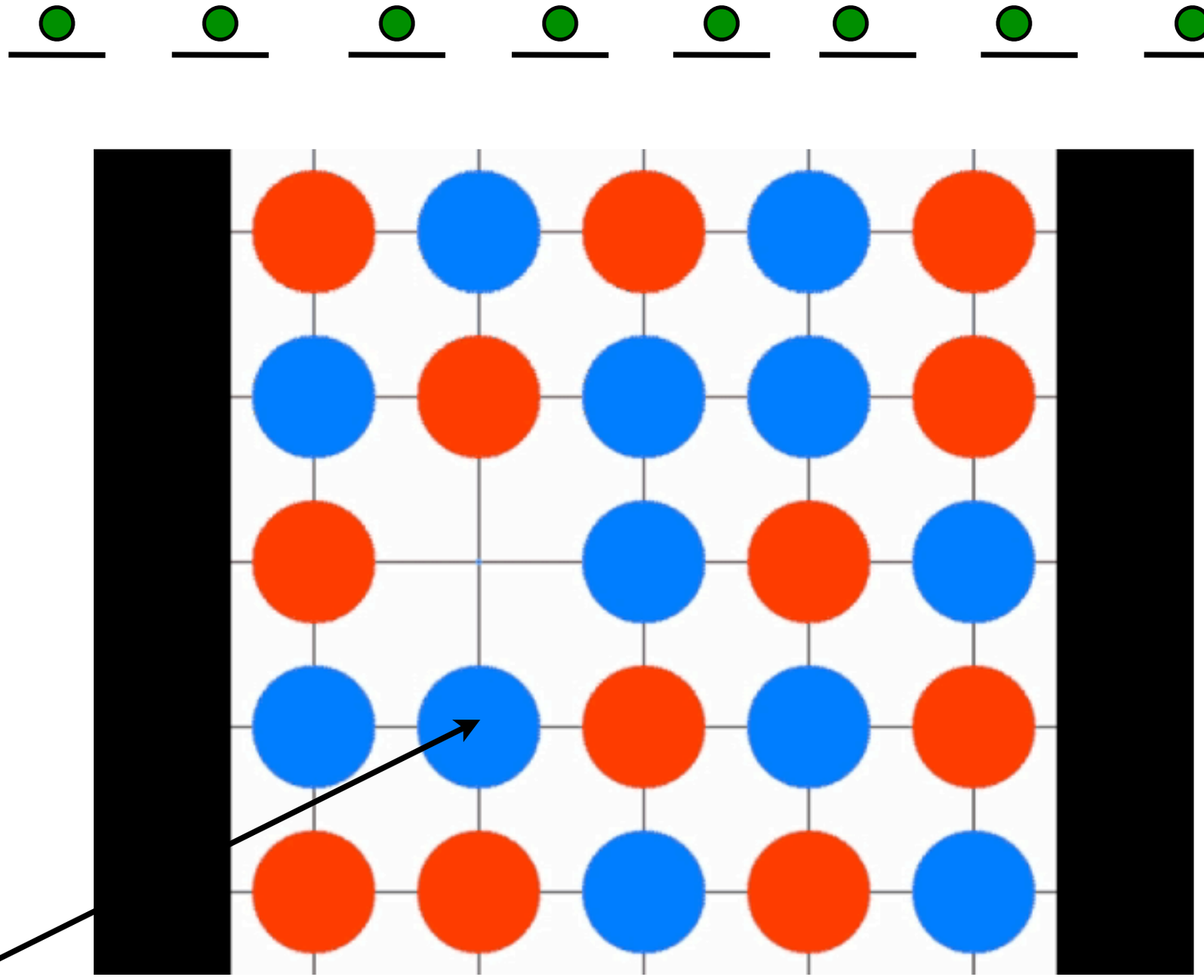


number of empty sites=x

$$N_{\text{eff}} \propto x$$



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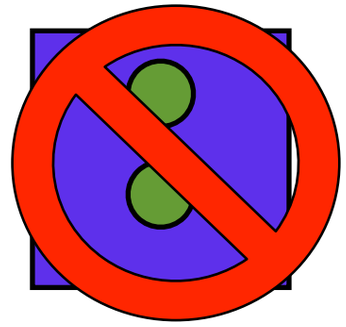


2 states

number of empty sites=x

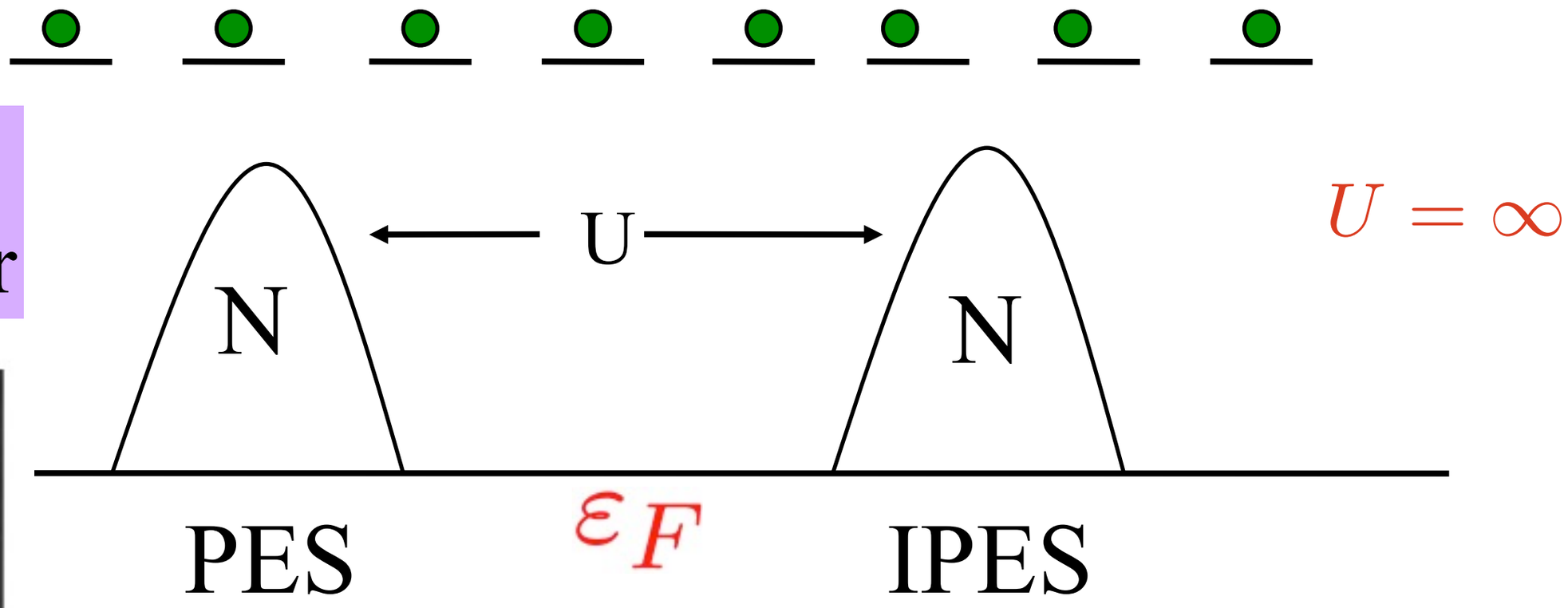
$$N_{\text{eff}} \propto x$$

Mott insulator

Mott insulator

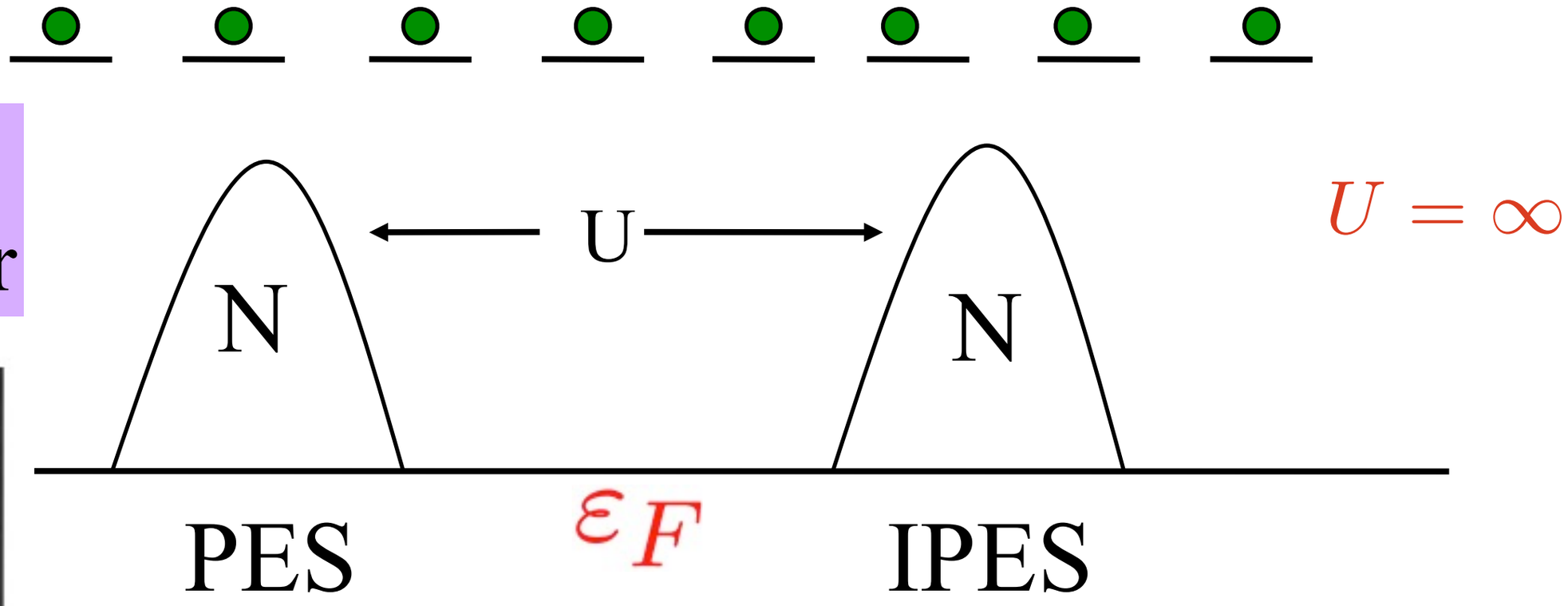
$U = \infty$

Mott insulator



$U = \infty$

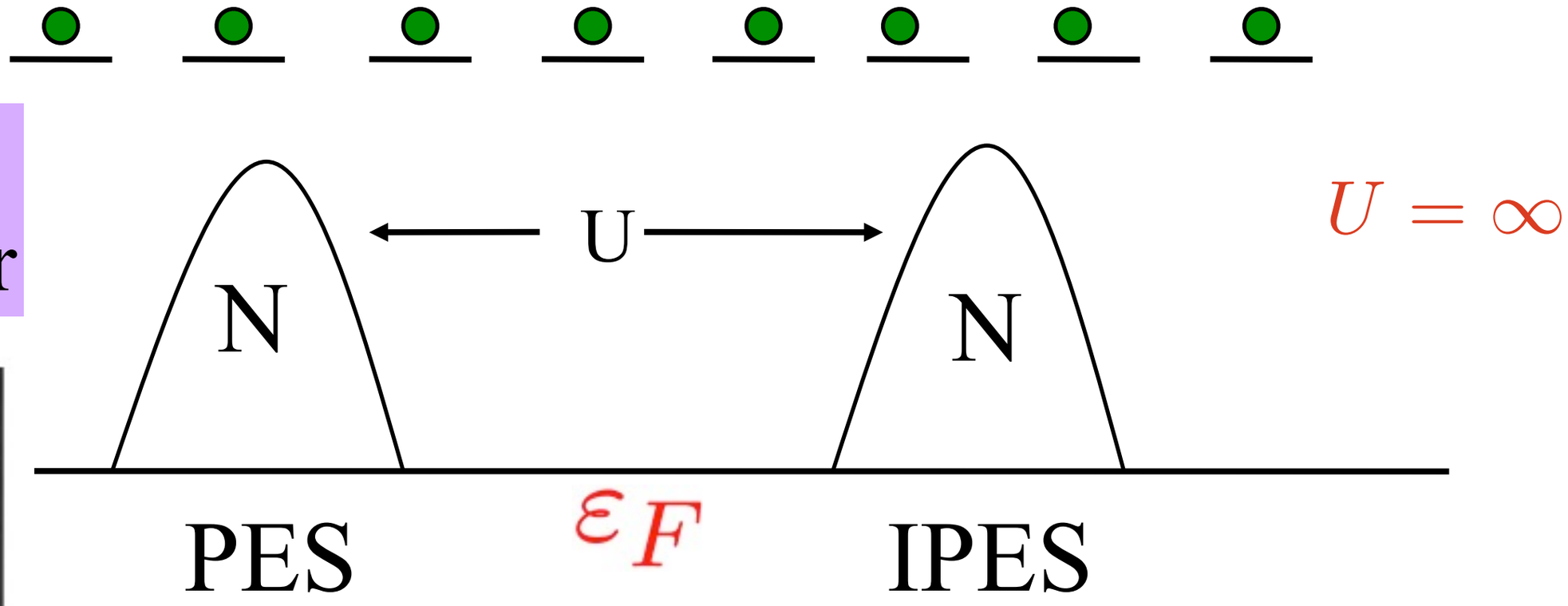
Mott insulator



$U = \infty$

no hopping:

Mott insulator



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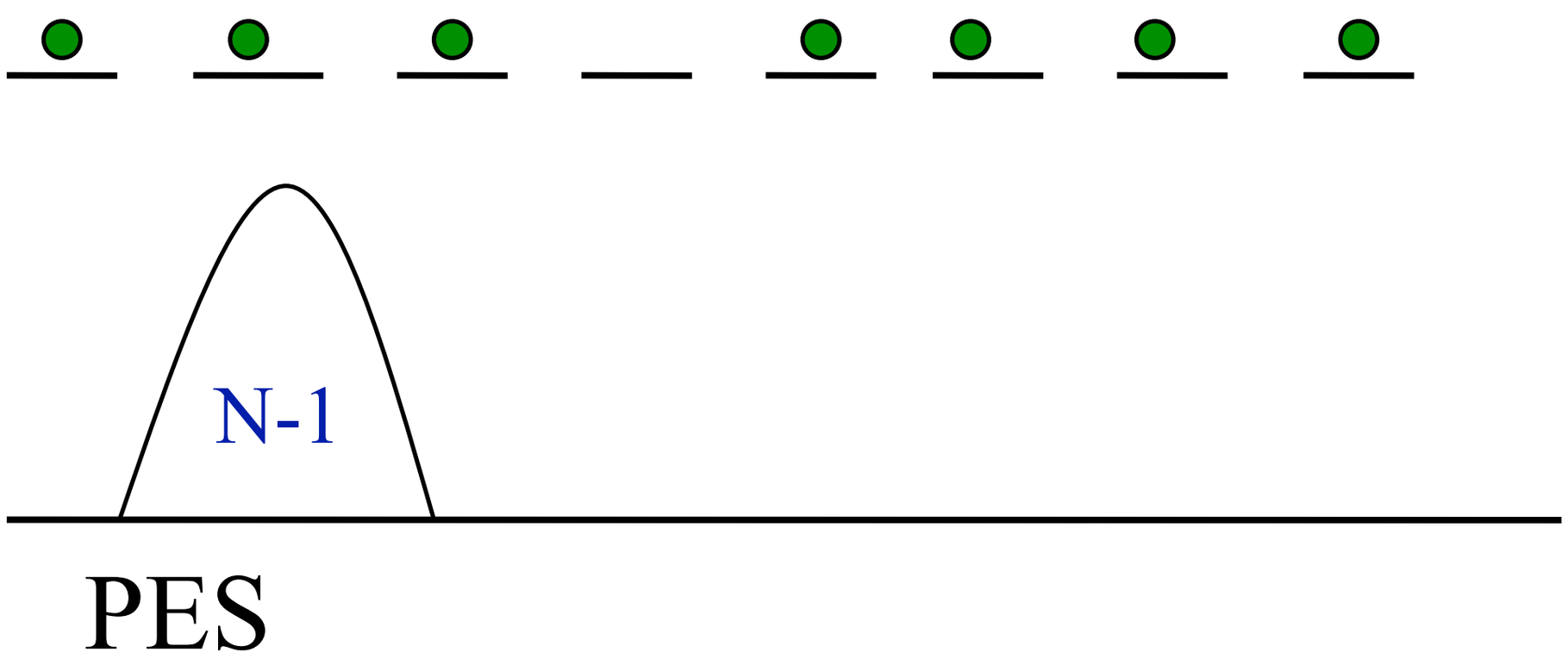
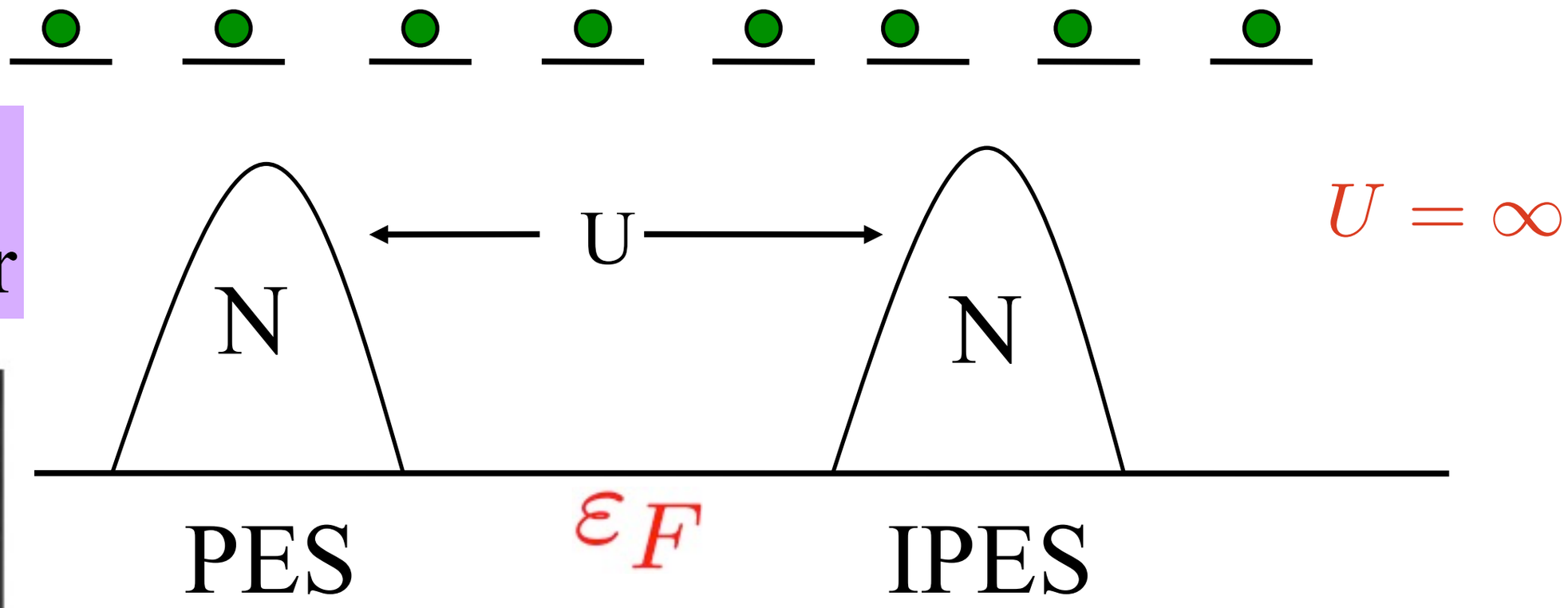


Mott insulator



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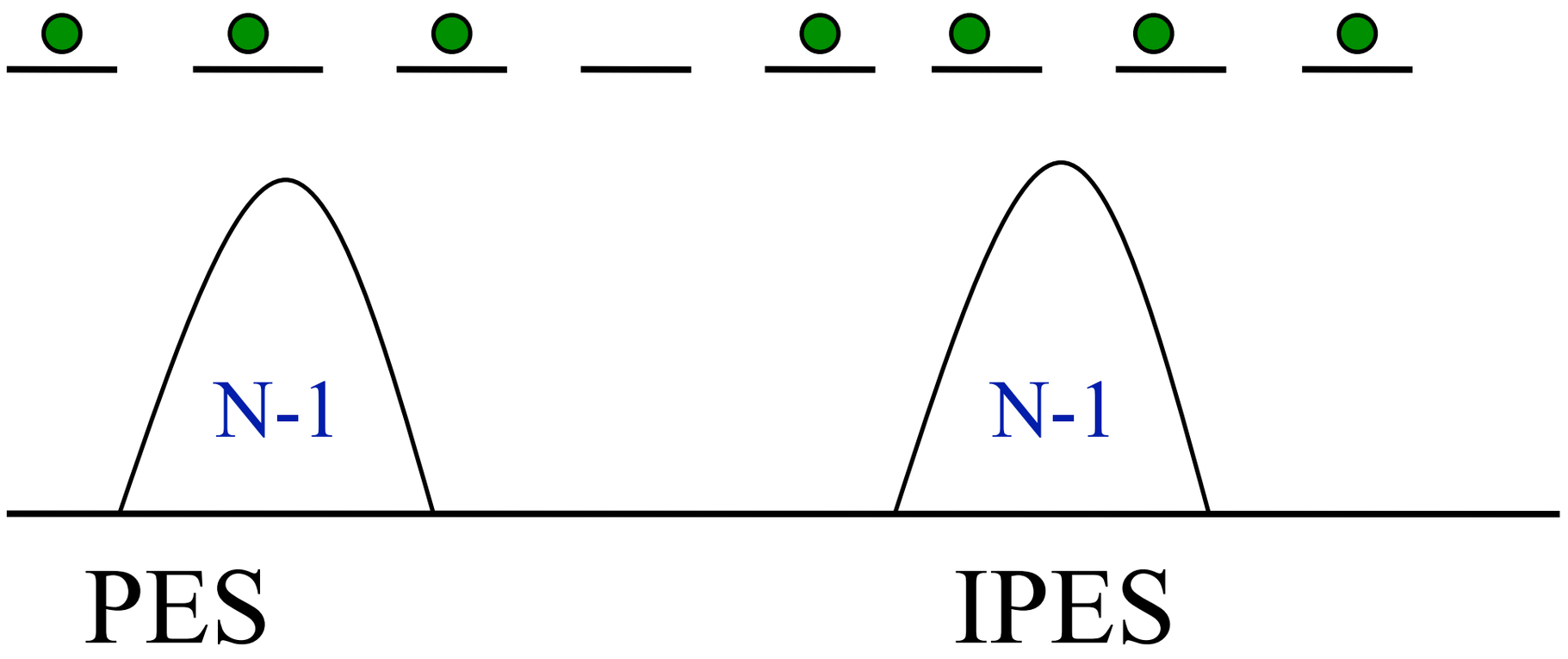

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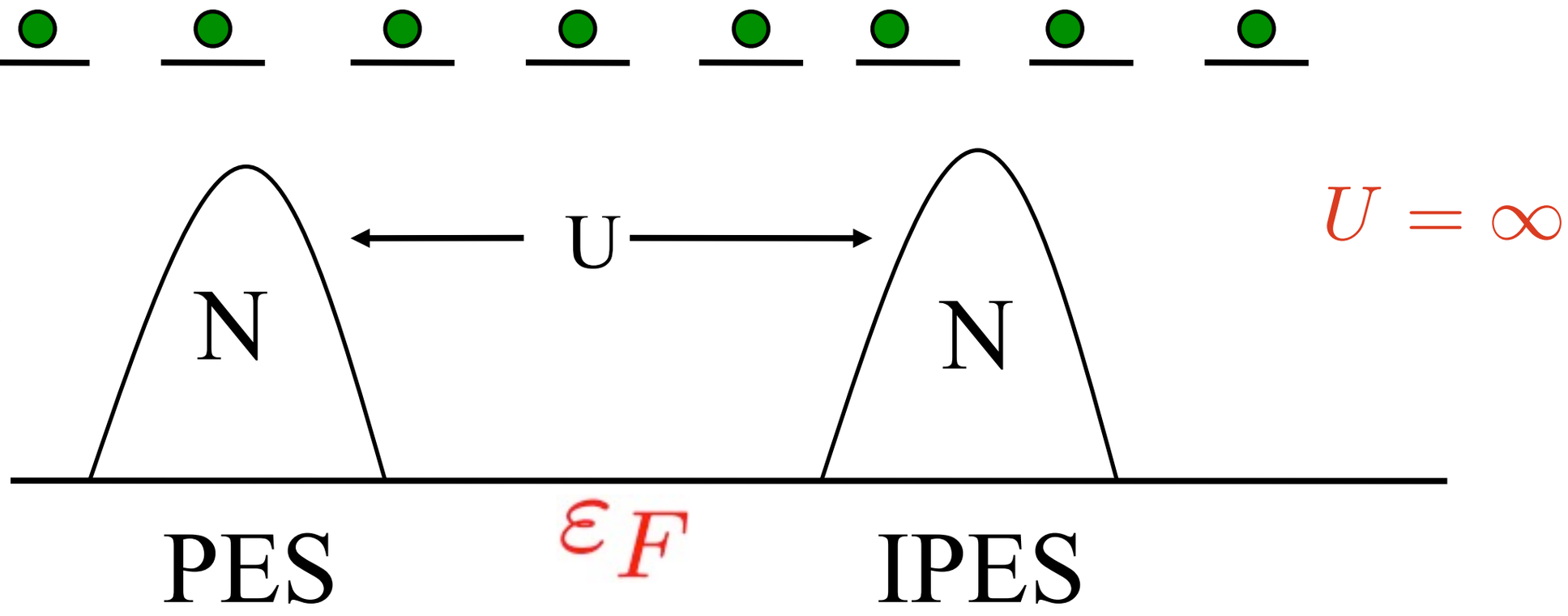

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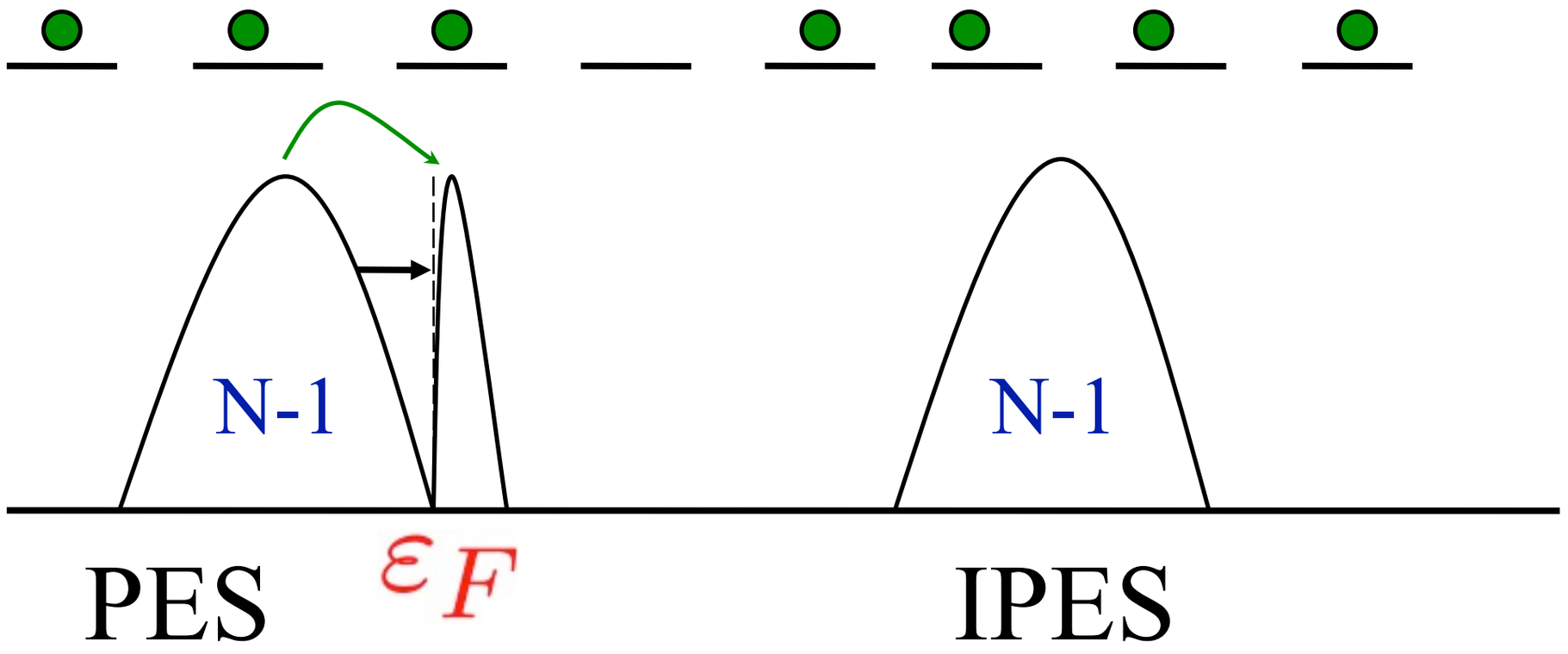

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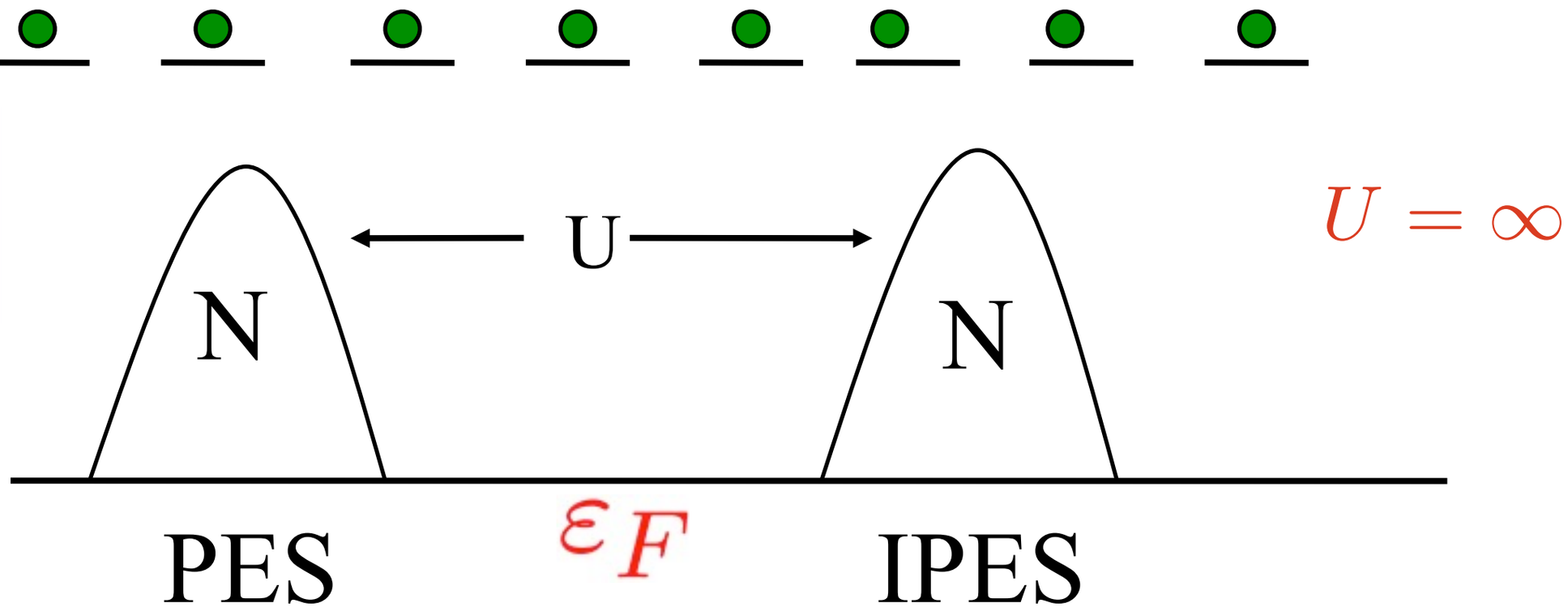

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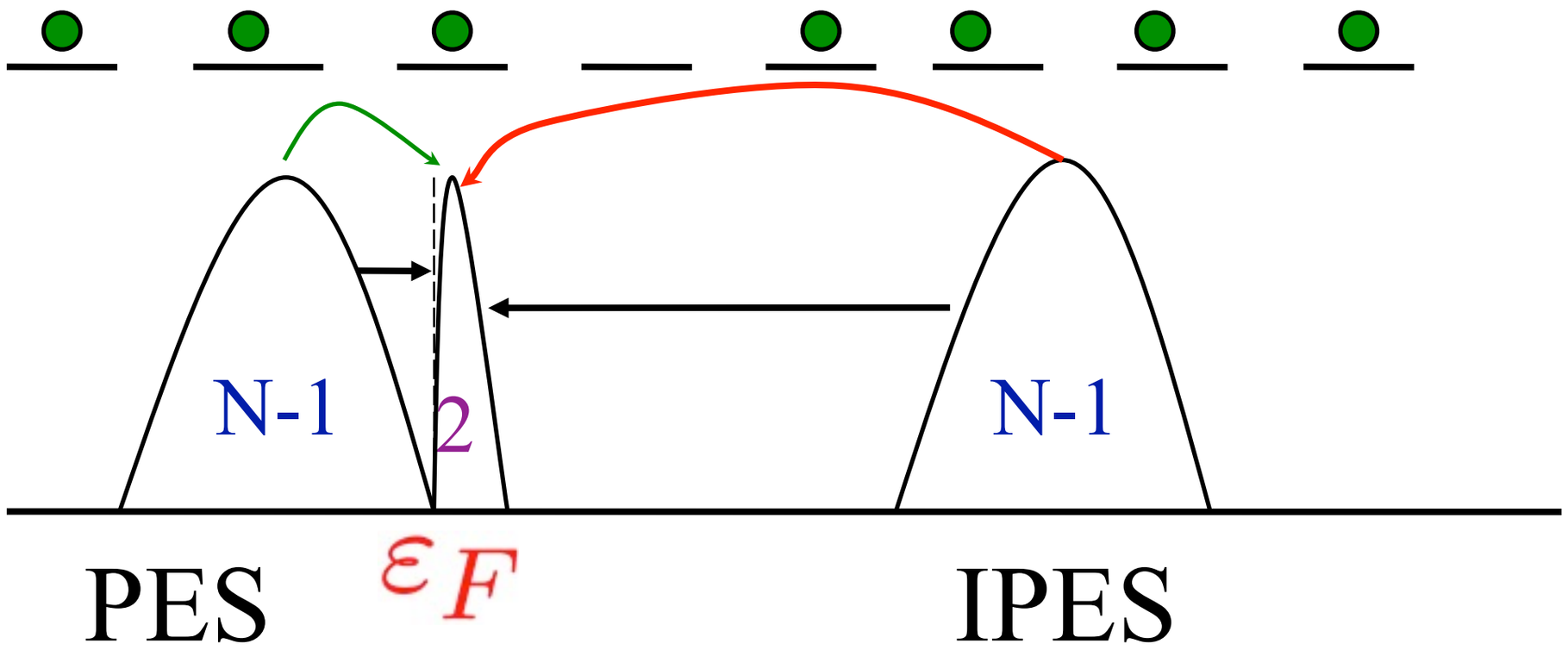
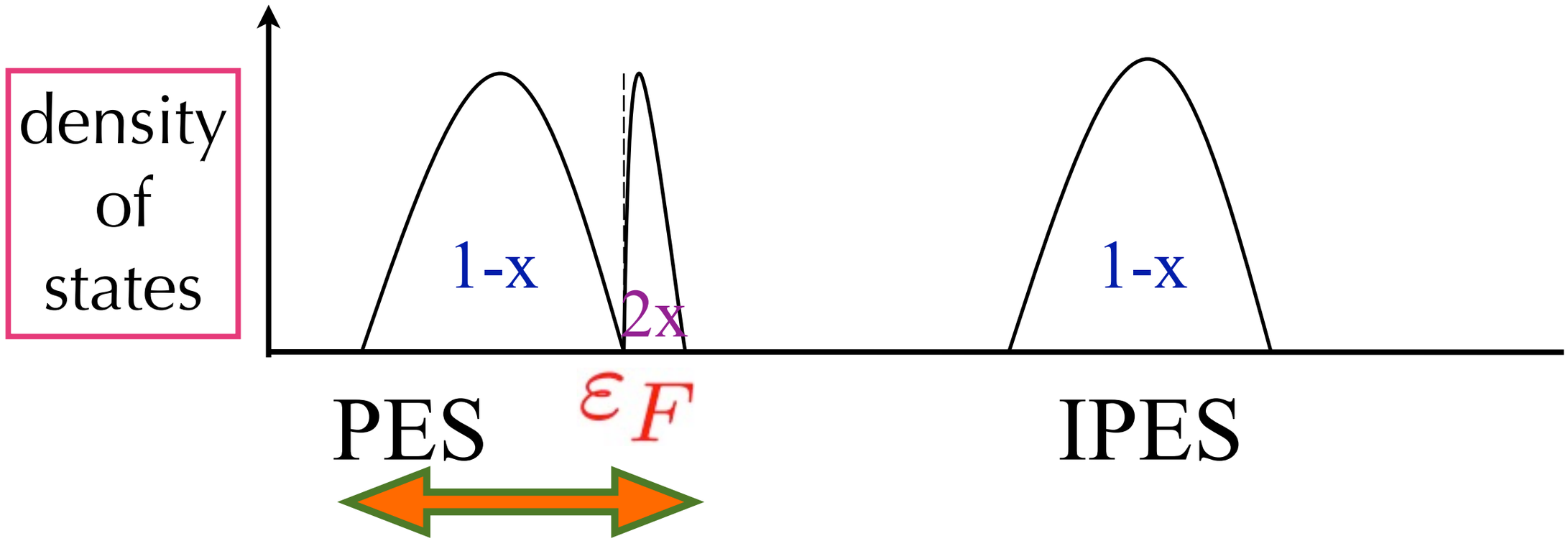


Sawatzky

atomic limit: x holes



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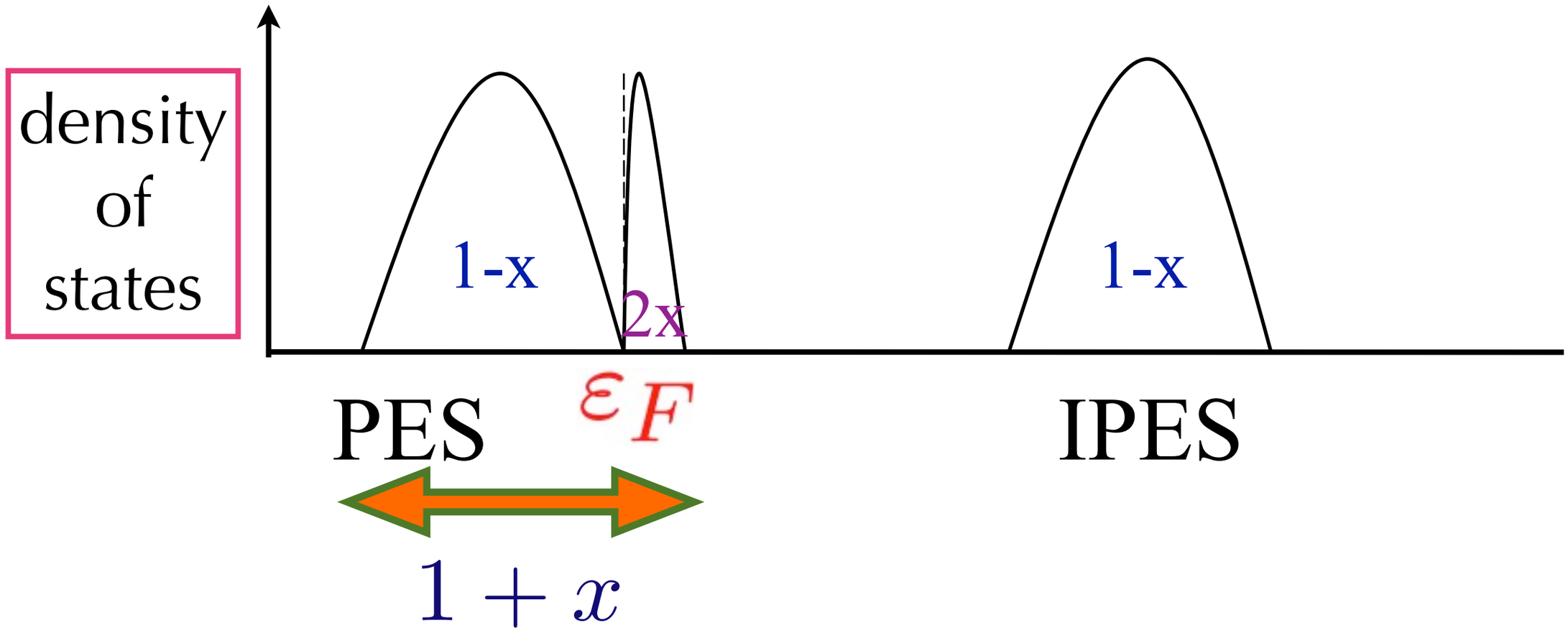

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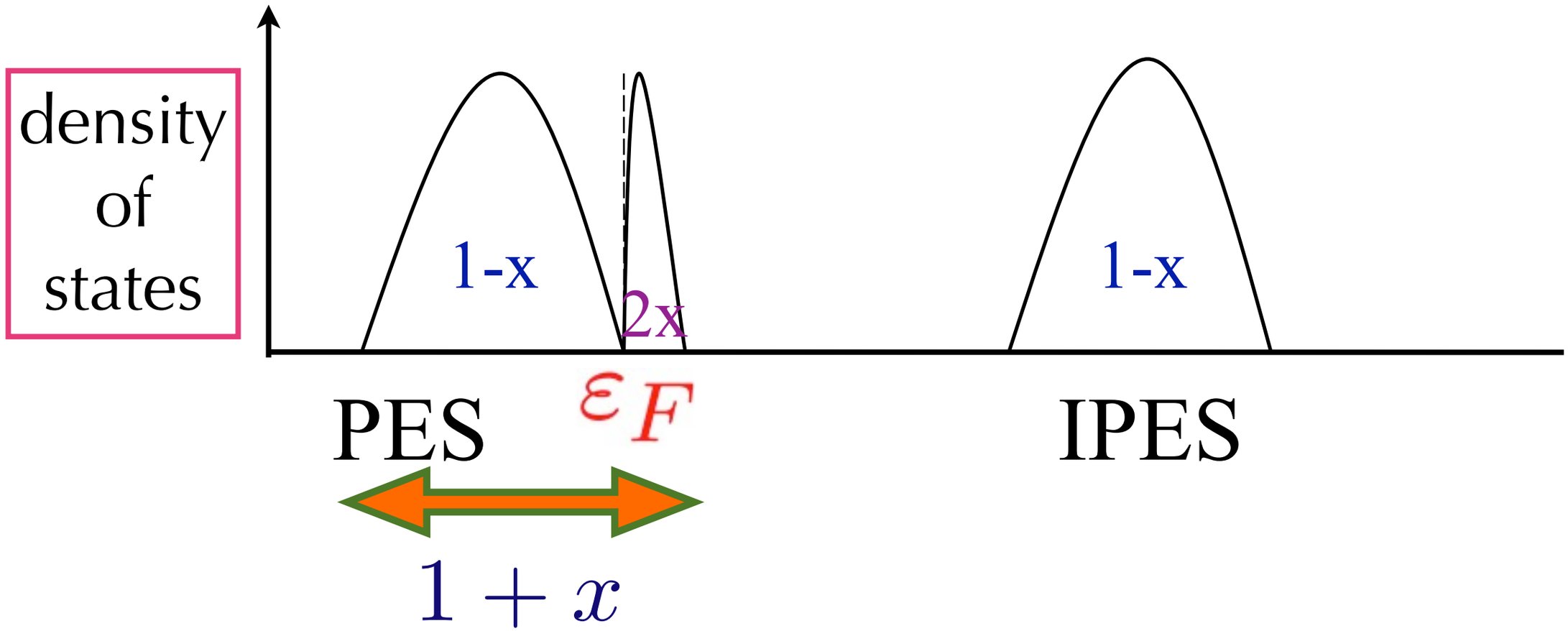
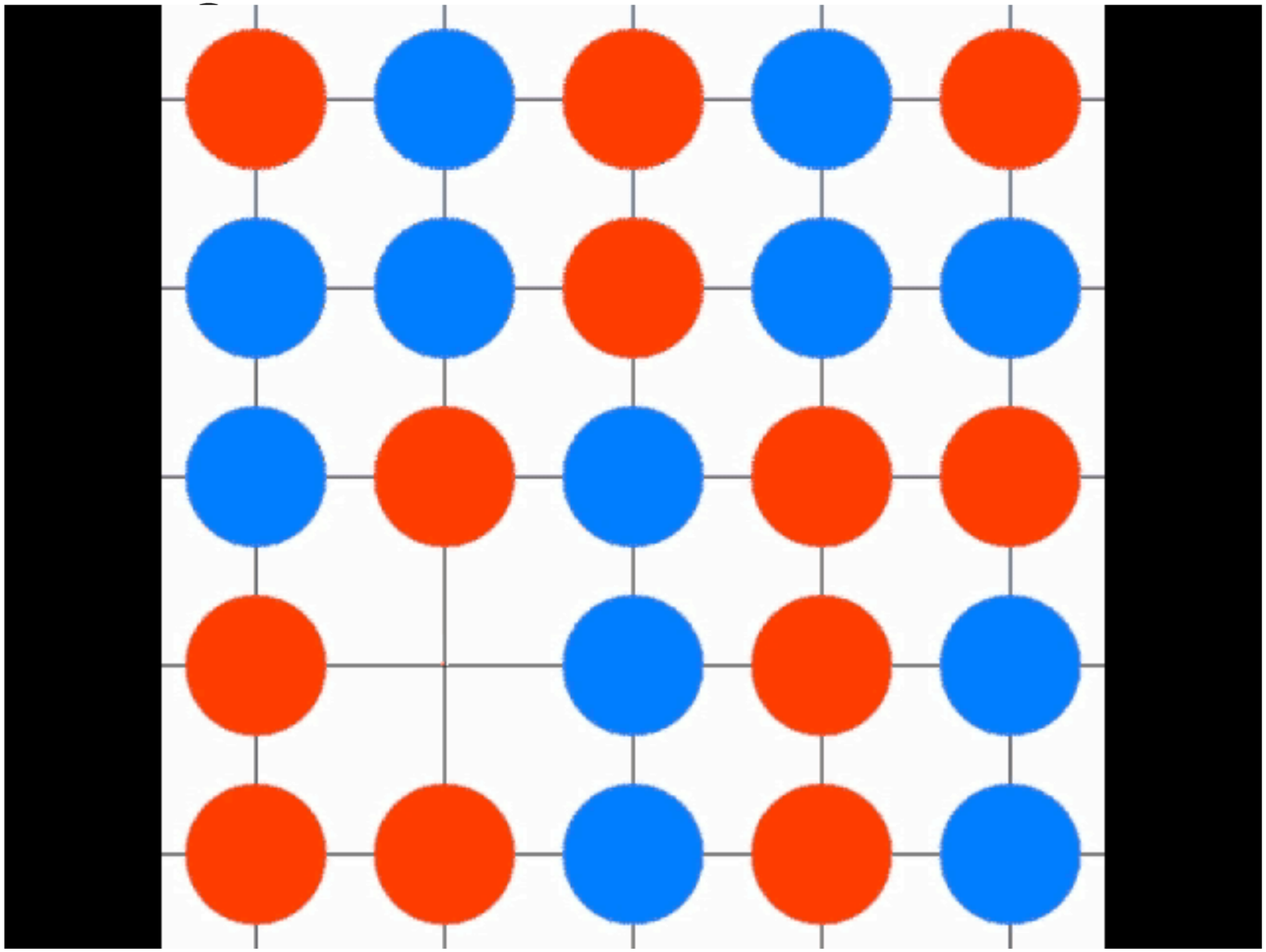


$1 + x$

2

U finite $U \gg t$



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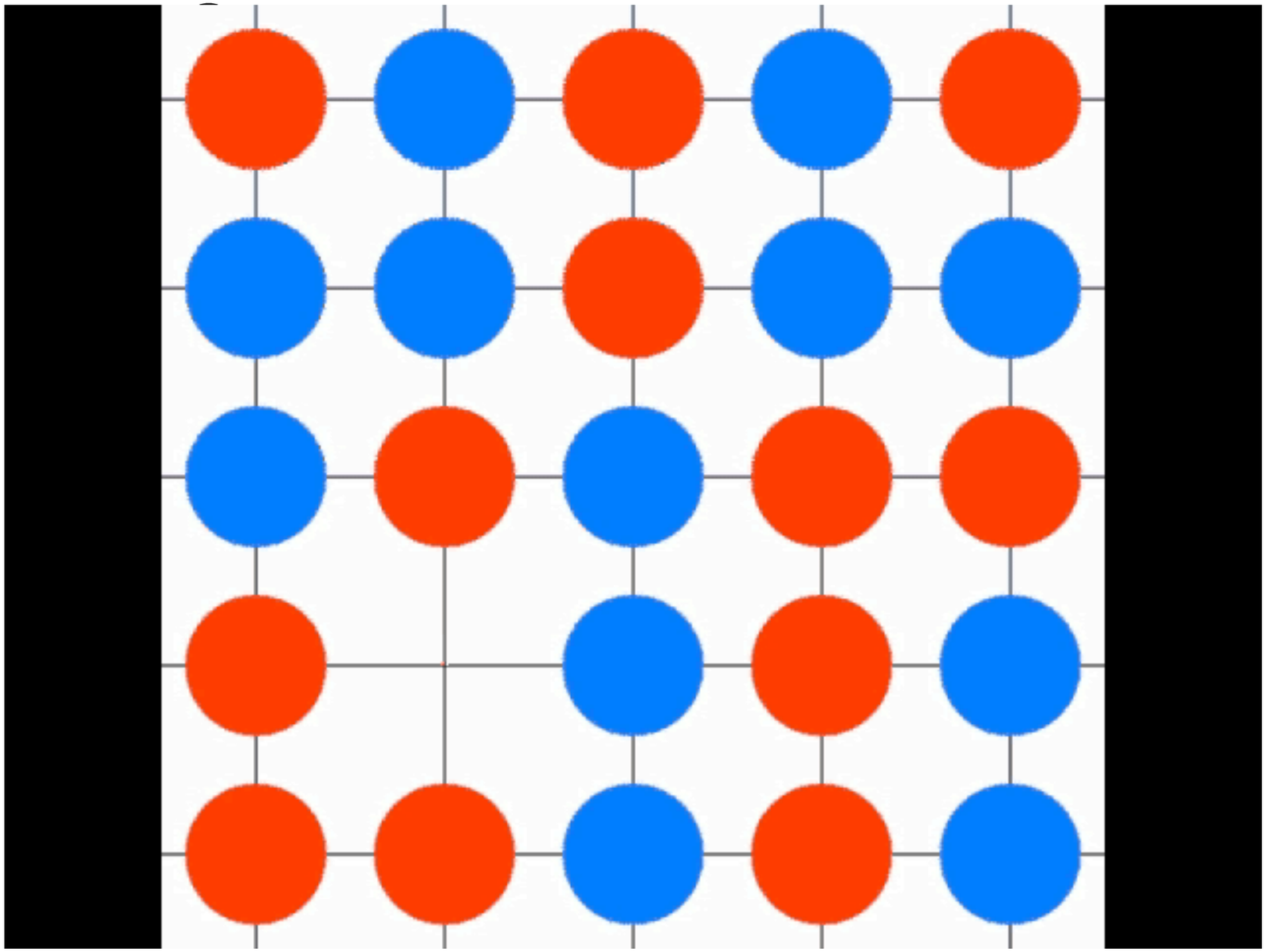
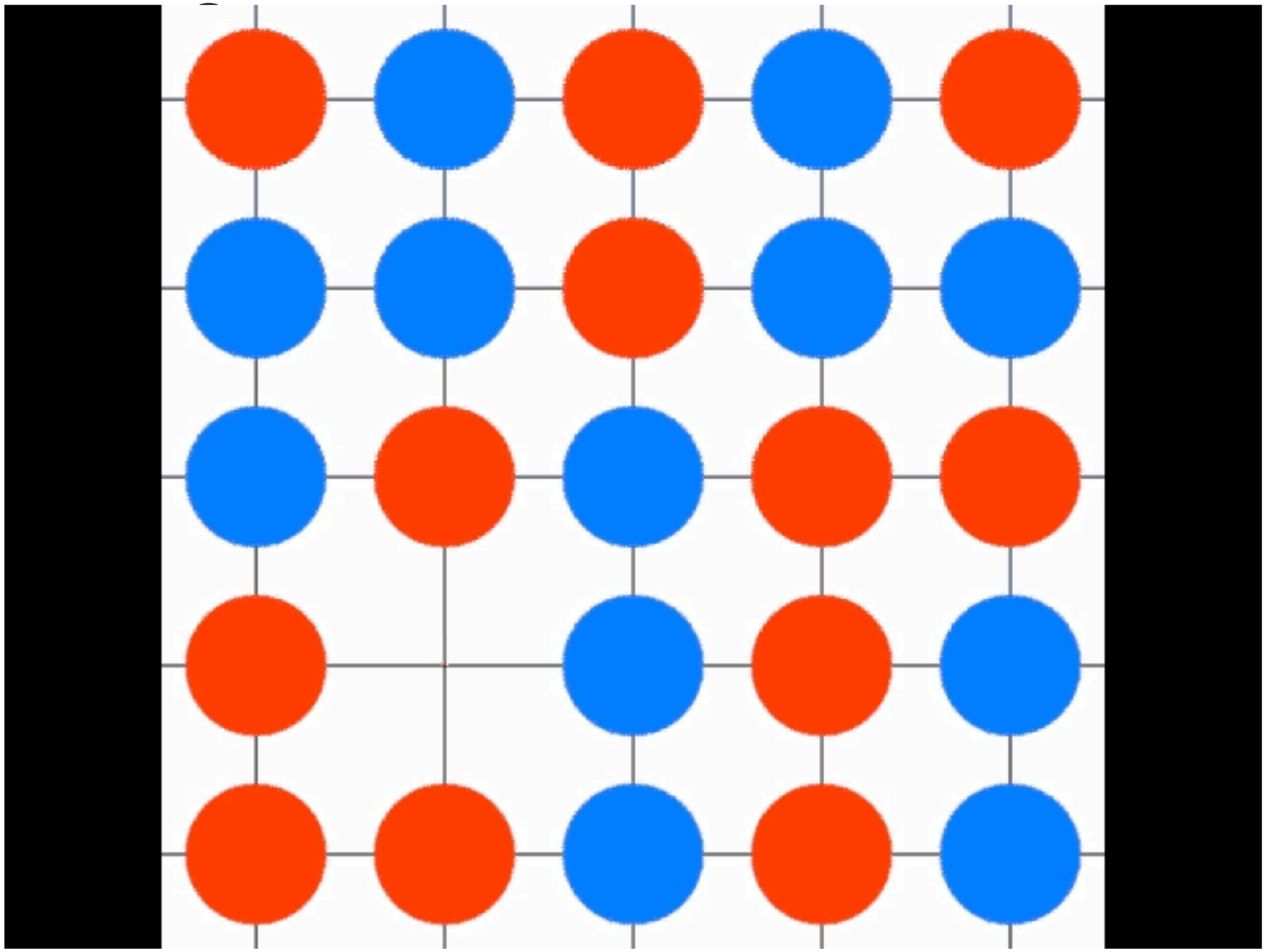


double occupancy in ground state!!

U finite $U \gg t$



double occupancy in ground state!!

$W_{\mathrm{PES}} > 1 + x$

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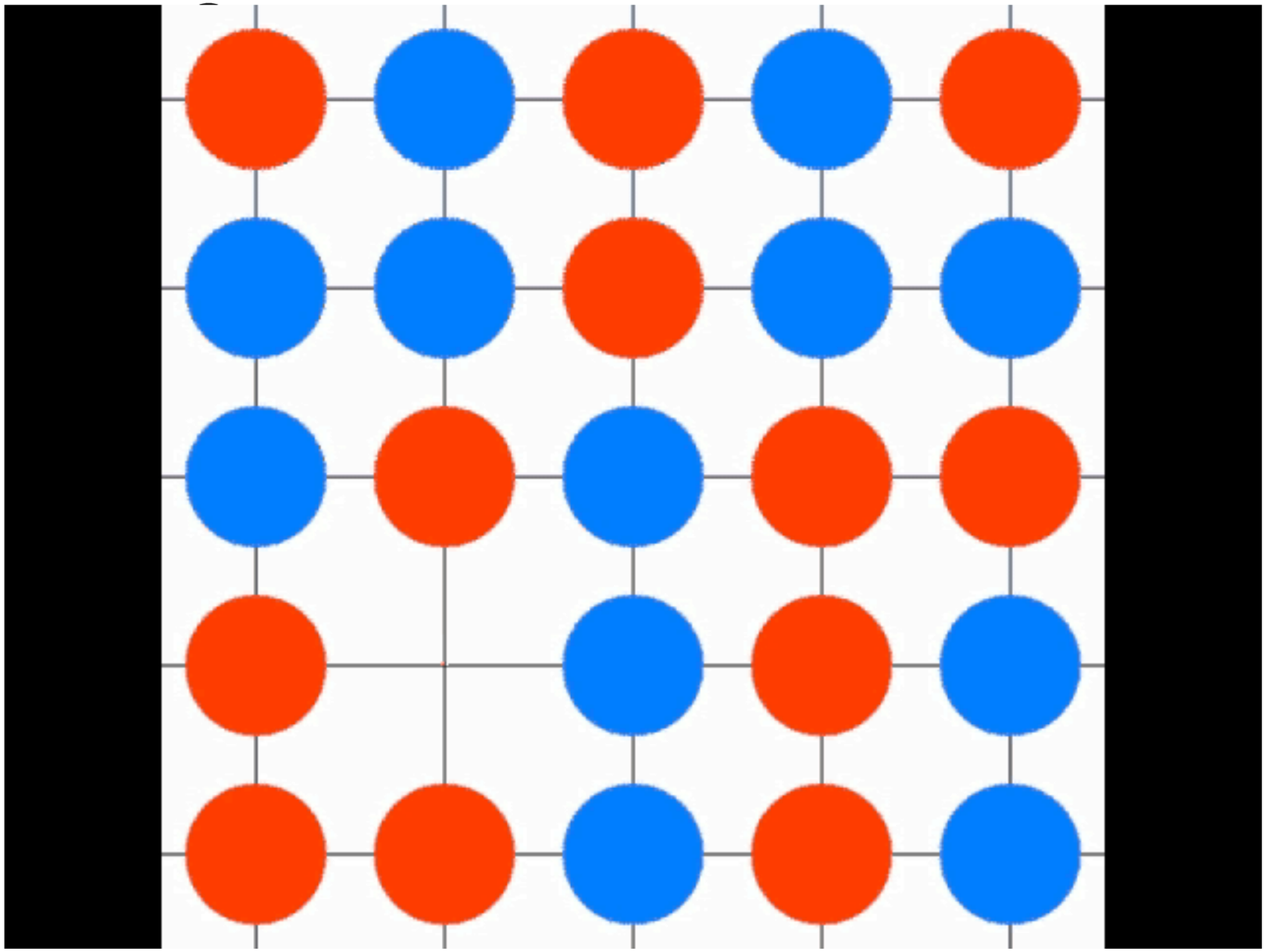


$N_{\text{eff}} \neq \#x$

double occupancy in ground state!!

$W_{\text{PES}} > 1 + x$

why is this a problem?

counting electron states

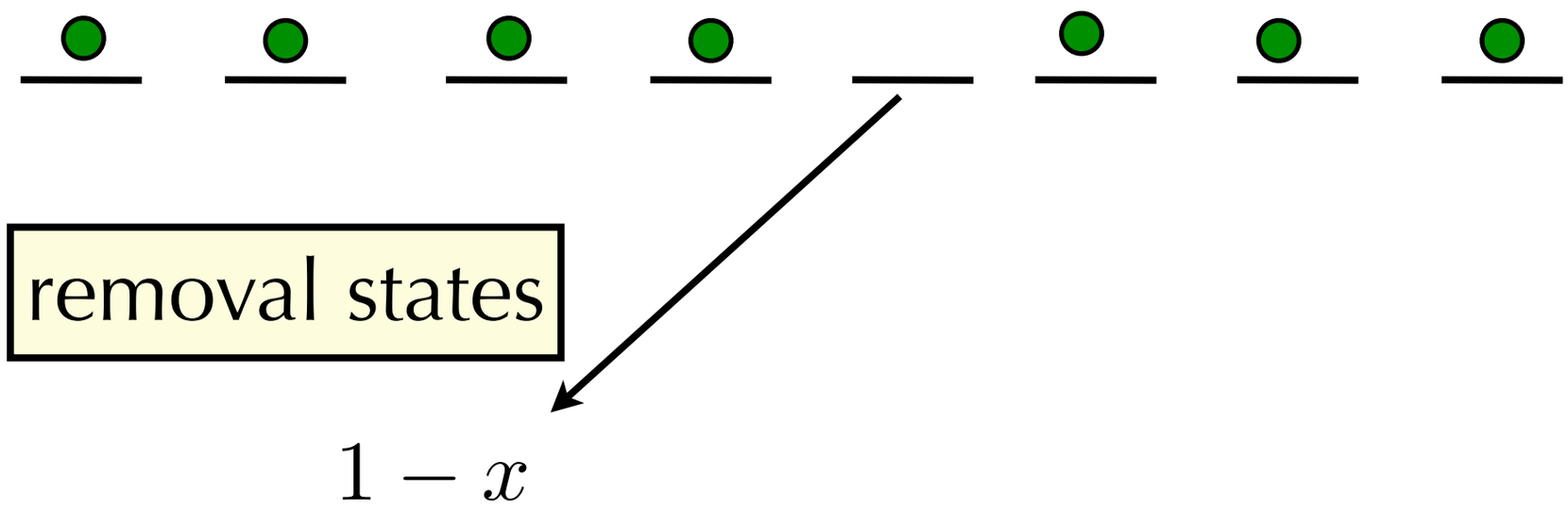
need to know: N (number of sites)

counting electron states

$$x = n_h/N$$

need to know: N (number of sites)

counting electron states



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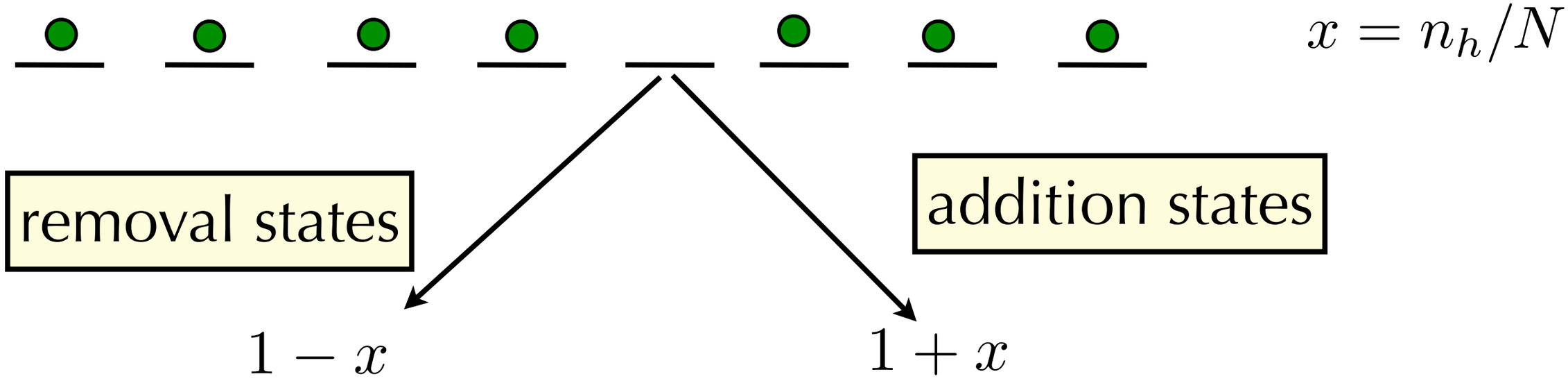
counting electron states



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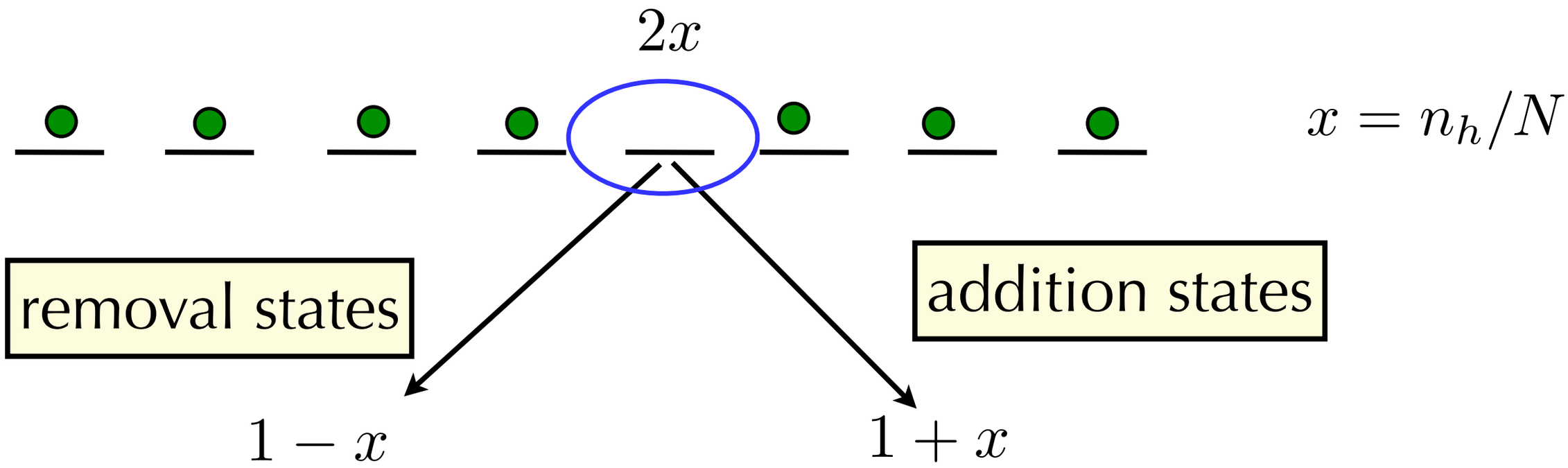
counting electron states



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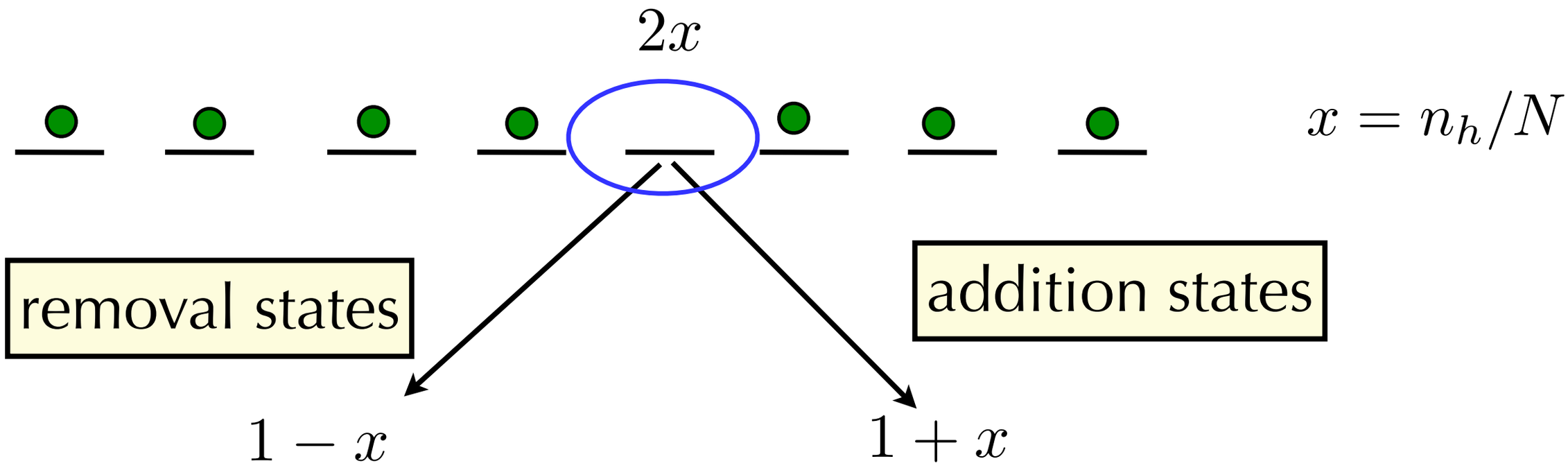
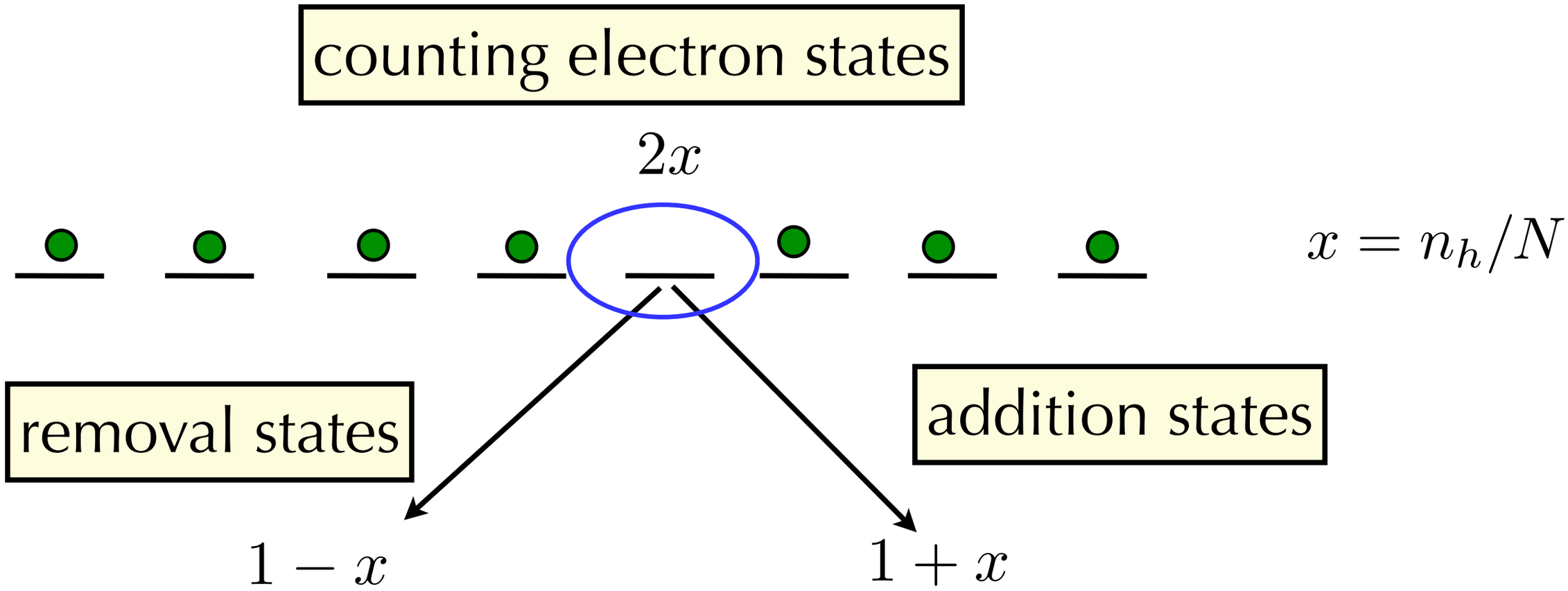
counting electron states



low-energy electron states

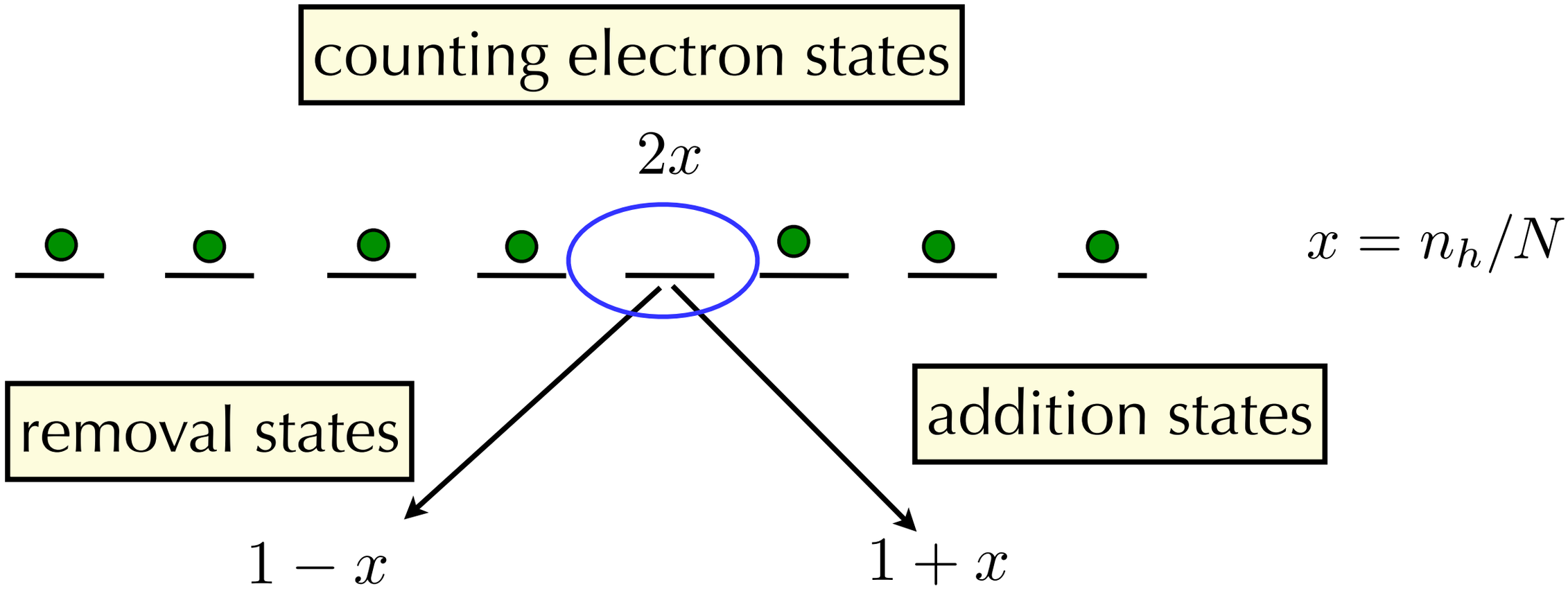
need to know: N (number of sites)



low-energy electron states

$$1 - x + 2x = 1 + x$$

need to know: N (number of sites)



low-energy electron states

$$1 - x + 2x = 1 + x$$

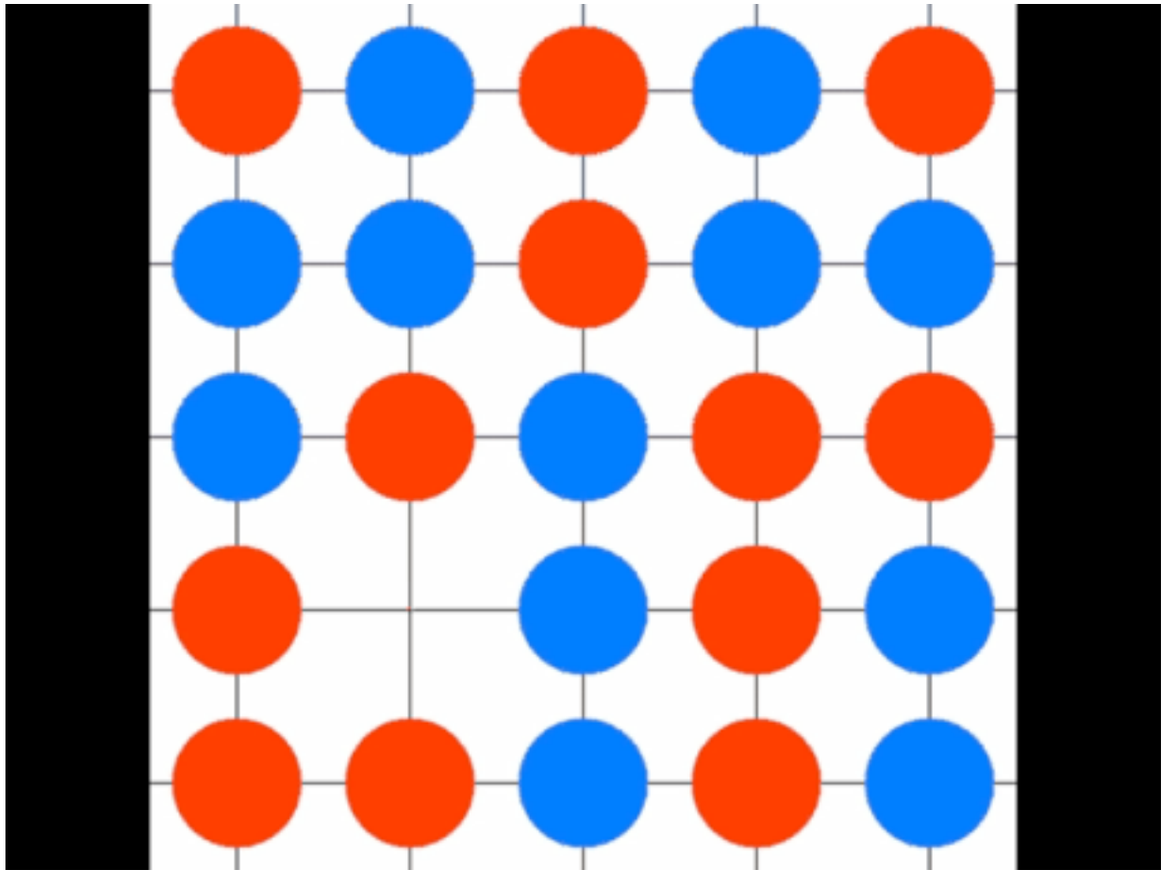
high energy

$$1 - x$$

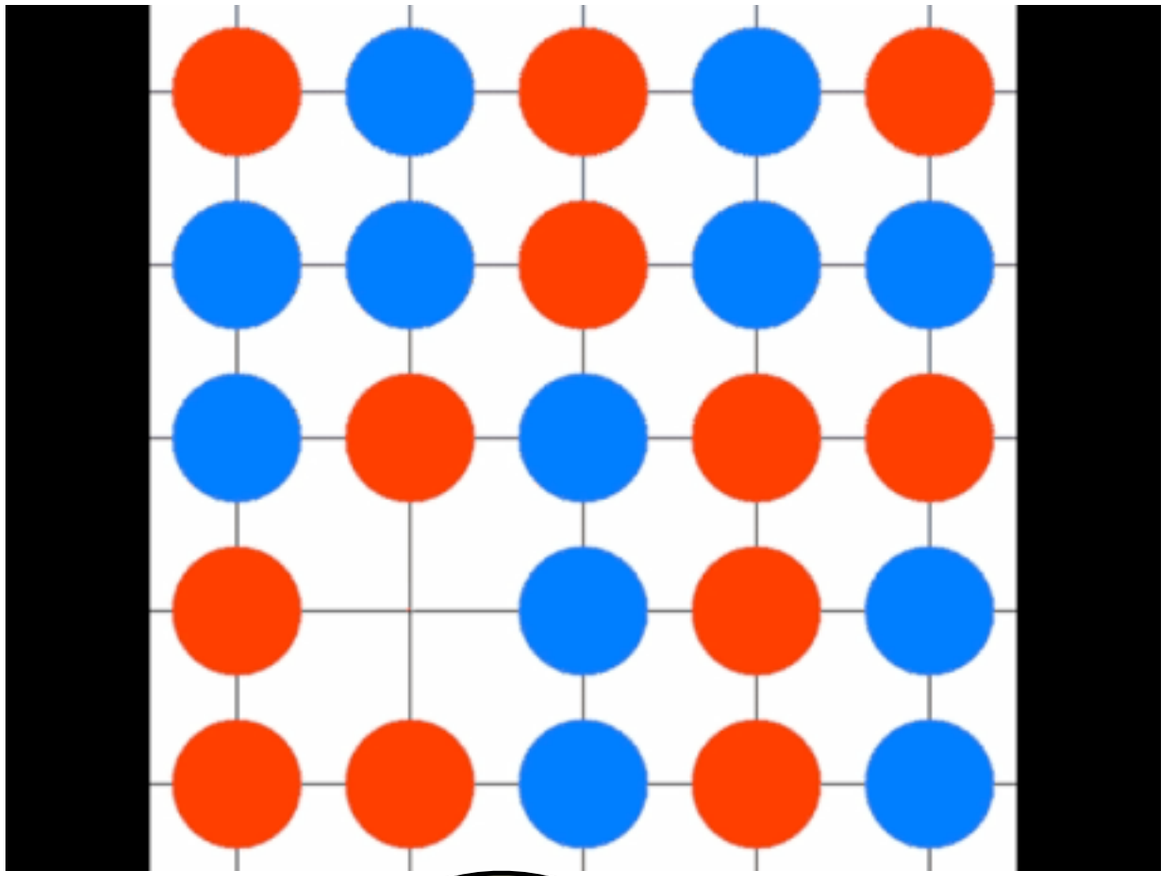
need to know: N (number of sites)

spectral function (dynamics)

spectral function (dynamics)

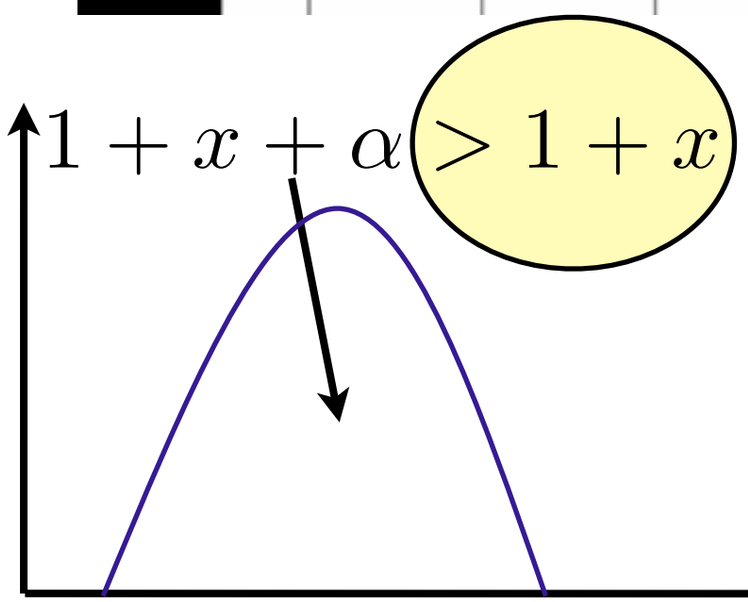


spectral function (dynamics)



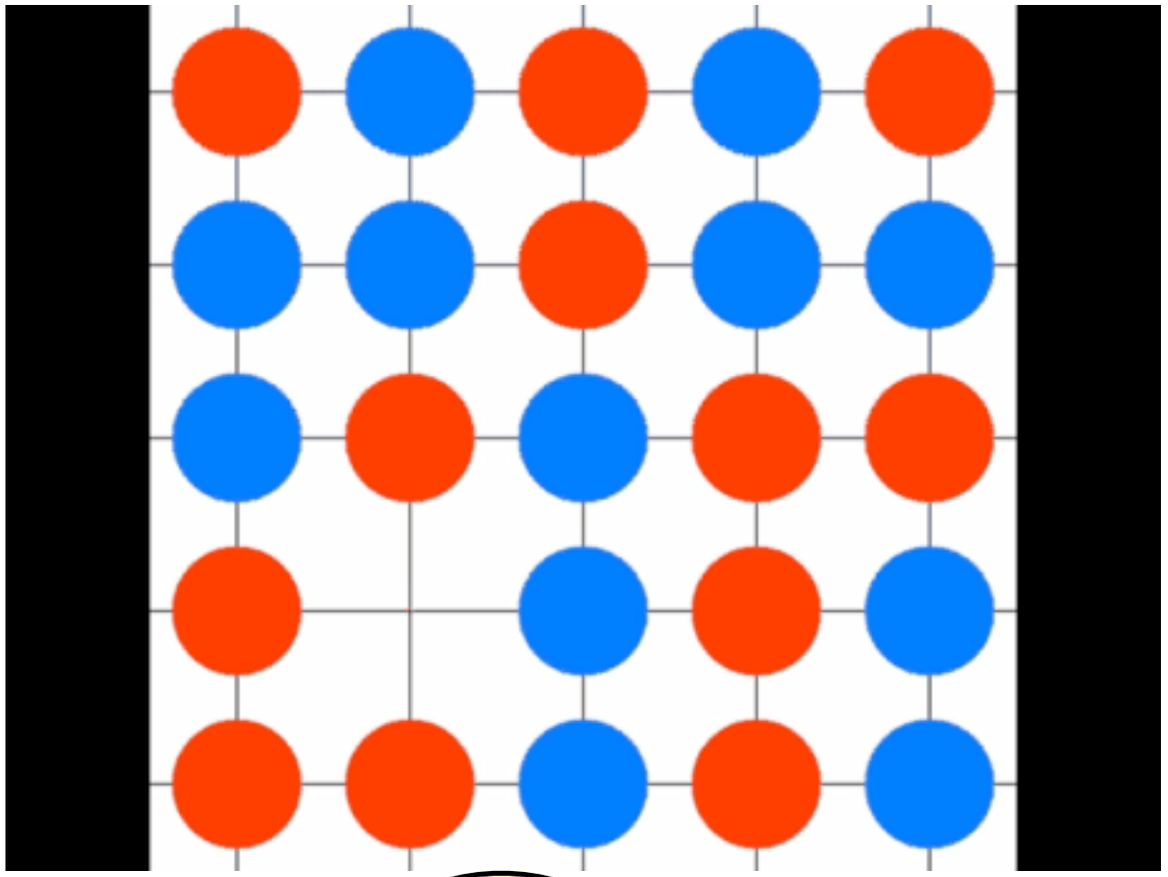
$U \gg t$

density of states



Harris and Lange (1967)

spectral function (dynamics)



$U \gg t$

density of states

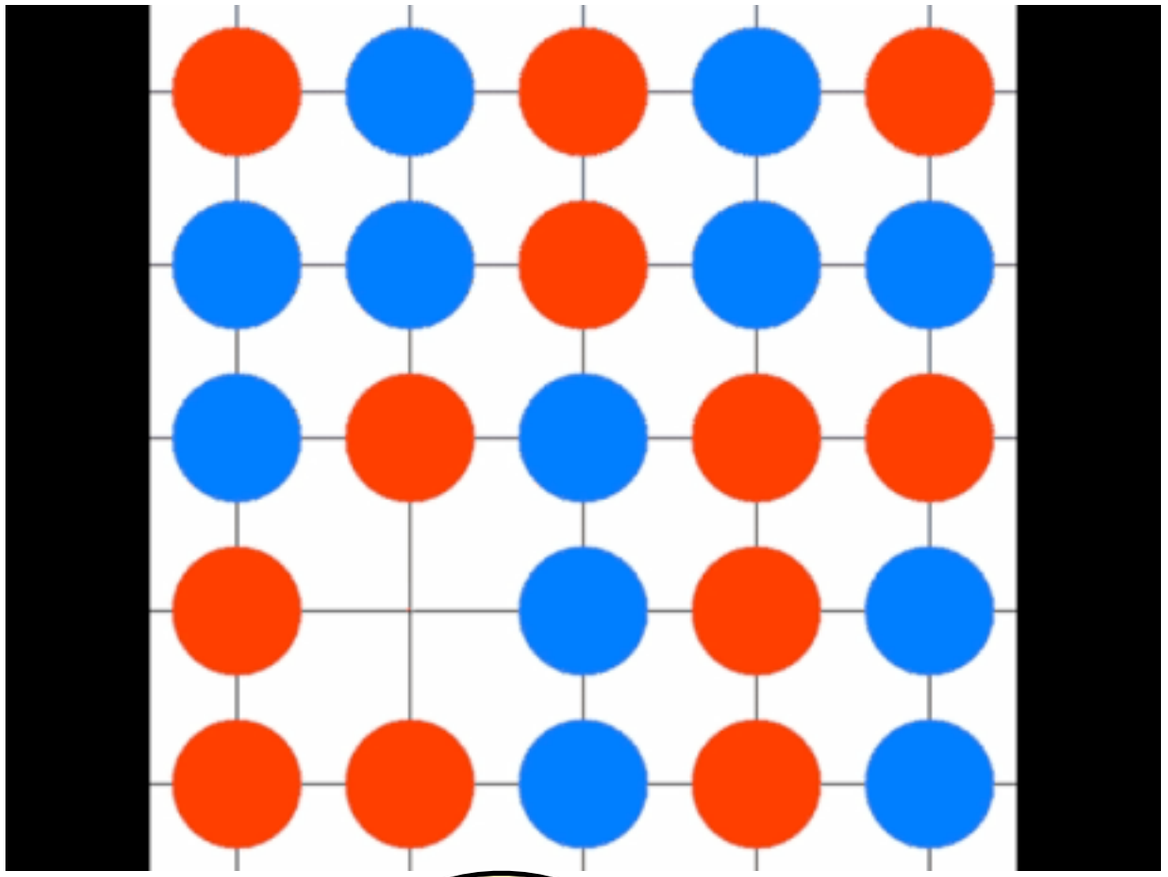
$1 + x + \alpha$ $> 1 + x$

$\alpha \propto t/U$

$1 - x - \alpha$

Harris and Lange (1967)

spectral function (dynamics)



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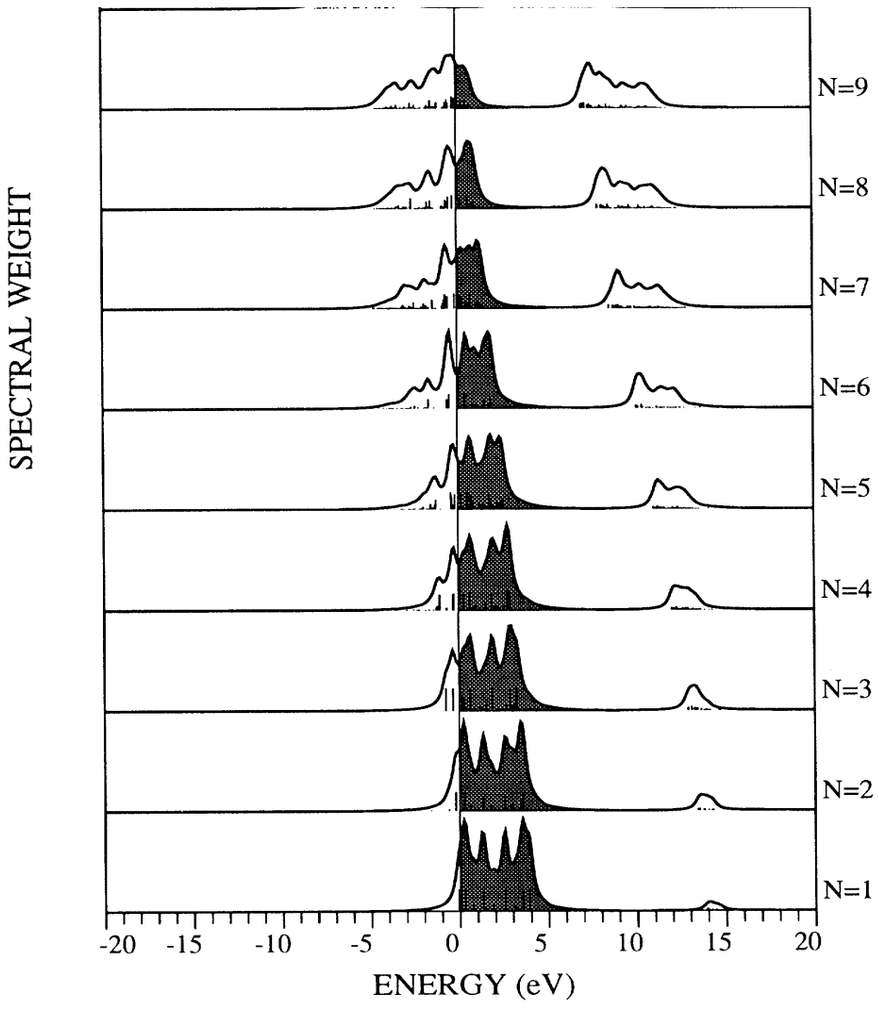
$1 - x - \alpha$

Harris and Lange (1967)

not exhausted by counting electrons alone

determination of N_{eff}

Hubbard model



x=0.1

x=0.2

x=0.3

x=0.4

x=0.5

x=0.6

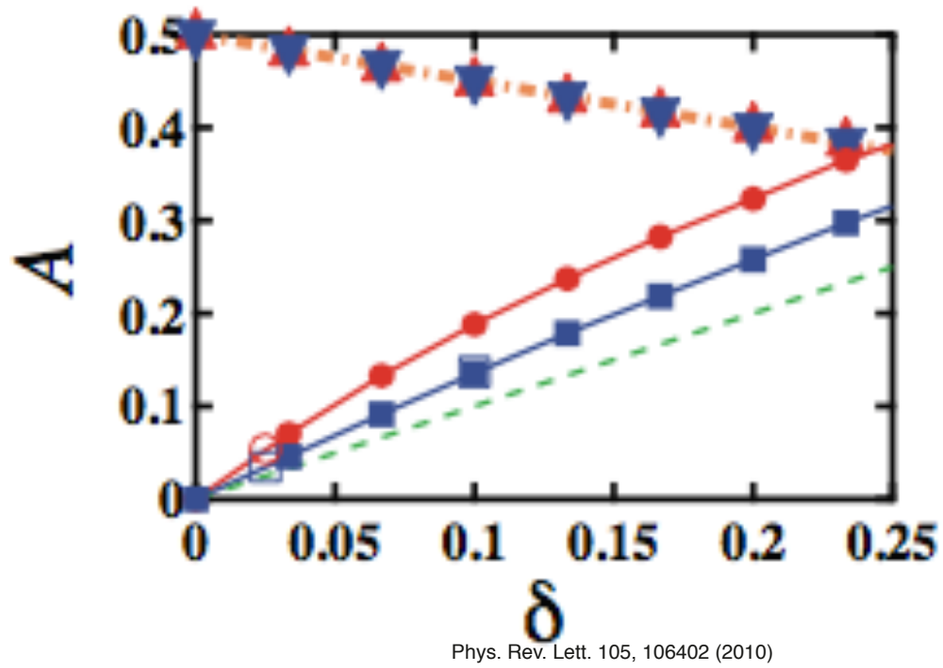
x=0.7

x=0.8

x=0.9

Sawatzky

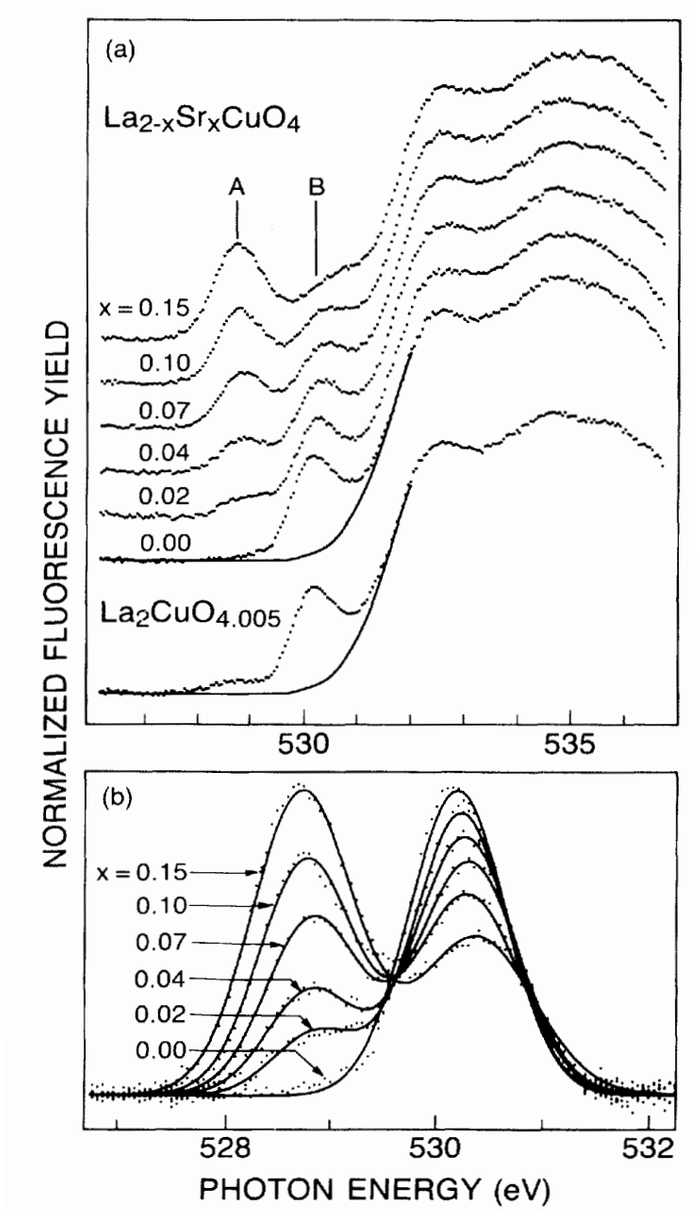
d=1

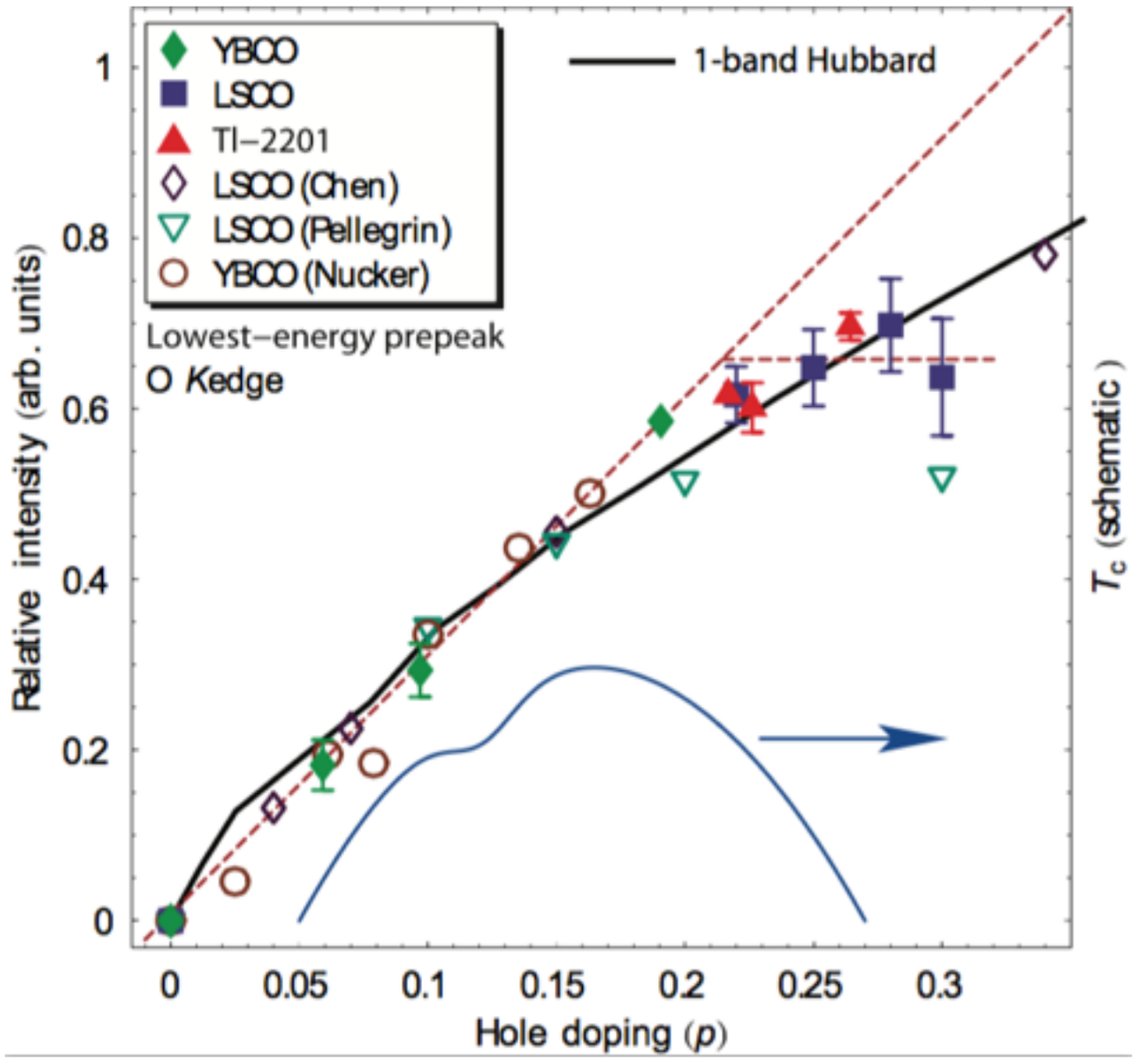


Phys. Rev. Lett. 105, 106402 (2010)

M. Kohno, PRL, 105, 106402

determination of $N_{\rm eff}$



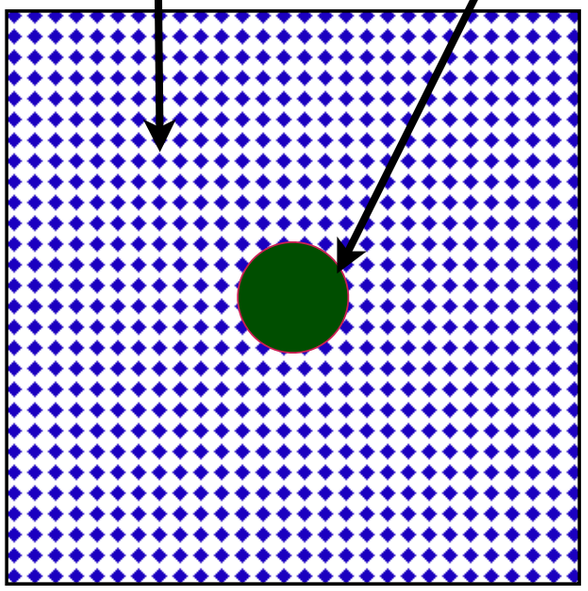


chen/Batlogg, 1990

non-interacting

interacting

DMFT on Hubbard model



DMFT

DMFT on Hubbard model

DMFT on Hubbard model

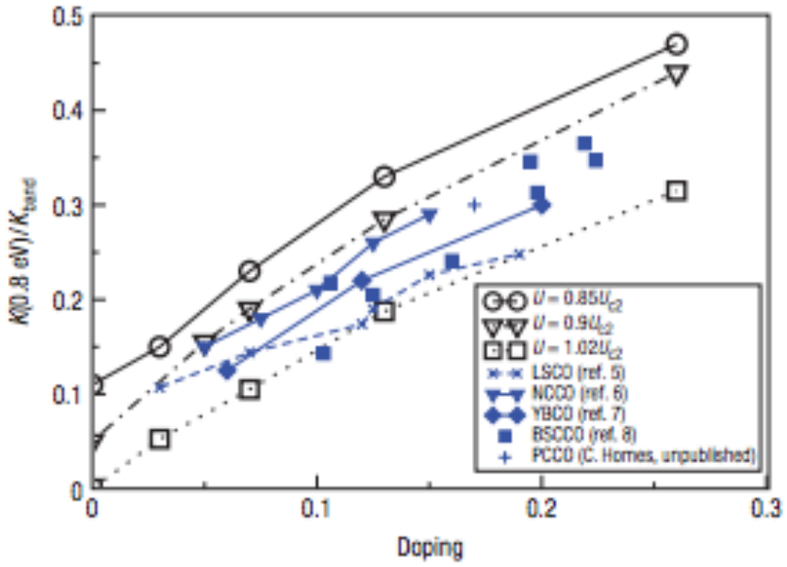
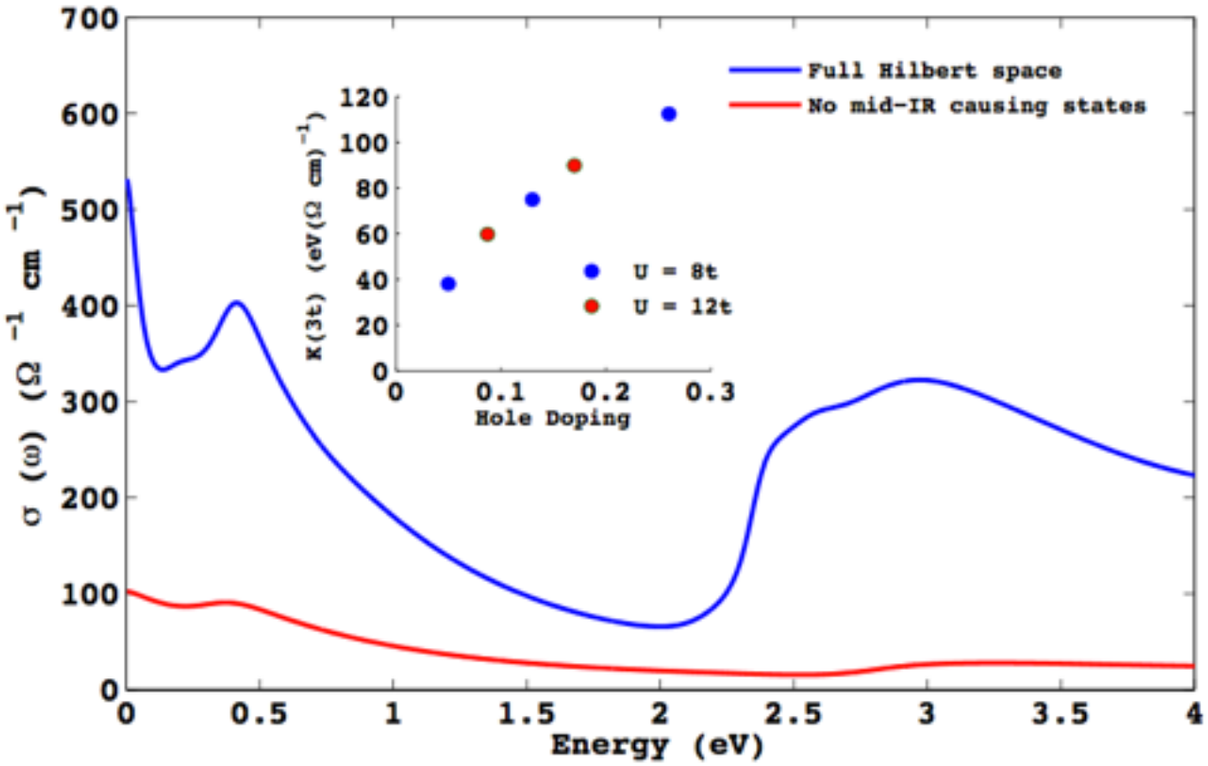
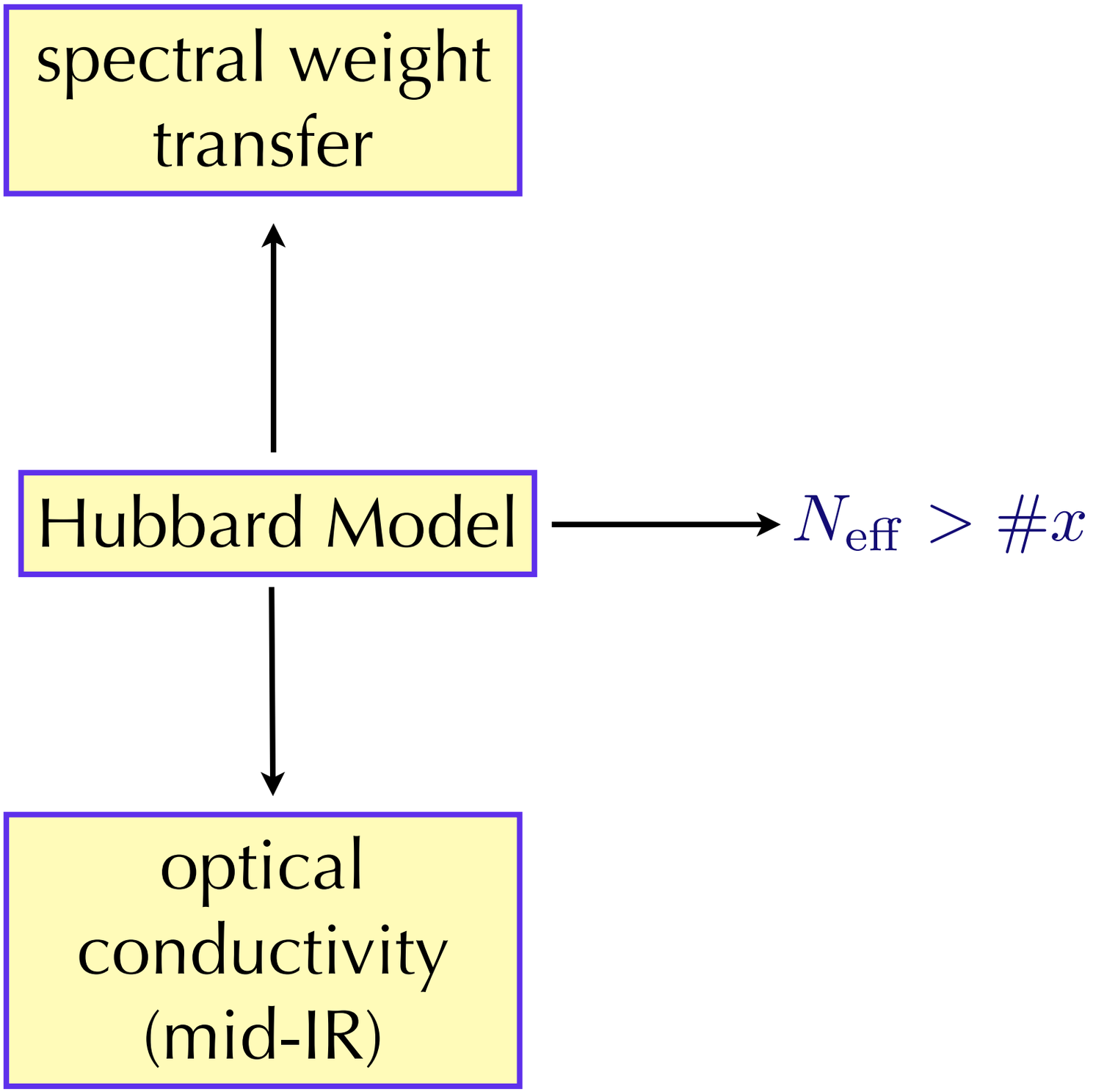


Figure 3 Comparison of measured and calculated optical spectral weight. Filled symbols: spectral weight obtained by integrating experimental conductivity up to 0.8 eV from references given. Open symbols: theoretically calculated spectral weight, integrated up to $W/4$. For $U = 0.85U_{c2}$ and $U = 0.9U_{c2}$, the band-theory estimate $W = 3$ eV is used to convert the calculation to physical units; for $U = 1.02U_{c2}$, the value $W = 2.25$ eV which reproduces the insulating gap is used.

Millis, 2008



chakraborty & Phillips, 2007


spectral weight transfer
Hubbard Model
$N_{\rm eff} > \#x$
optical conductivity (mid-IR)

is there
anything else?

yes

Quantum critical behaviour in a high-T_c superconductor

D. van der Marel[1]*, H. J. A. Molegraaf[1]*, J. Zaanen[2], Z. Nussinov[2]*, F. Carbone[1]*, A. Damascelli[3]*, H. Eisaki[3]*, M. Greven[3], P. H. Kes[2] & M. Li[2]

[1]*Materials Science Centre, University of Groningen, 9747 AG Groningen, The Netherlands*
[2]*Leiden Institute of Physics, Leiden University, 2300 RA Leiden, The Netherlands*
[3]*Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA*

Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

yes

Quantum critical behaviour in a high-T_c superconductor

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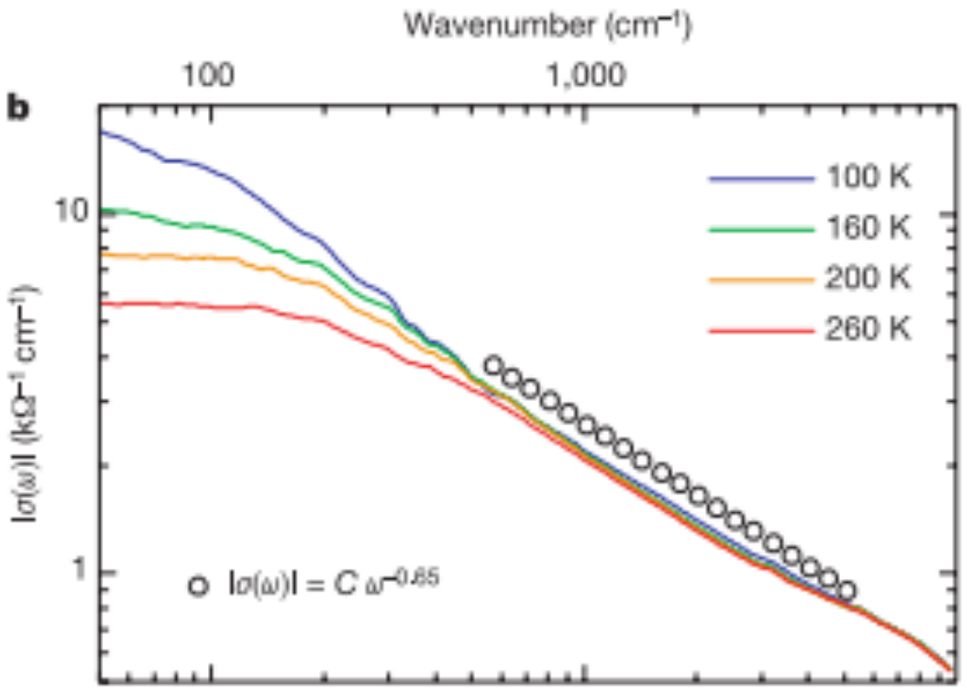
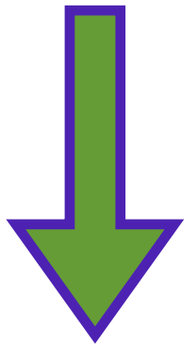


Drude conductivity

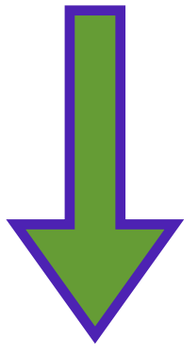
$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

$$\sigma(\omega) = C\omega^{-\frac{2}{3}}$$

criticality



scale invariance



power law correlations

scale-invariant propagators

$$\left(\frac{1}{p^2}\right)^{\alpha}$$

scale-invariant propagators

$$\left(\frac{1}{p^2}\right)^{\alpha}$$

Anderson: use Luttinger Liquid propagators

$$G^R \propto \frac{1}{(\omega - v_s k)^{\eta}}$$

compute conductivity without vertex corrections (PWA)

is flawed. In fact, in the Luttinger liquid such direct calculations are not to be trusted very firmly, since it is the nature of the Luttinger liquid that vertex corrections, if they must be included, will be singular; conventional transport theory is not applicable, and special methods such as the above are necessary.

$$\sigma(\omega) \propto \frac{1}{\omega} \int dx \int dt G^e(x,t) G^h(x,t) e^{i\omega t} \propto (i\omega)^{-1+2\eta}$$

problems

problems

1.) cuprates are not 1-dimensional

problems

1.) cuprates are not 1-dimensional

2.) vertex corrections matter

problems

1.) cuprates are not 1-dimensional

2.) vertex corrections matter

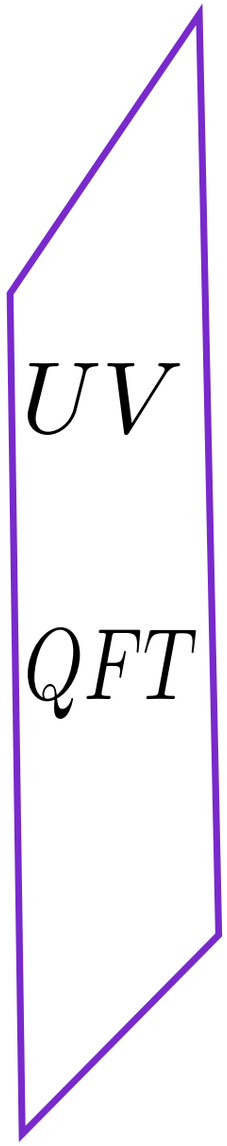
$$\left.\begin{aligned} \sigma &\propto G^2 \Gamma^\mu \Gamma^{\mu\nu} \\ [G] &= L^{d+1-d_U} \\ [\Gamma^\mu] &= L^{2d_U - d} \\ [\Gamma^{\mu\nu}] &= L^{2d_U - d + 1} \end{aligned}\right\} \quad [\sigma] = L^{3-d}$$

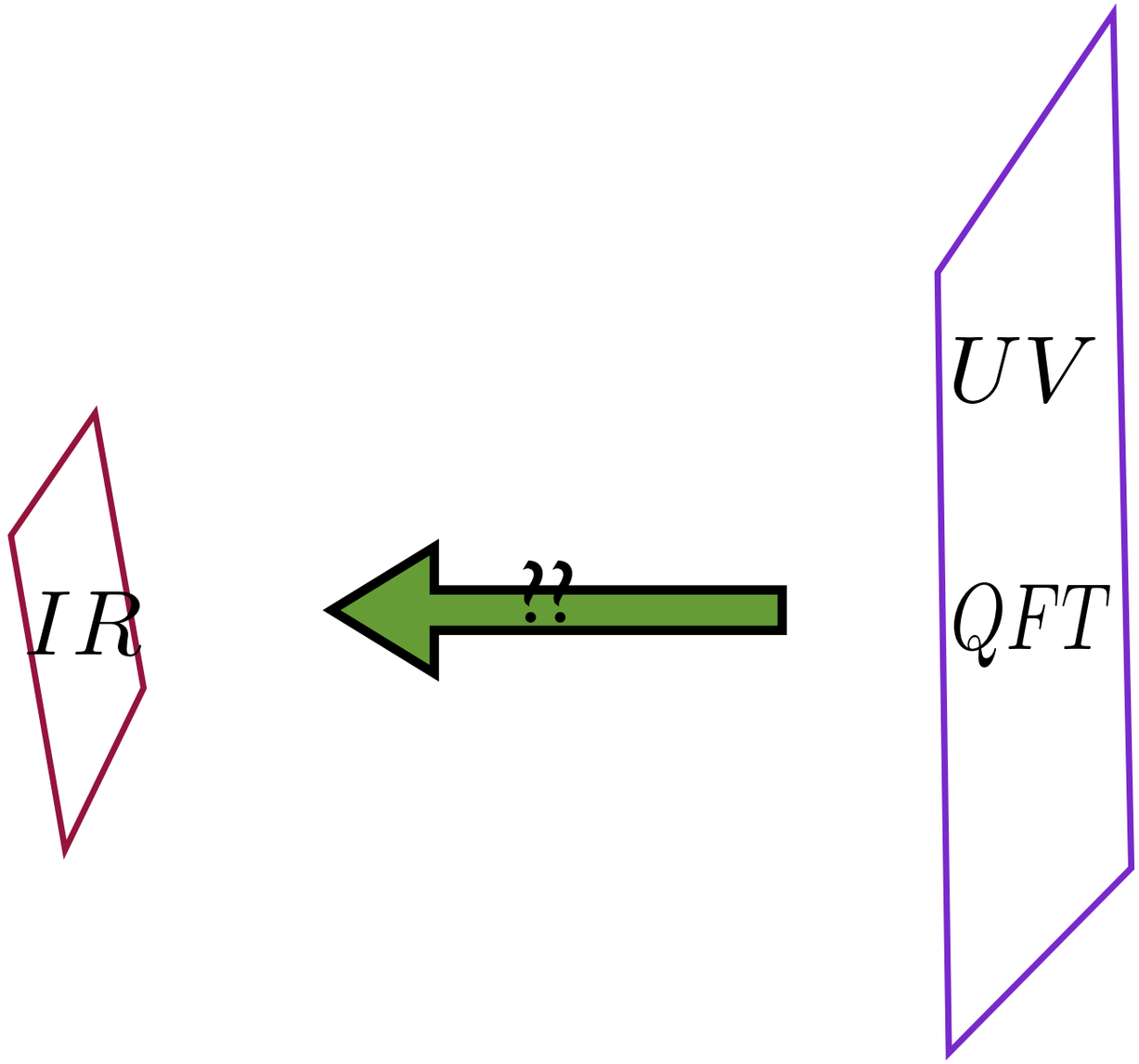
independent of d_U

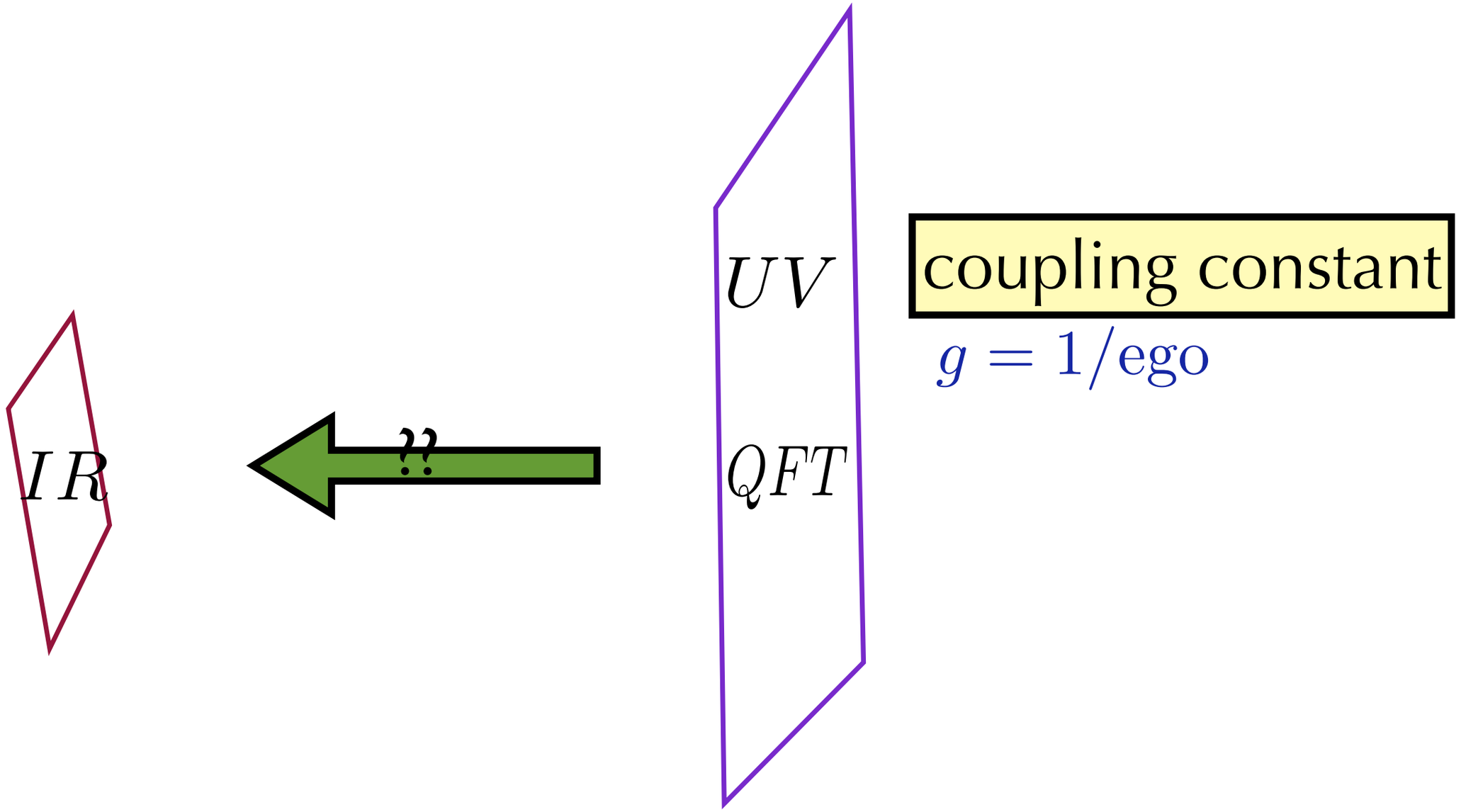
power law?

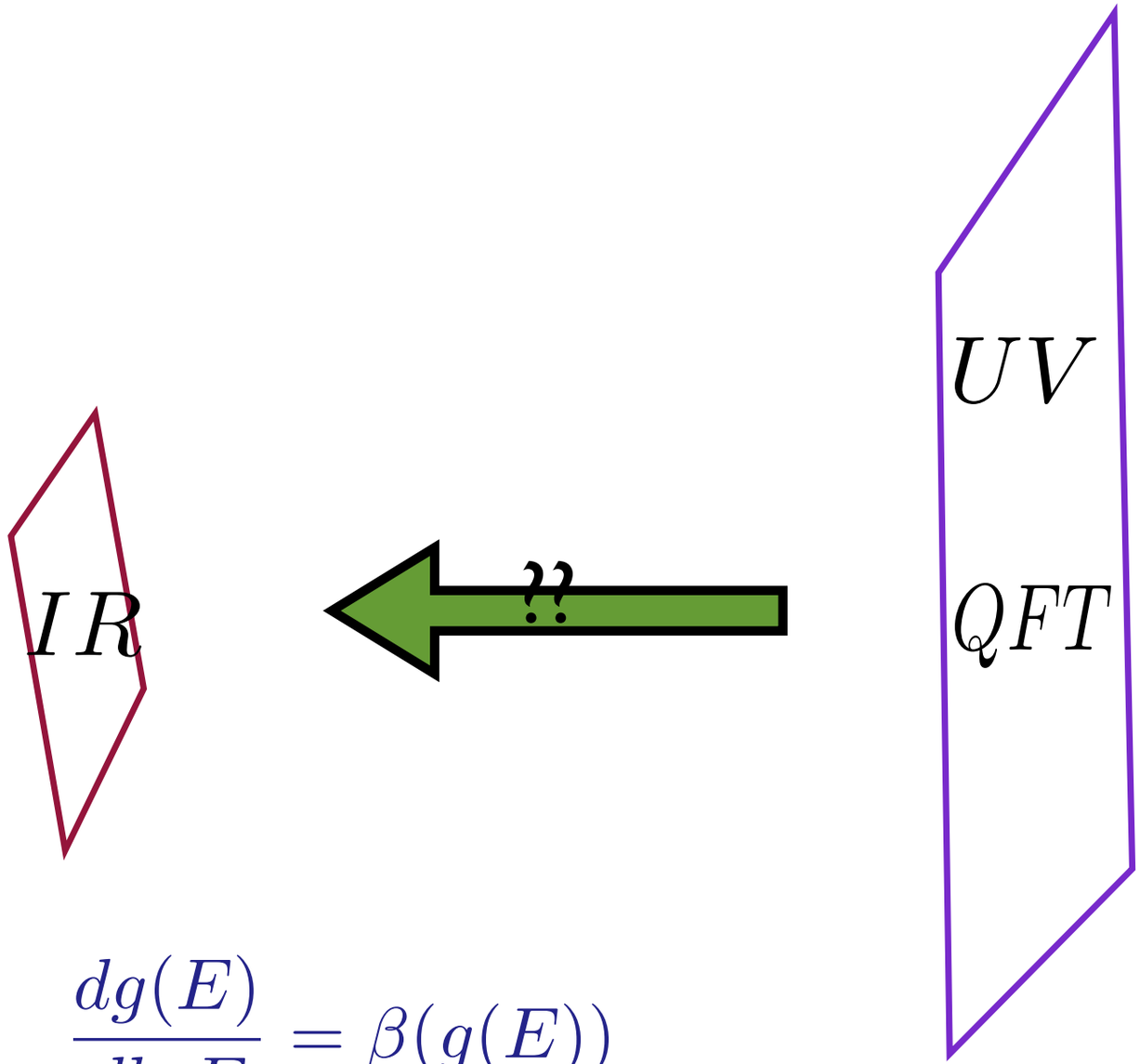
power law?

Could string theory be the answer?


UV
QFT


UV
IR
??
QFT


UV
coupling constant
$g = 1/\mathrm{ego}$
IR
??
QFT

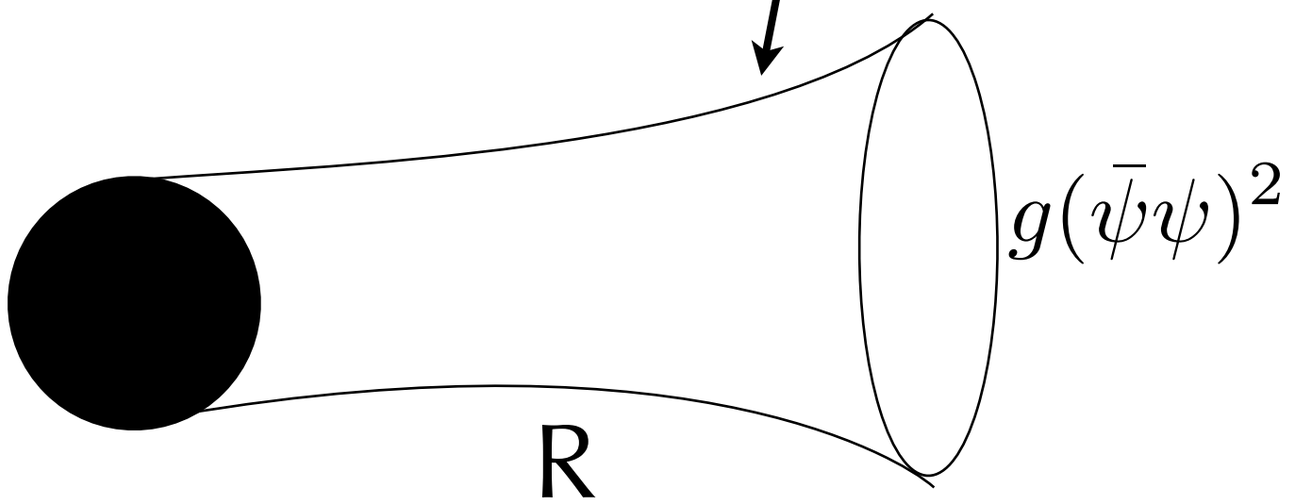


$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

locality in energy

replace coupled theory with geometry

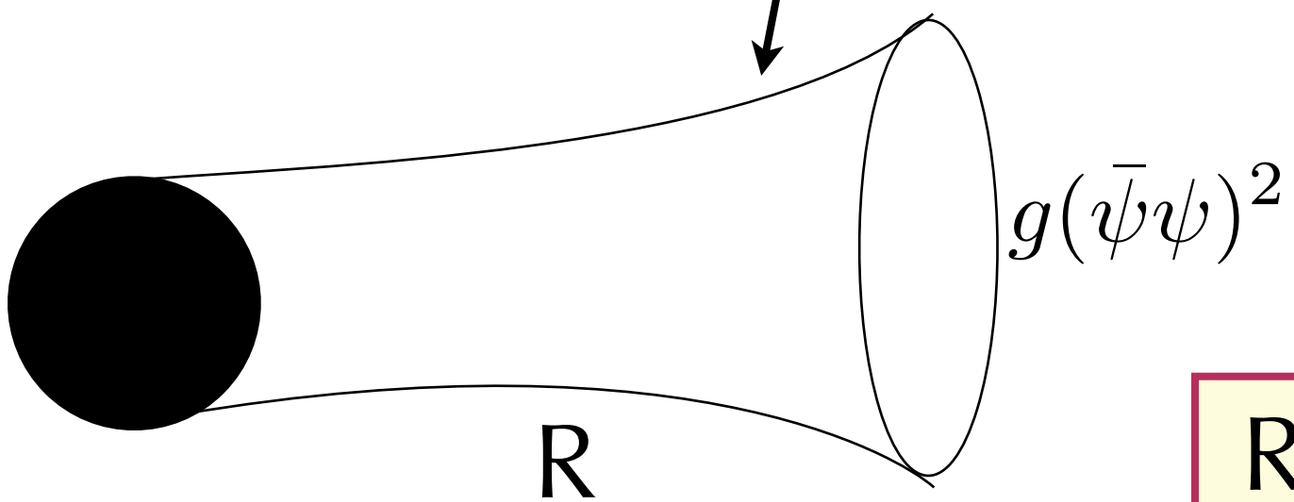
scale-invariant anti de-Sitter



$$R \propto \sqrt{g}$$

replace coupled theory with geometry

scale-invariant anti de-Sitter

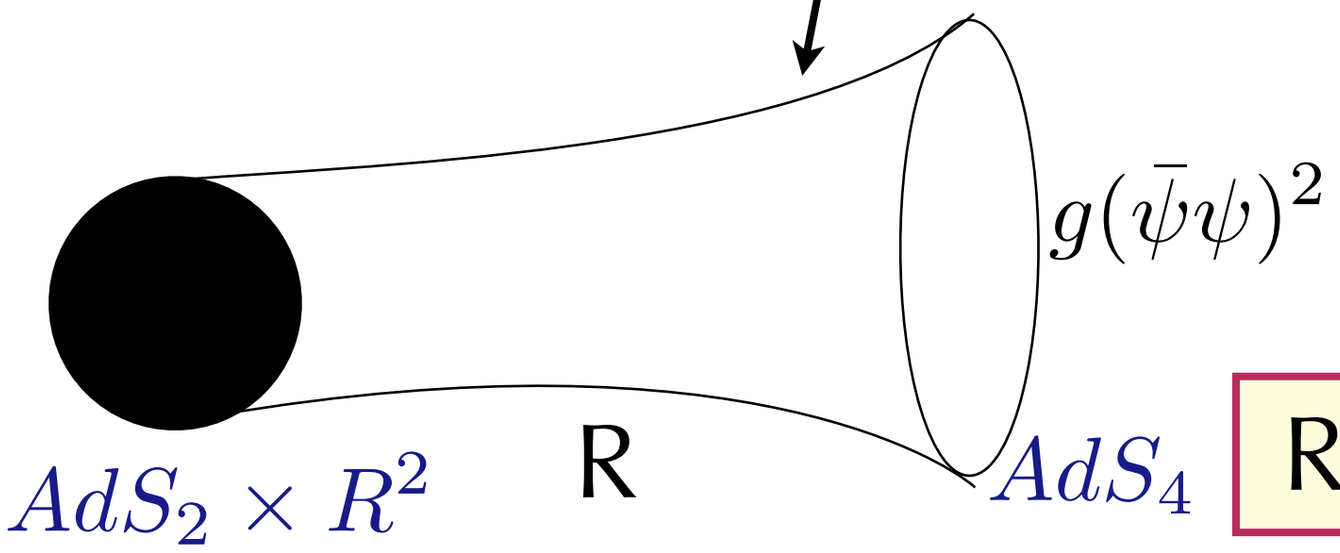


RG=GR

$$R \propto \sqrt{g}$$

replace coupled theory with geometry

scale-invariant anti de-Sitter



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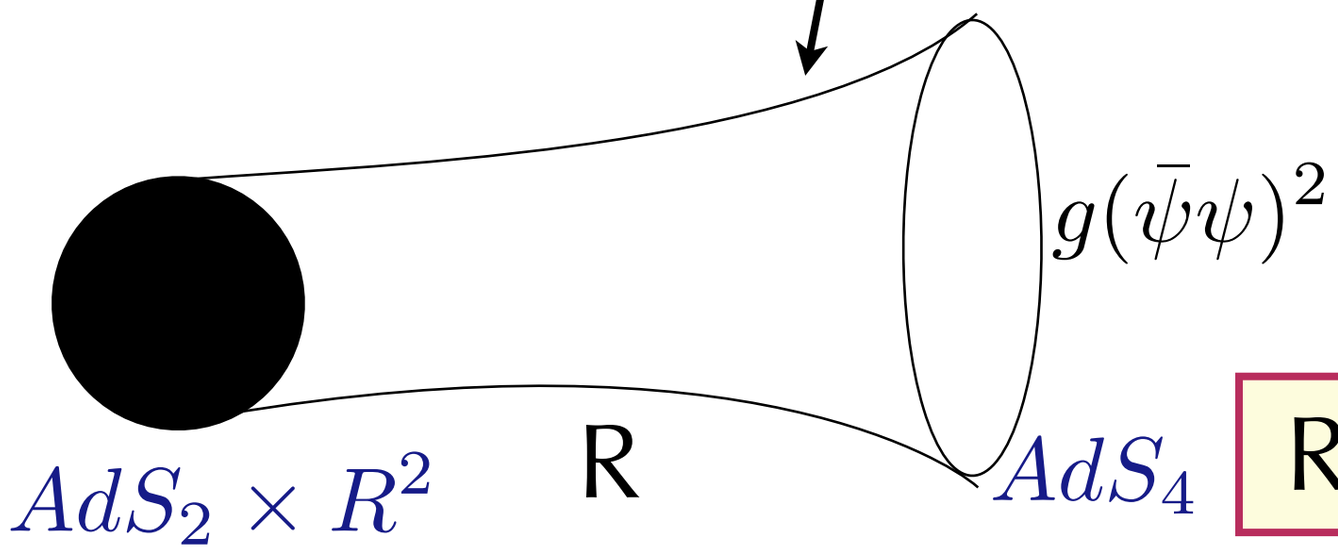


RG=GR

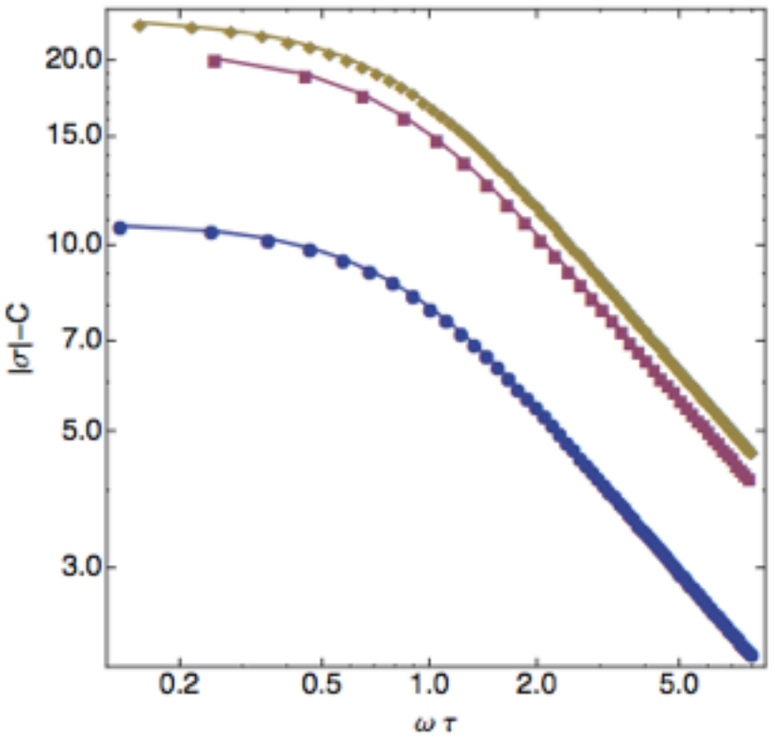
$$R \propto \sqrt{g}$$

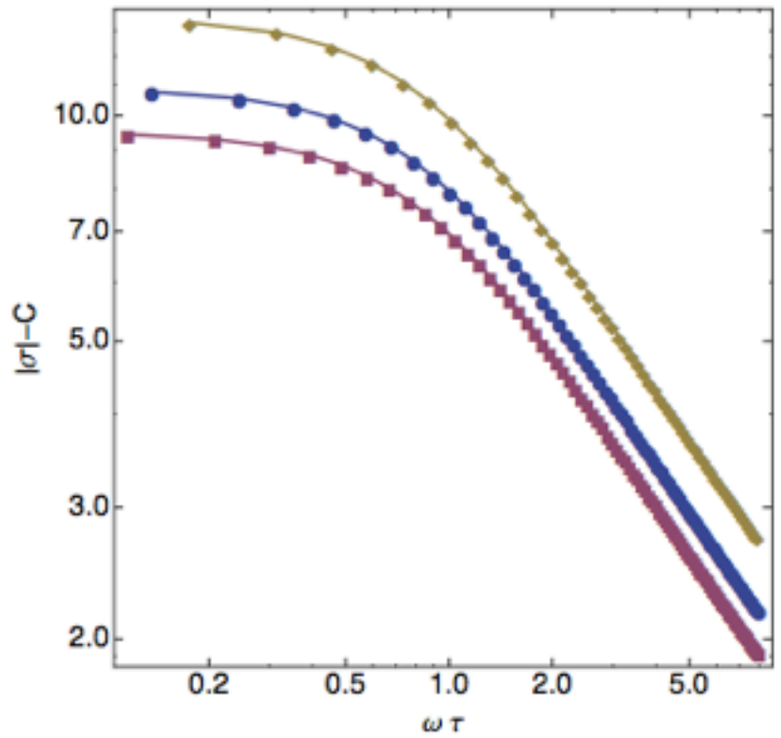
cannot describe systems at g=0!

optical conductivity from a gravitational lattice

G. Horowitz *et al*., Journal of High Energy Physics, 2012

optical conductivity from a gravitational lattice





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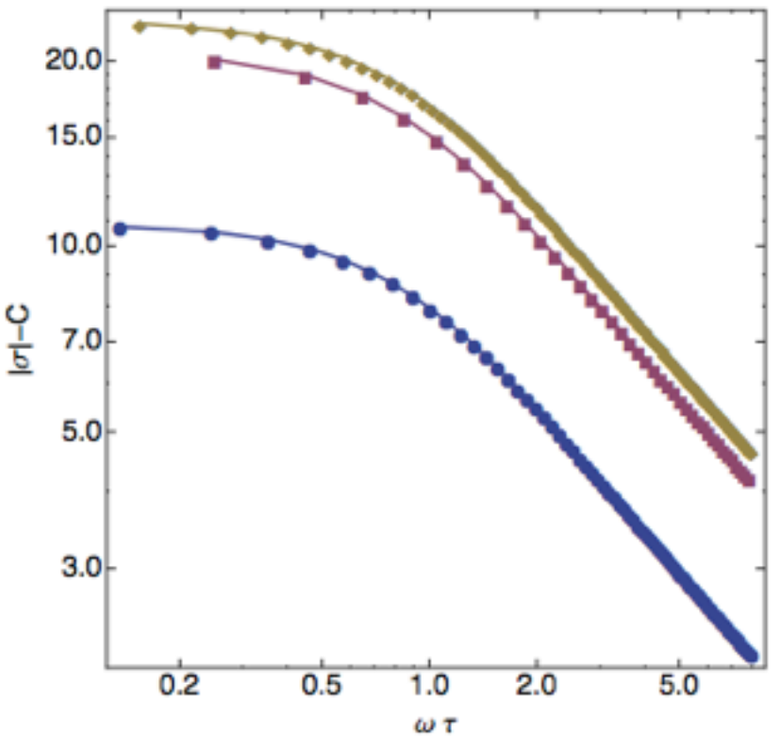
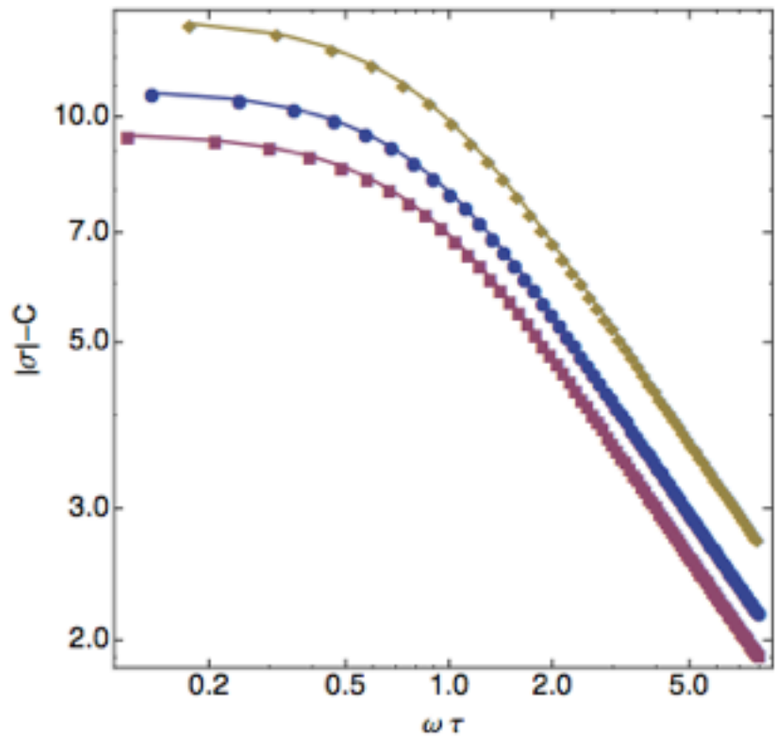
optical conductivity from a gravitational lattice

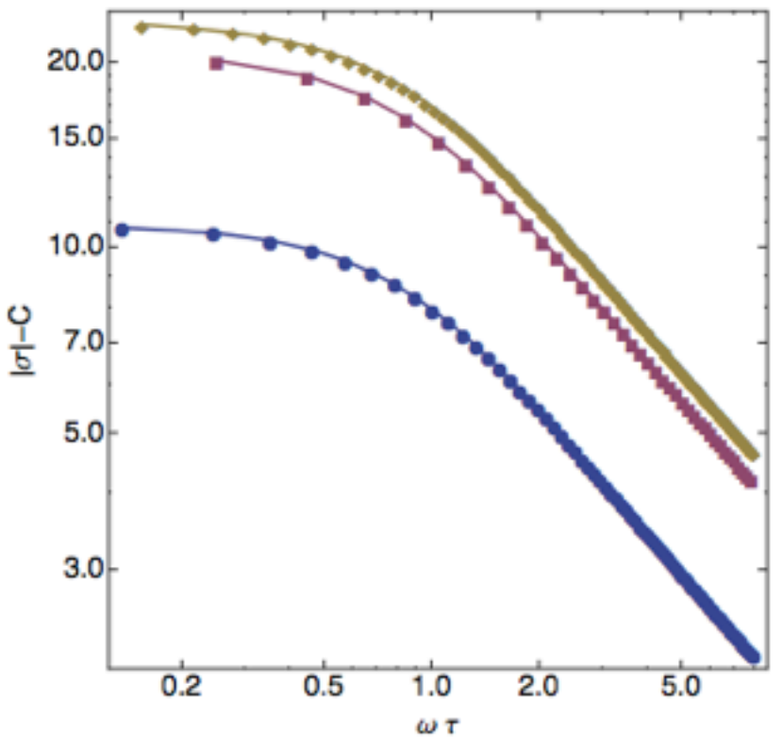


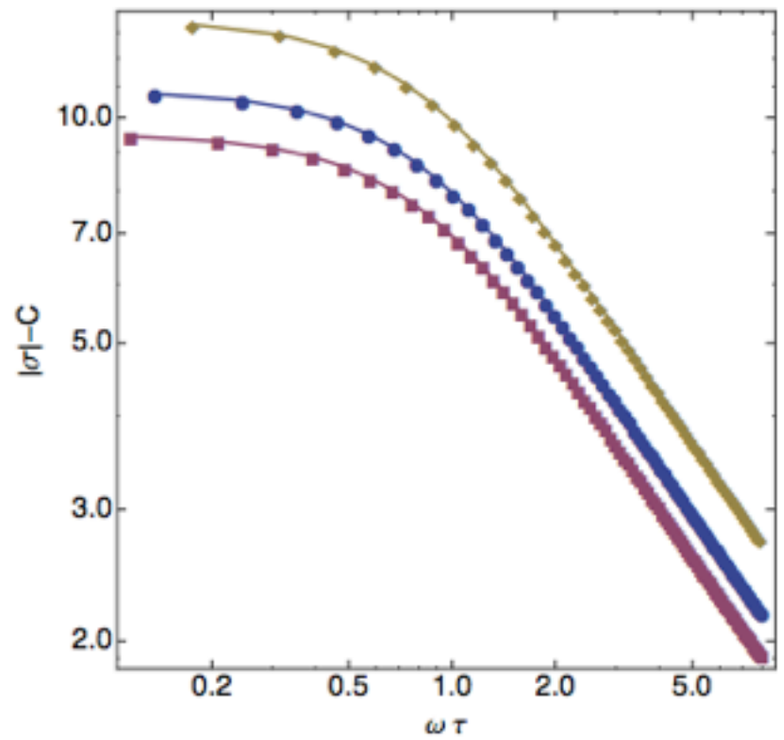


log-log plots for various parameters

G. Horowitz *et al*., Journal of High Energy Physics, 2012

optical conductivity from a gravitational lattice





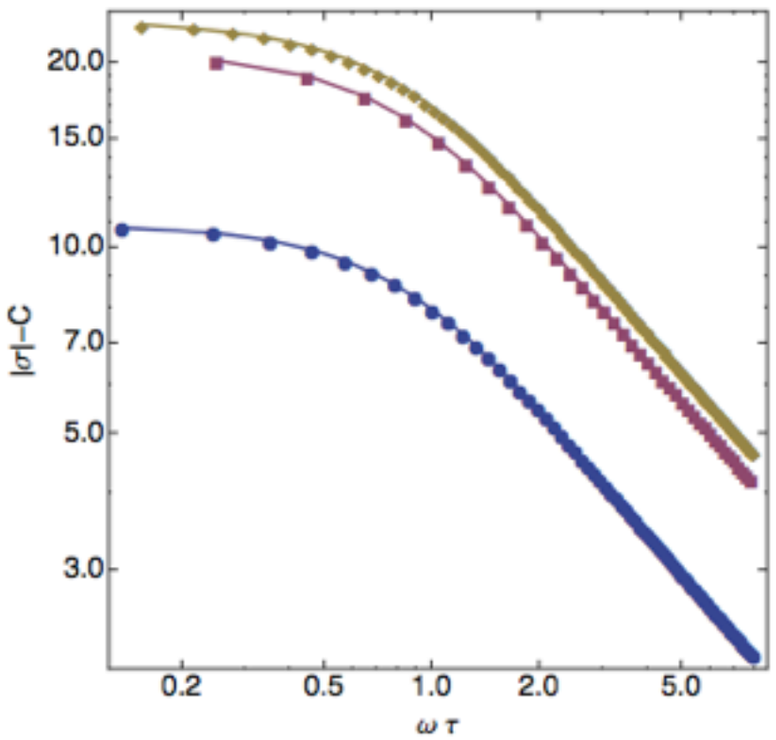
log-log plots for various parameters

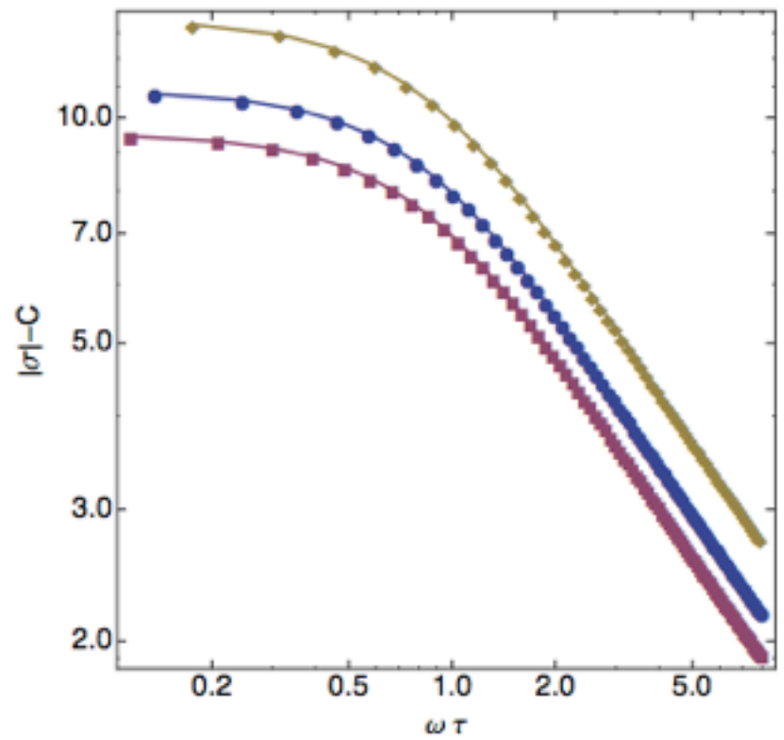
$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

for $0.2 \lesssim \omega\tau \lesssim 0.8$

G. Horowitz *et al*., Journal of High Energy Physics, 2012

optical conductivity from a gravitational lattice





log-log plots for various parameters

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

for $0.2 \lesssim \omega\tau \lesssim 0.8$

a remarkable claim!
replicates features of the strange metal? how?

G. Horowitz *et al*., Journal of High Energy Physics, 2012

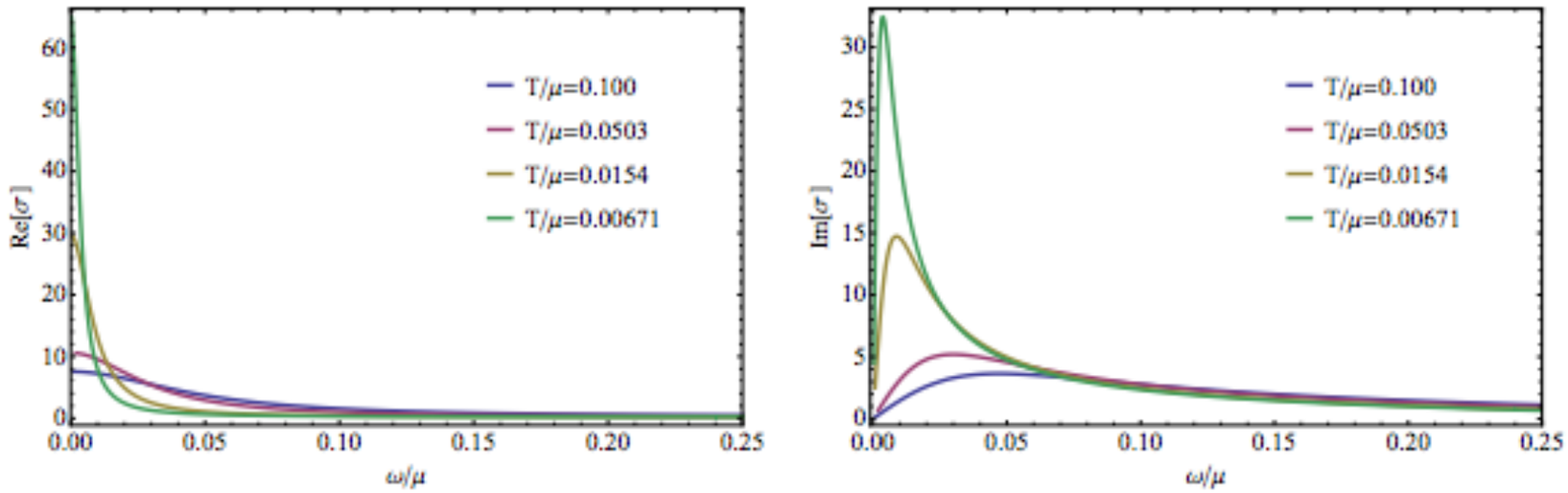
Einstein-Maxwell equations + non-uniform charge density $= B\omega^{-2/3}$

not so fast!

Donos and Gauntlett (gravitational crystal)

Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$



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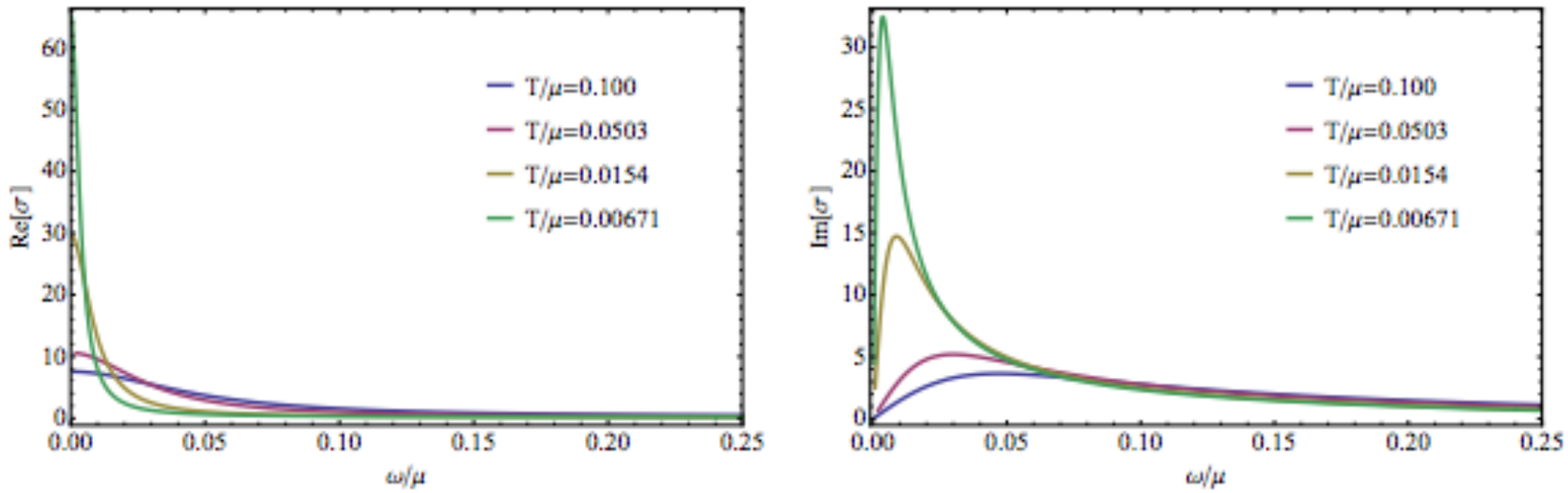

no power law!!

Donos and Gauntlett (gravitational crystal)

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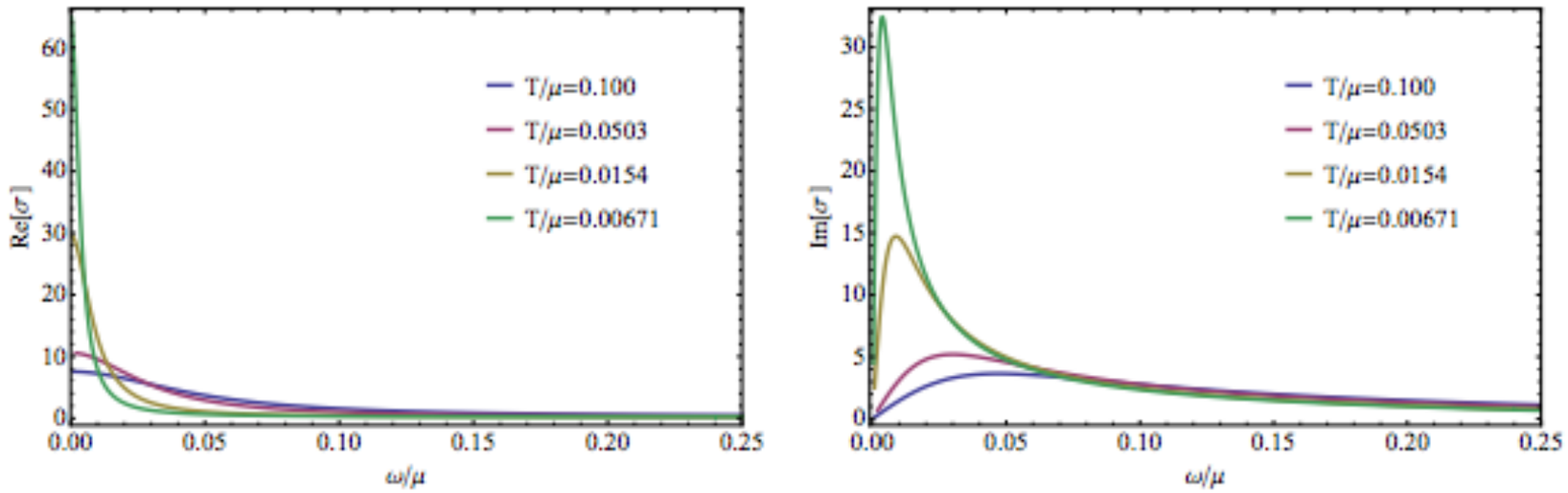

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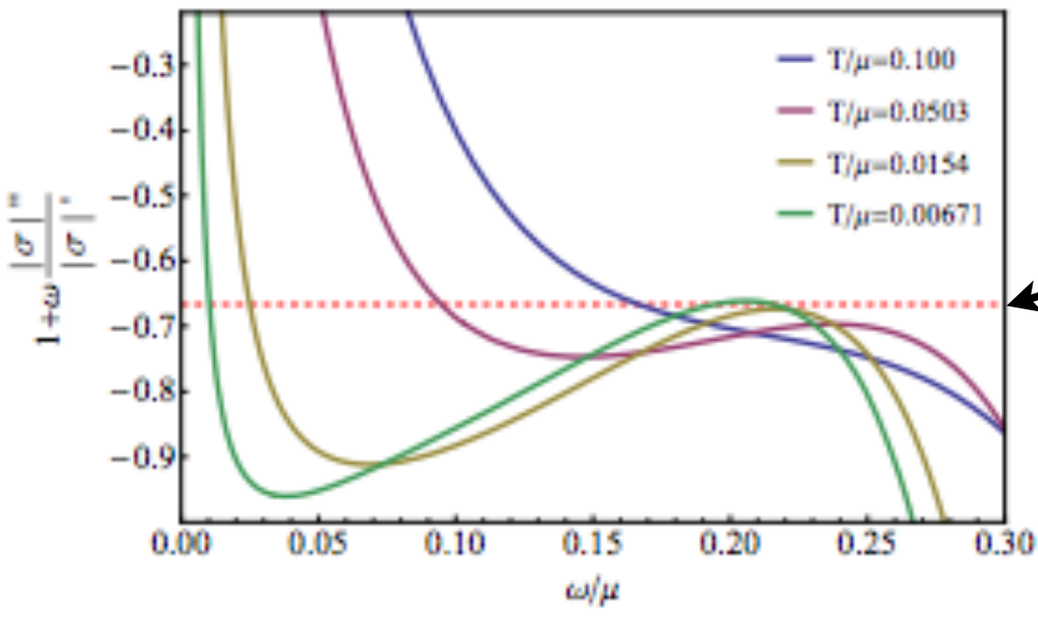


$$B\omega^{-2/3}$$

who is correct?

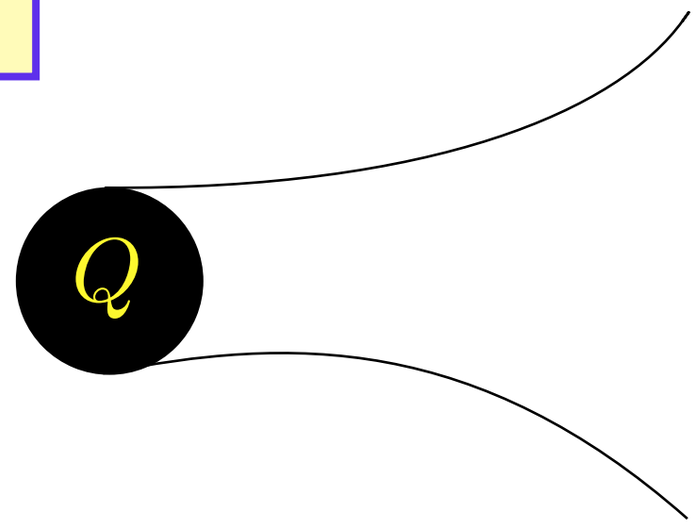
who is correct?

let's redo the calculation

conductivity within AdS

$(g_{\rm ab}, V(\Phi), A_t)$

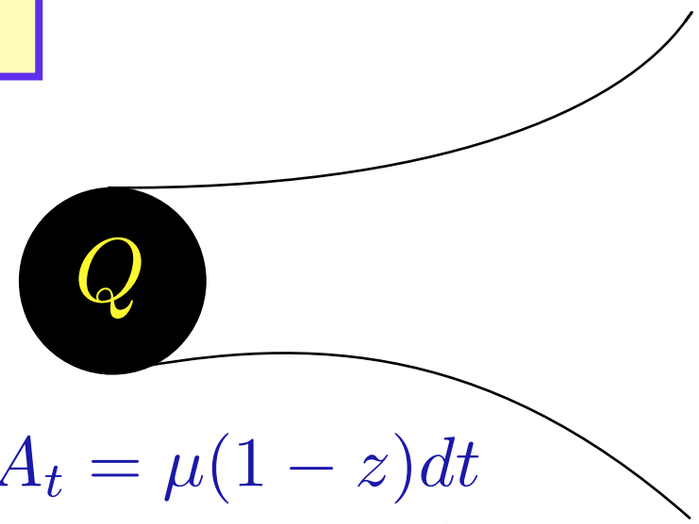
(metric , potential , gaugefield)



conductivity within AdS

$$(g_{\mathrm{ab}}, V(\Phi), A_t)$$

$$(\text{metric}\,, \text{potential}\,, \text{gaugefield})$$



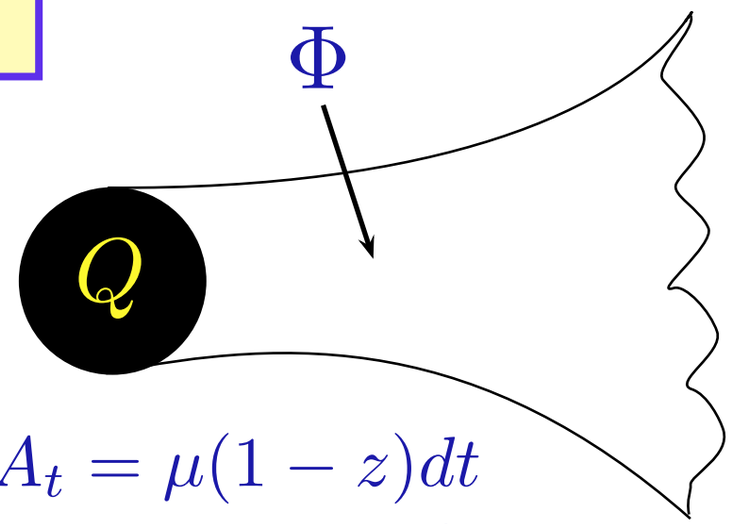
$$A_t = \mu(1-z)dt$$

$$\rho = \lim_{z\to 0} \sqrt{g} F^{tz}$$

conductivity within AdS

$$(g_{\rm ab}, V(\Phi), A_t)$$

$$(\text{metric}\,, \text{potential}\,, \text{gaugefield})$$



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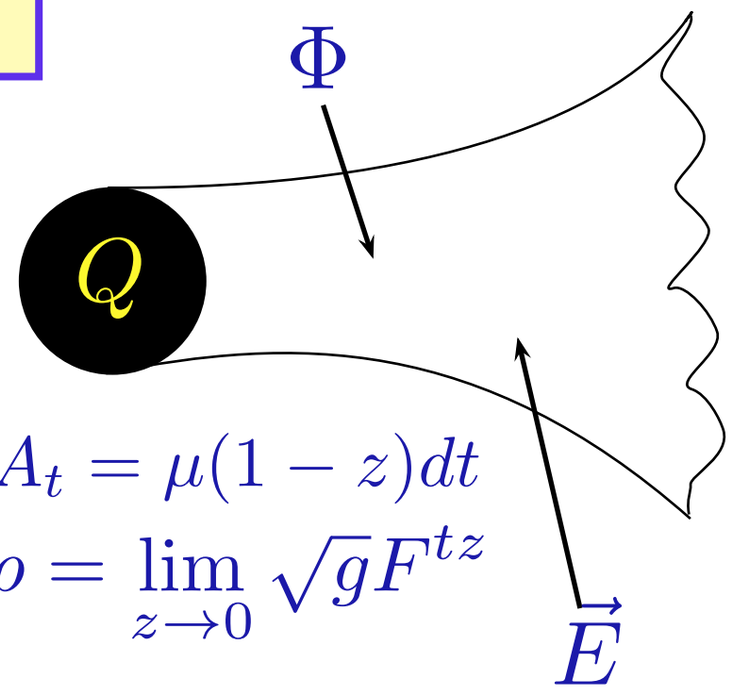
perturb with electric field



conductivity within AdS

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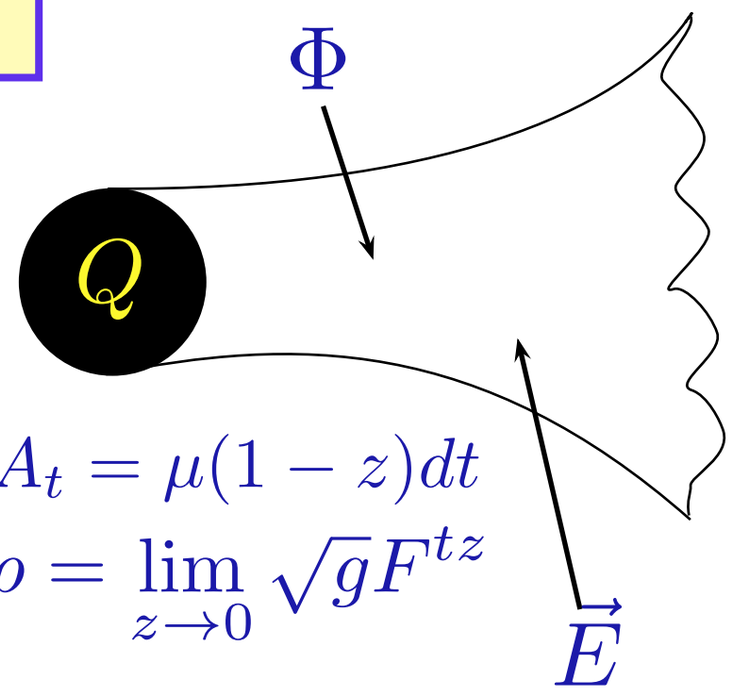


$$g_{\rm ab} = \bar{g}_{\rm ab} + h_{\rm ab}$$

$$A_a = \bar{A}_a + b_a$$

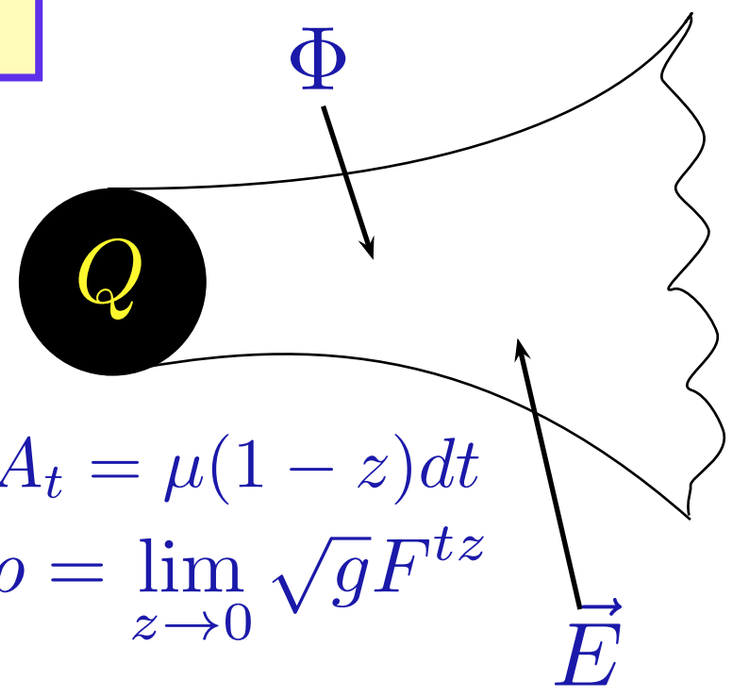
$$\Phi_i = \bar{\Phi}_i + \eta_i$$

conductivity within AdS

$(g_{\rm ab}, V(\Phi), A_t)$

(metric , potential , gaugefield)

perturb with electric field



$$g_{\rm ab} = \bar{g}_{\rm ab} + h_{\rm ab}$$

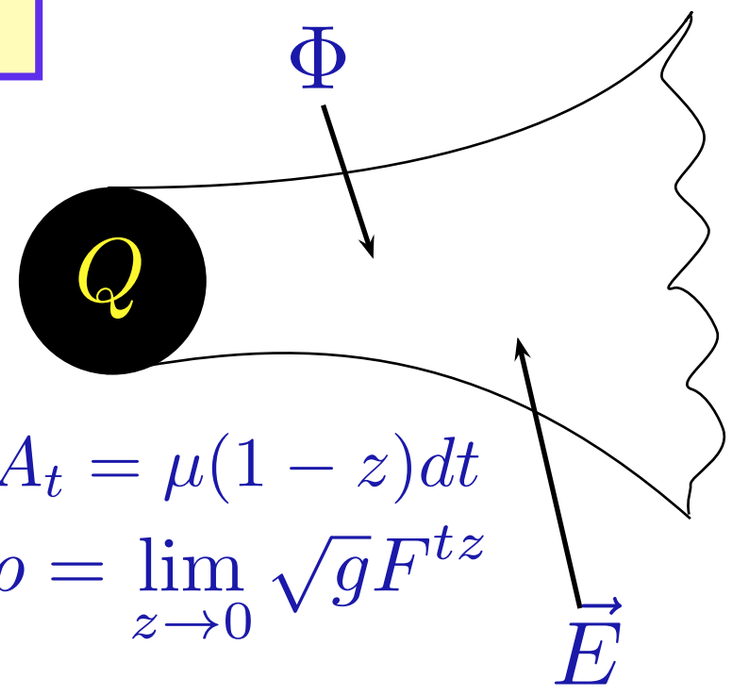
$$A_a = \bar{A}_a + b_a$$

$$\Phi_i = \bar{\Phi}_i + \eta_i$$

$$\delta A_x = \frac{E}{i\omega} + J_x(x,\omega) z + O(z^2)$$

conductivity within AdS

$$(g_{\rm ab}, V(\Phi), A_t)$$
$$(\text{metric}\,, \text{potential}\,, \text{gaugefield})$$



perturb with electric field

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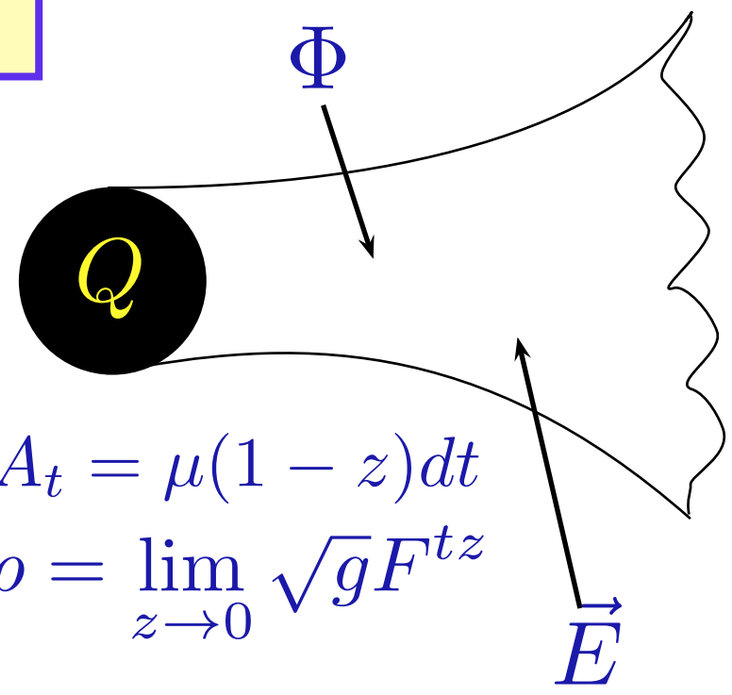
$$\delta A_x = \frac{E}{i\omega} + J_x(x,\omega)z + O(z^2)$$

solve equations of motion with gauge invariance

conductivity within AdS

$(g_{\rm ab}, V(\Phi), A_t)$

(metric , potential , gaugefield)



perturb with electric field

$$g_{\rm ab} = \bar{g}_{\rm ab} + h_{\rm ab}$$

$$A_a = \bar{A}_a + b_a$$

$$\Phi_i = \bar{\Phi}_i + \eta_i$$

$$\delta A_x = \frac{E}{i\omega} + J_x(x,\omega) z + O(z^2)$$

solve equations of motion with gauge invariance

$$\sigma = J_x(x,\omega)/E$$

model

RNAdS

$$ds^2 = \frac{L^2}{r^2 f\left(\frac{r_H}{r}\right)} dr^2 + \frac{r^2}{L^2}\left(-f\left(\frac{r_H}{r}\right) dt^2 + dx^2 + dy^2\right),$$

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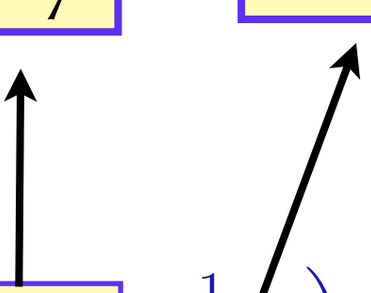

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$$\mathcal{L}(\phi) = \sqrt{-g}[-|\partial\phi|^2 - V(|\phi|)]$$

HST vs. DG

HST vs. DG

Horowitz, Santos, Tong (HST)

$$V(\Phi) = -\Phi^2/L^2$$

$$\Phi = z\Phi^{(1)} + z^2\Phi^{(2)} + \cdots ,$$

$$\Phi^{(1)}(x) = A_0 \cos(kx)$$

inhomogeneous in x

$$m^2 = -2/L^2$$

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DG

$$V\left(|\Phi|^2\right)$$

$$\Phi(z,x) = \phi(z)e^{ikx}$$

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radial gauge

Our Model

$$\mathcal{L}_\Phi = (\nabla\Phi_1)^2 + (\nabla\Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2)$$

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$\theta = 0$

HST

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$\theta = 0$ → HST

$\theta = \frac{\pi}{2}$ → DG

Einstein-De Turck EOM

$$G^H_{ab} = G_{ab} - \nabla_{(a}\xi_{b)},$$
$$\xi^a = g^{cd}\left(\Gamma^a_{cd}(g) - \Gamma^a_{cd}(\overline{g})\right).$$

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metric ansatz

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$$P(z) = 1 + z + z^2 - \frac{\mu_1^2}{2}z^3.$$

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RN-AdS when

$$Q_{tt} = Q_{zz} = Q_{yy} = 1 \qquad \Phi = 0 \qquad a_t = \mu_1 = \mu$$

Dirchlet boundary conditions

$$Q_{tt}(0,x) = Q_{zz}(0,x) = Q_{xx}(0,x) = Q_{yy}(0,x) = 1$$

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Newton-Raphson on grid

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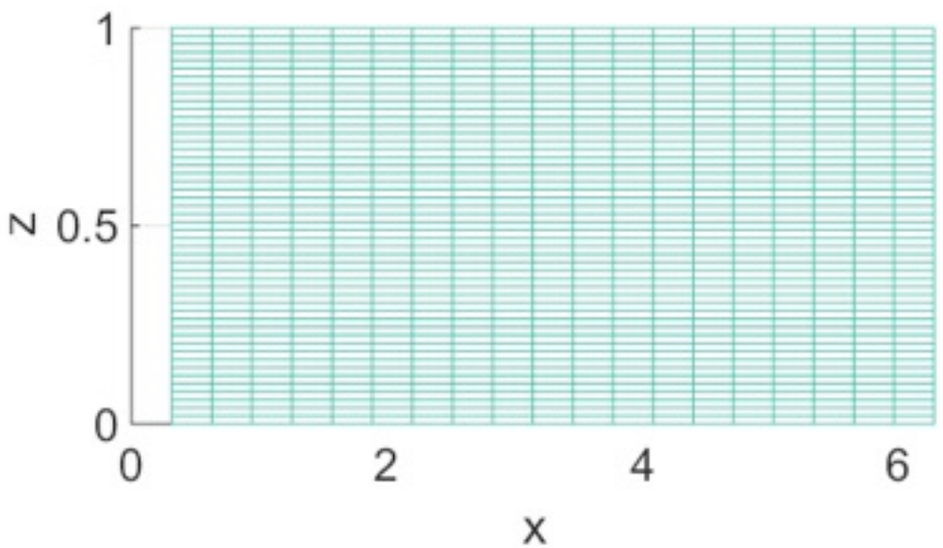
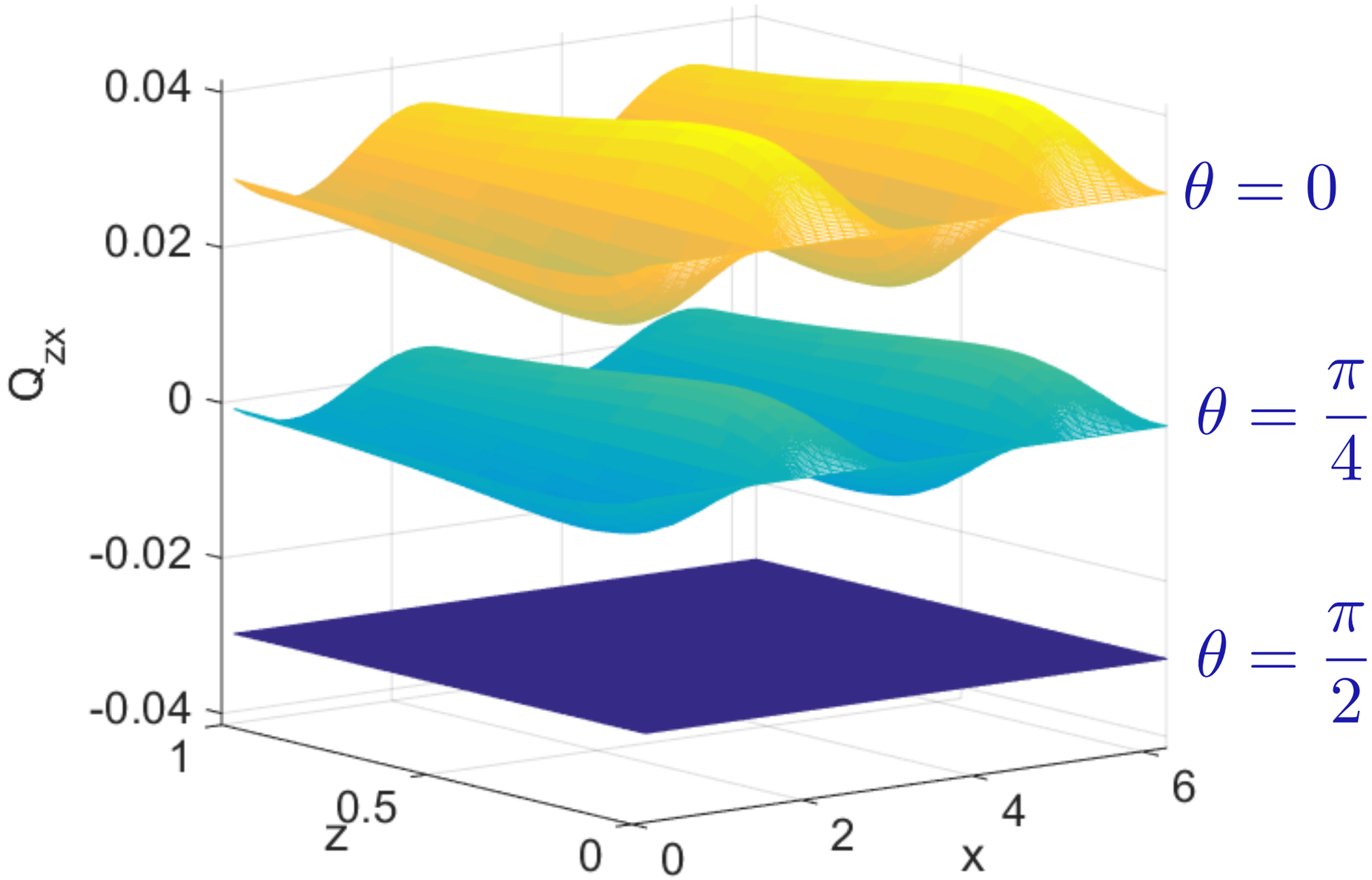


$$z \in [0,1], x \in (0, 2\pi)$$

$A_0 = 0.75,\ k = 1,\ \mu = 1.4,\ T/\mu = 0.115$



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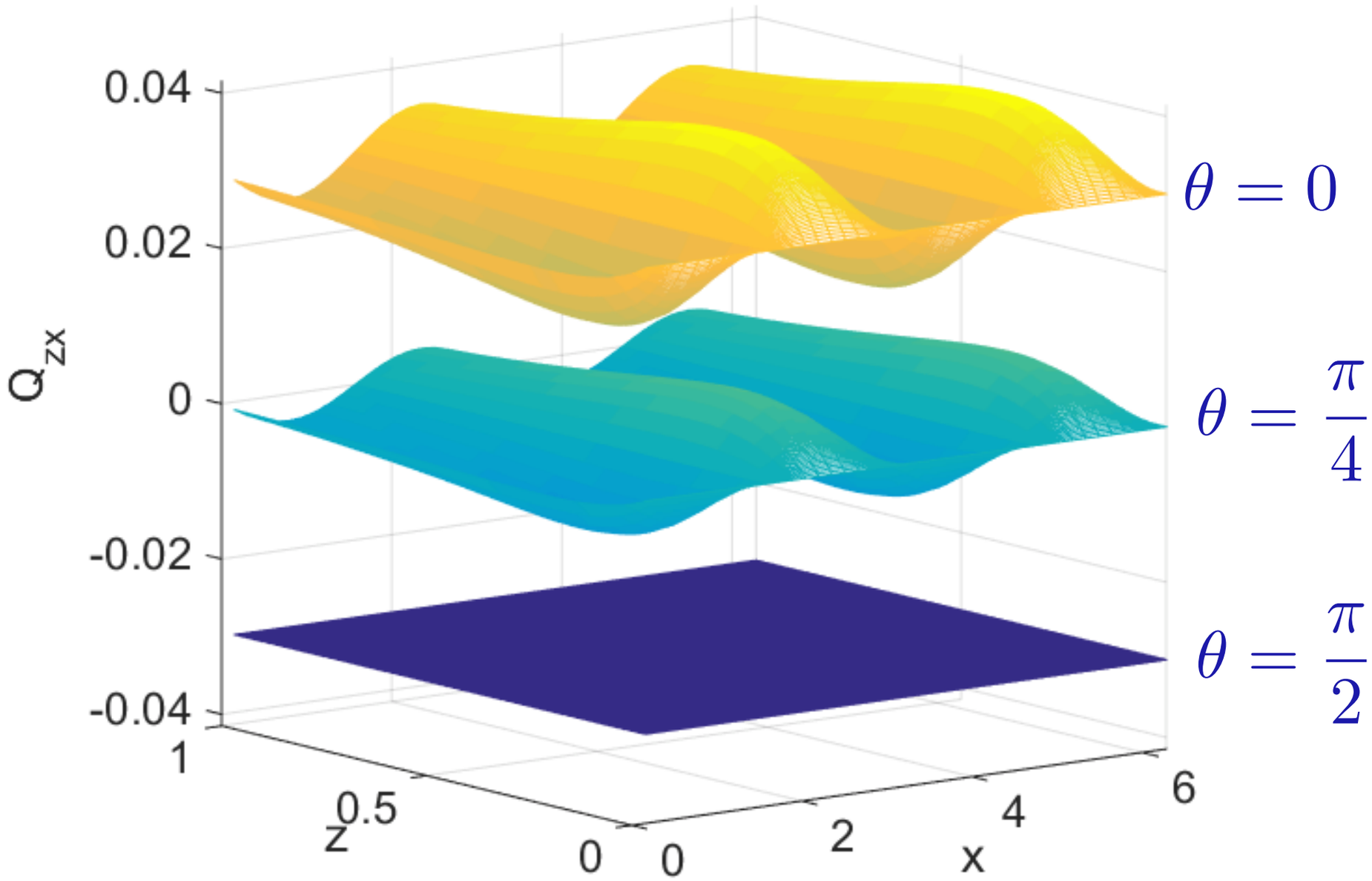
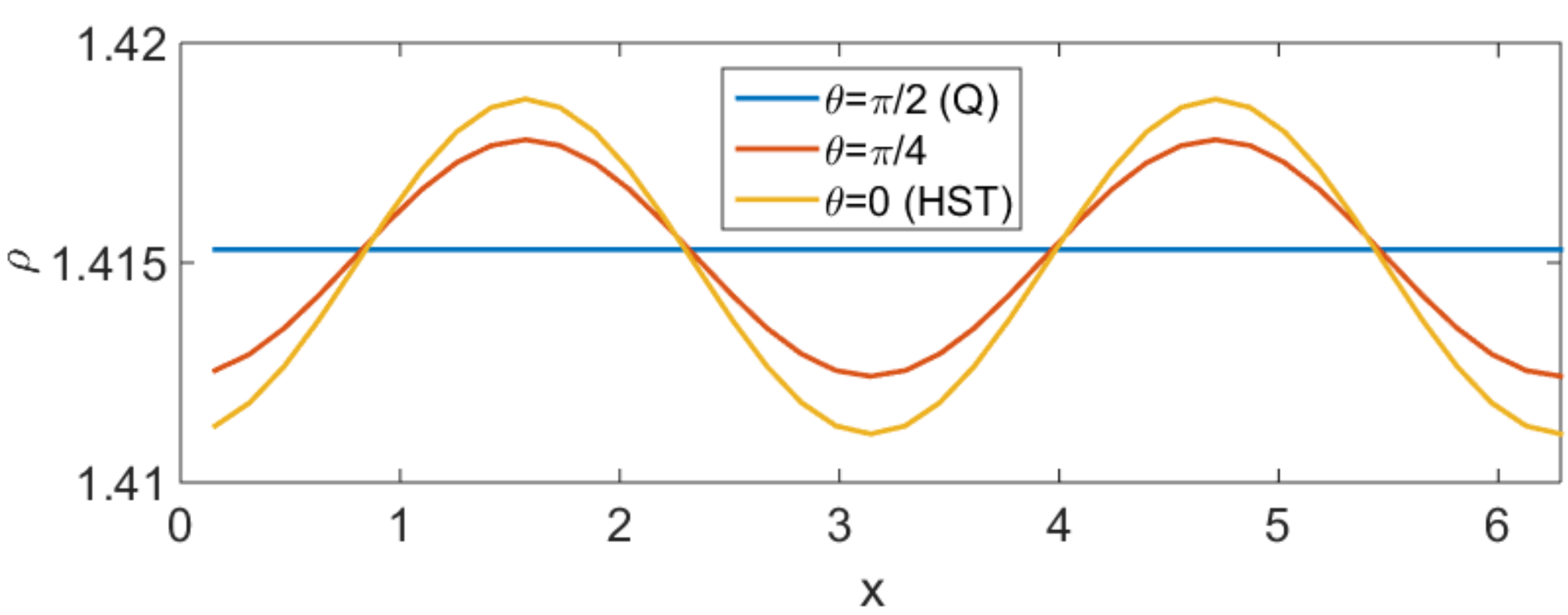


translational invariance is broken in metric in multiples of 2k

charge density

$$\rho = \lim_{z \to 0} \sqrt{-g} F^{tz}$$



perturb with electric field

$$g_{\mathrm{ab}} = \bar{g}_{\mathrm{ab}} + h_{\mathrm{ab}}$$

$$A_a = \bar{A}_a + b_a$$

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gauge invariance

$$\delta g_{ab} + \mathcal{L}_\zeta \overline{g}_{ab} = 0,$$

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$$\delta \Phi + \mathcal{L}_\zeta \overline{\Phi} = 0,$$

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solve equations without mistakes!!

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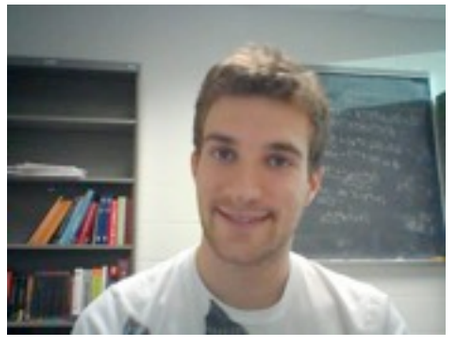
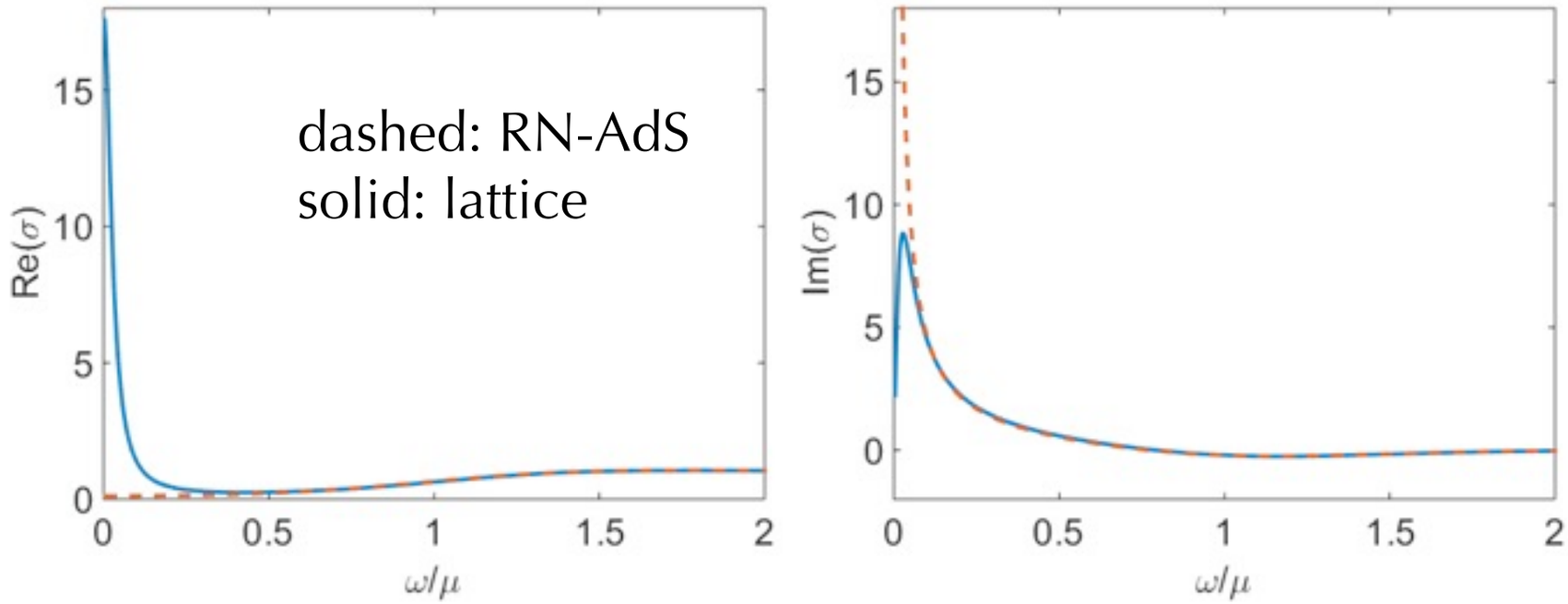


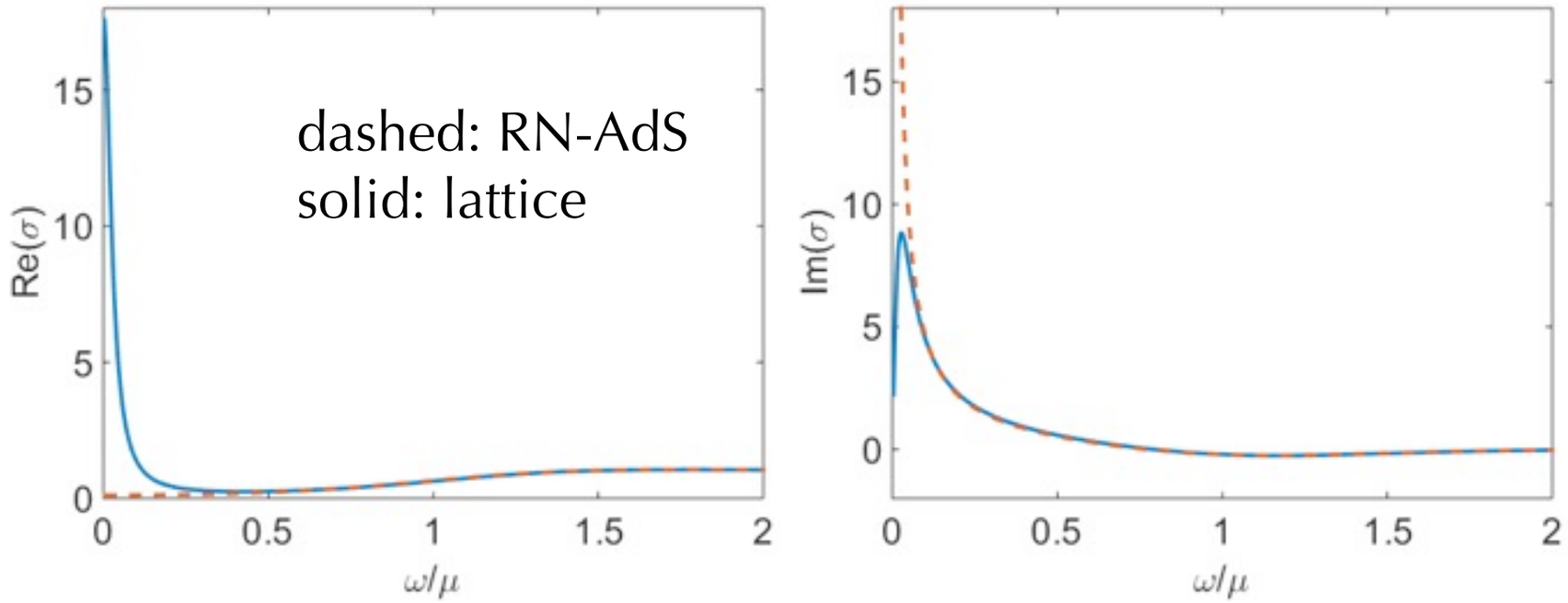
Brandon Langley

solve equations without mistakes!!



- high-frequency behavior is identical
- low-frequency RN has $\mathrm{Re}(\sigma) \sim \delta(\omega),\ \mathrm{Im}(\sigma) \sim 1/\omega$
- low-frequency lattice has Drude form

sample conductivity plots



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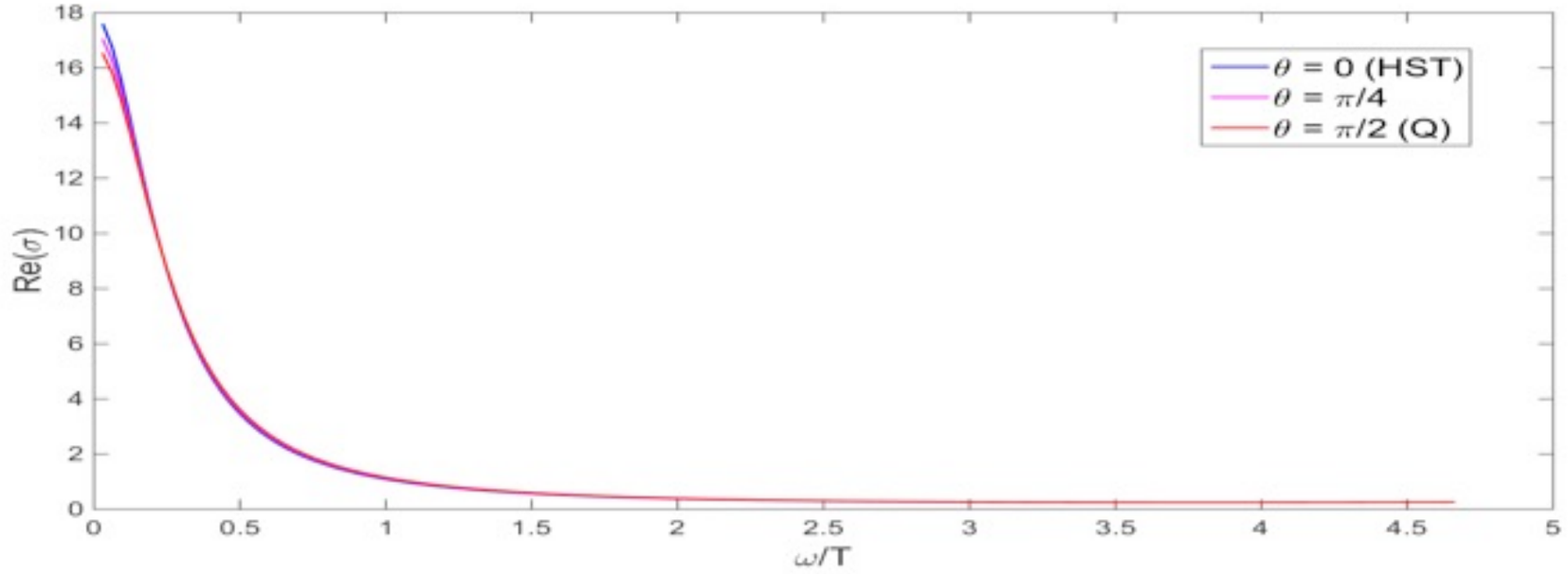

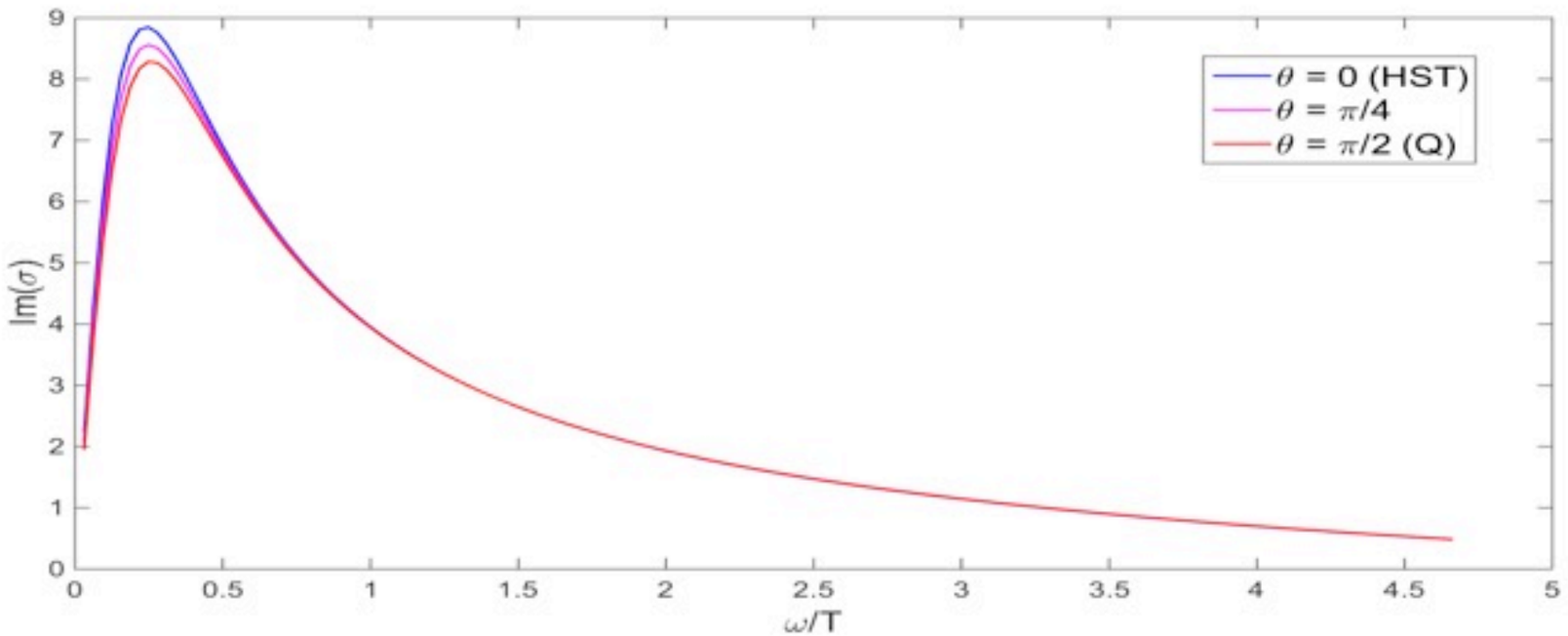

Results

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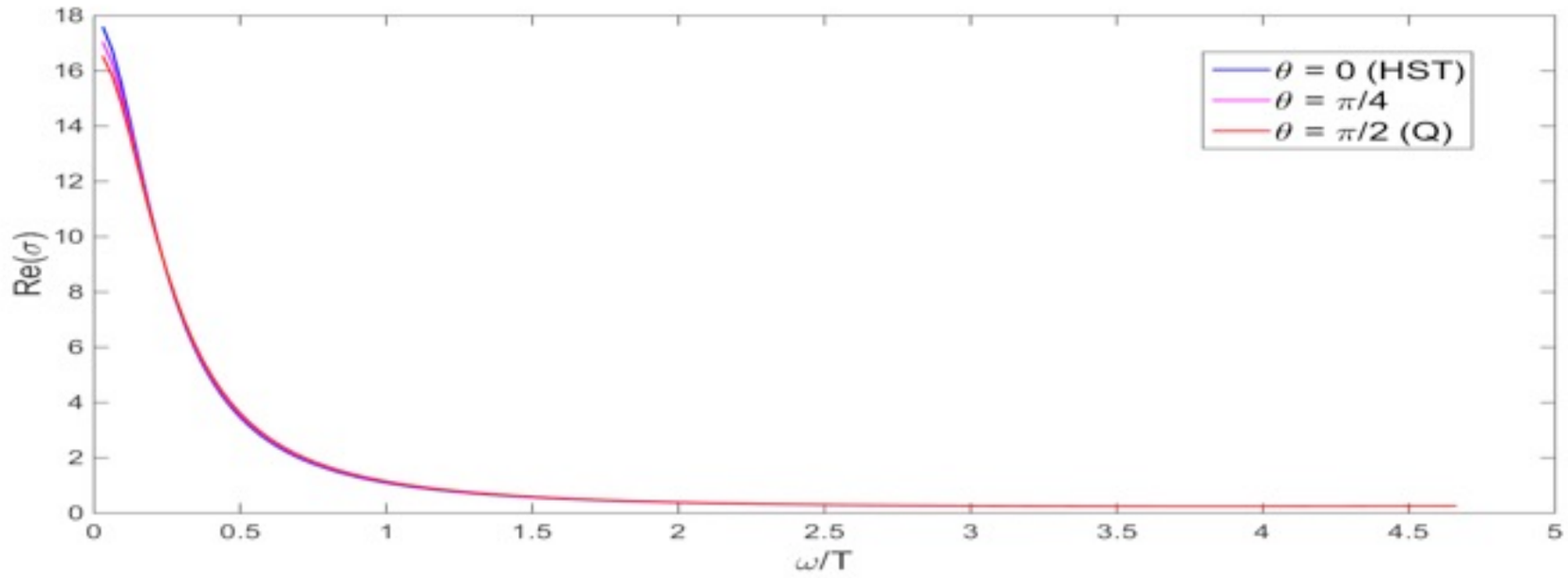

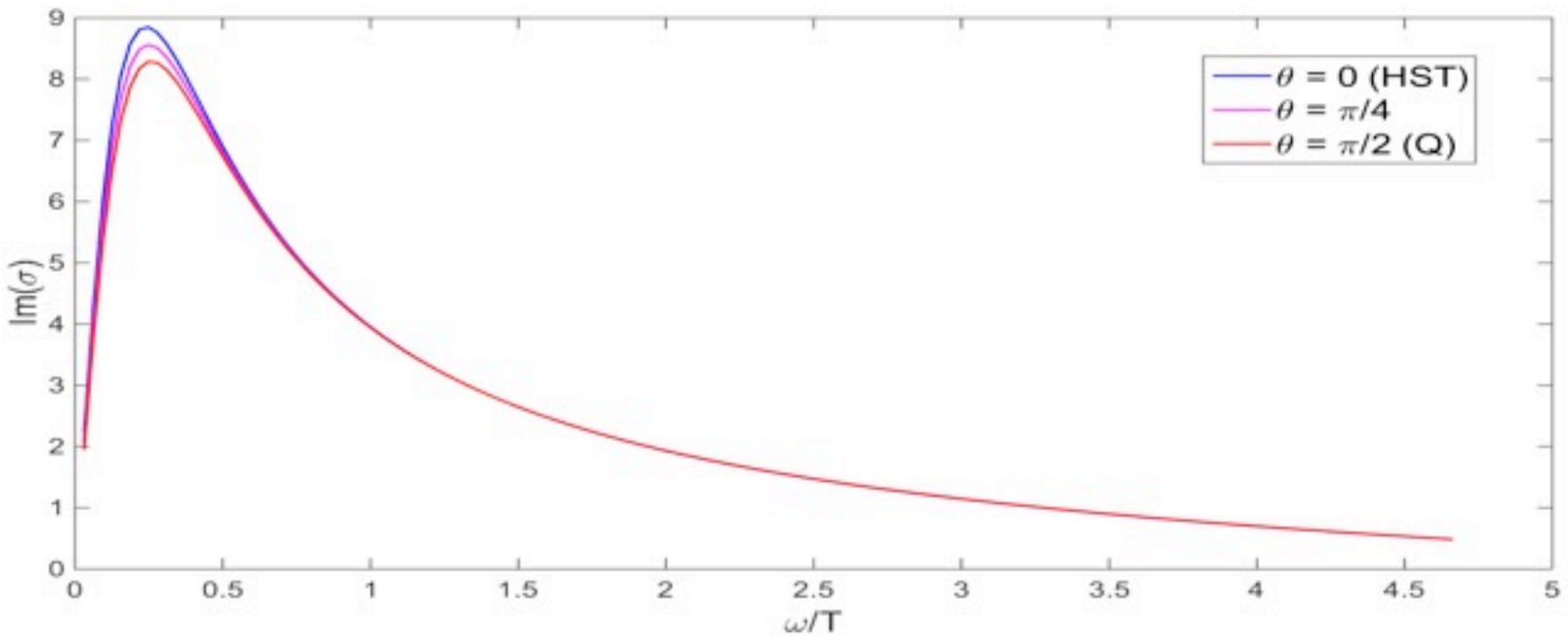

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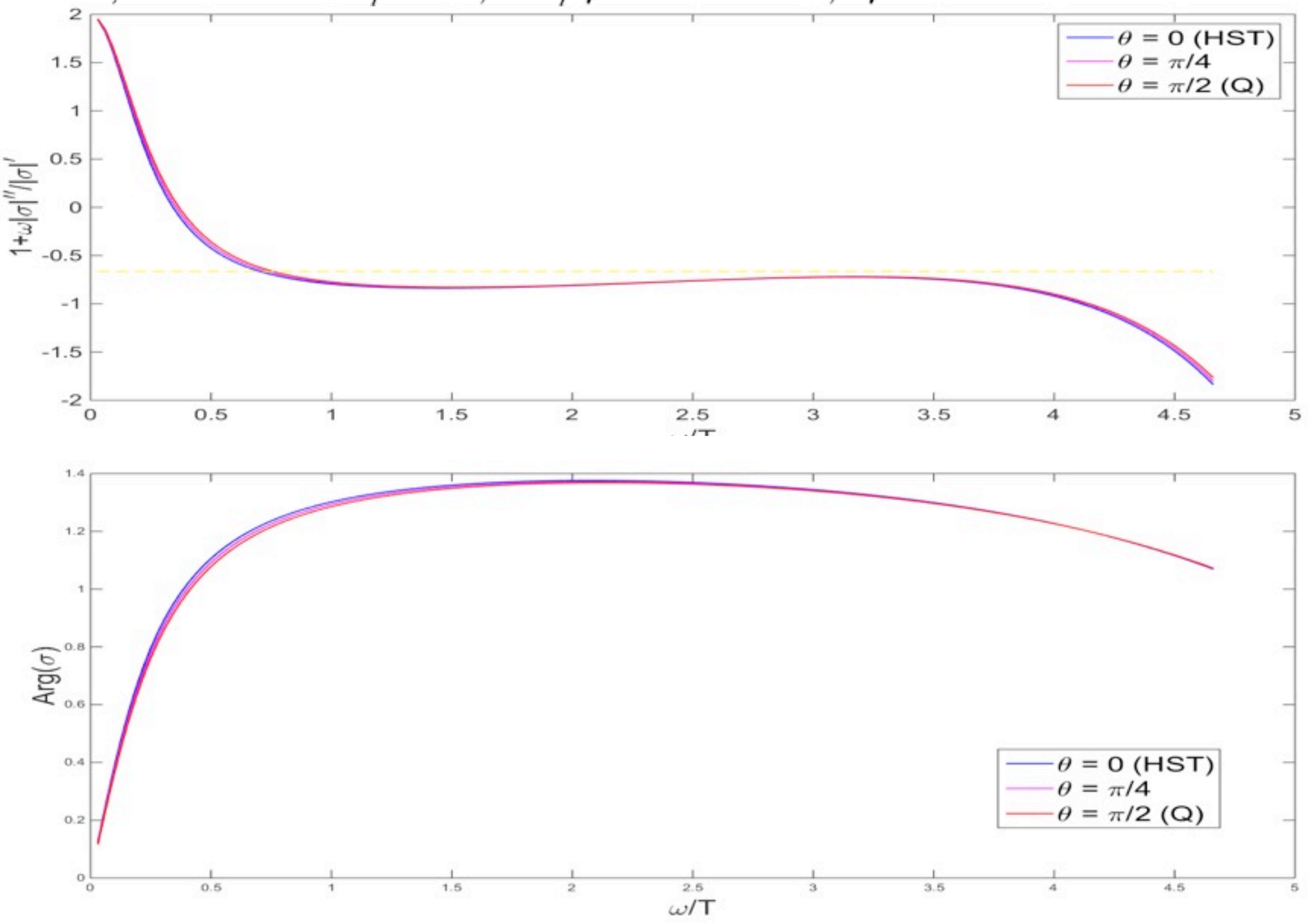

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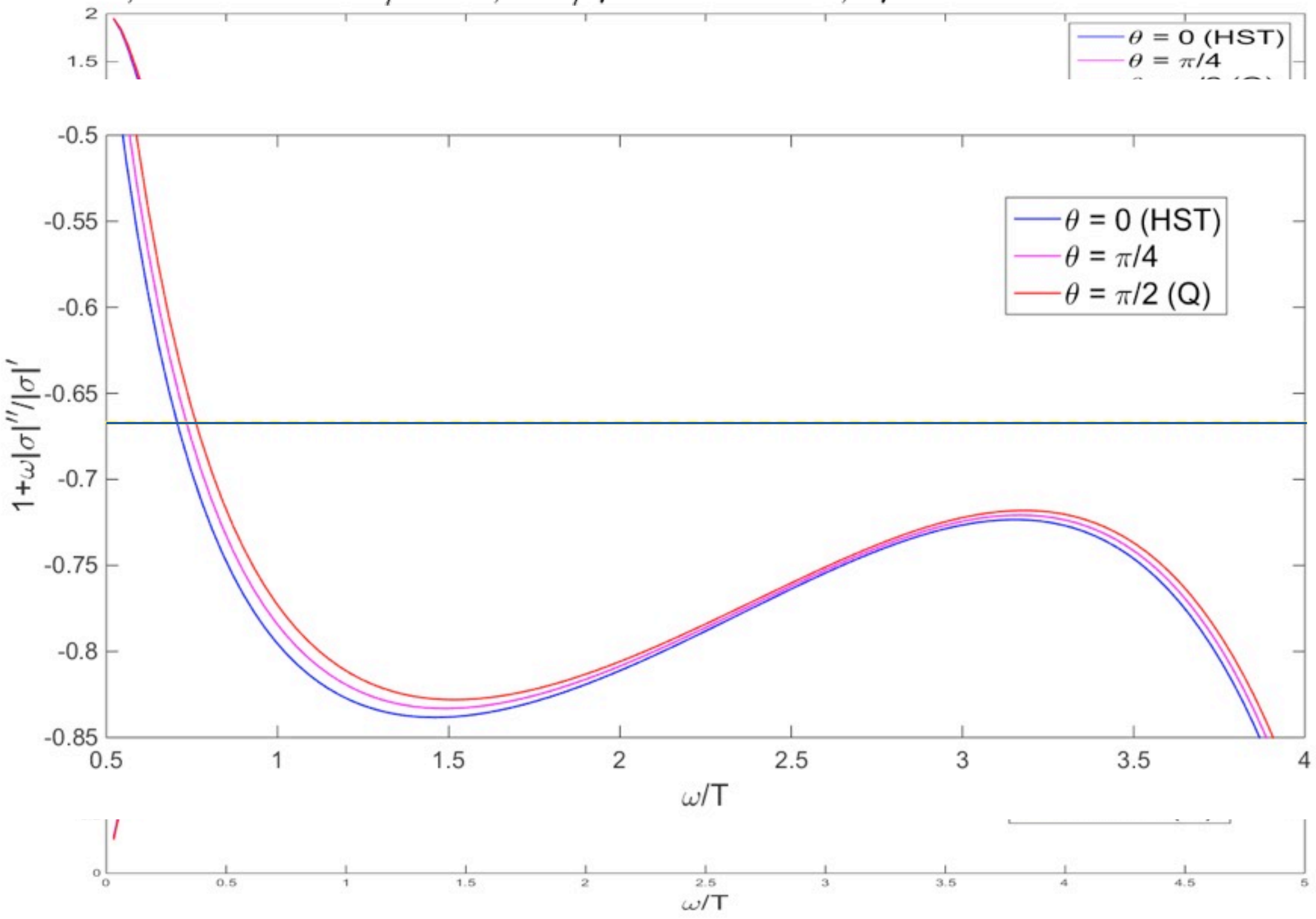

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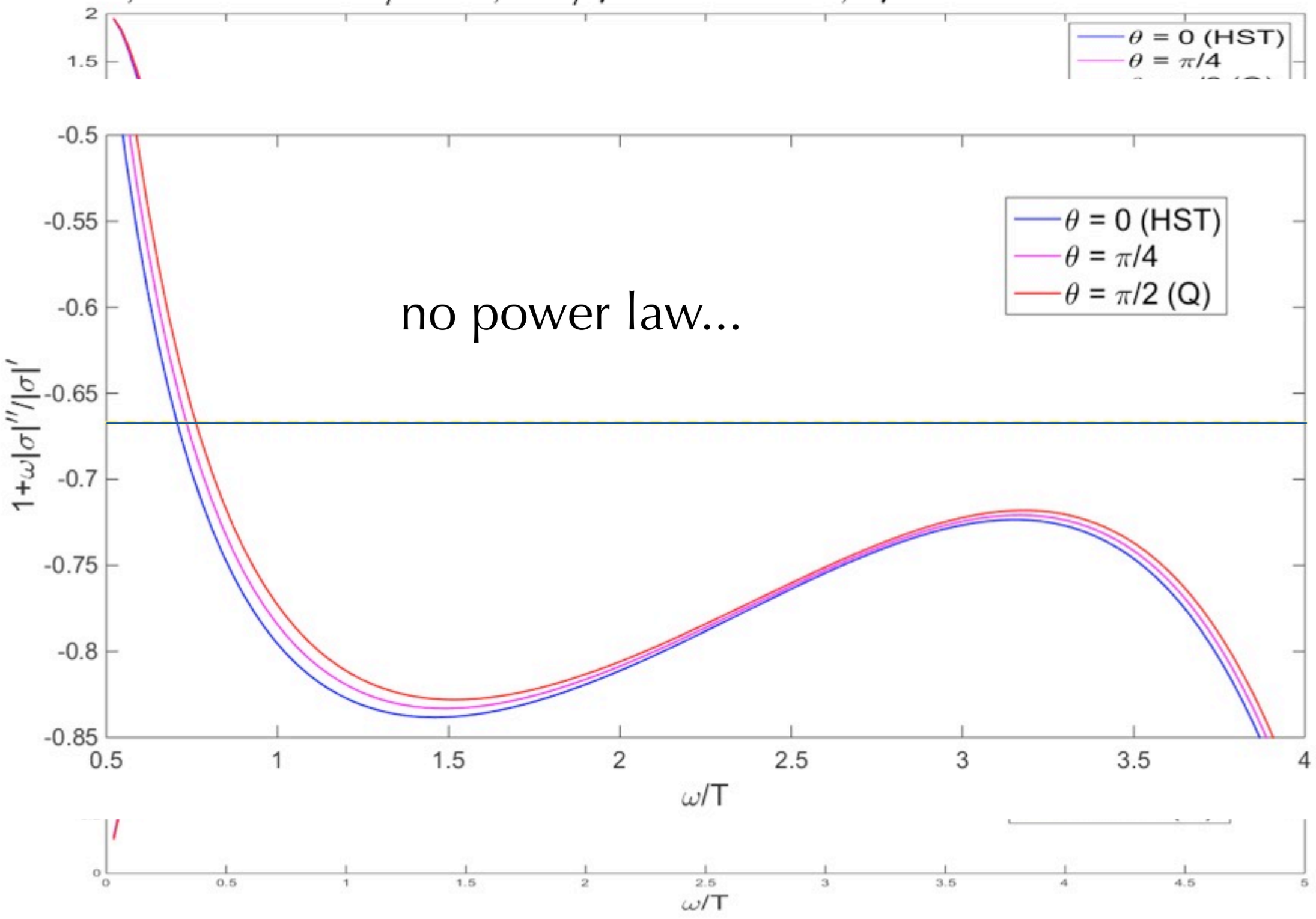

no power law...

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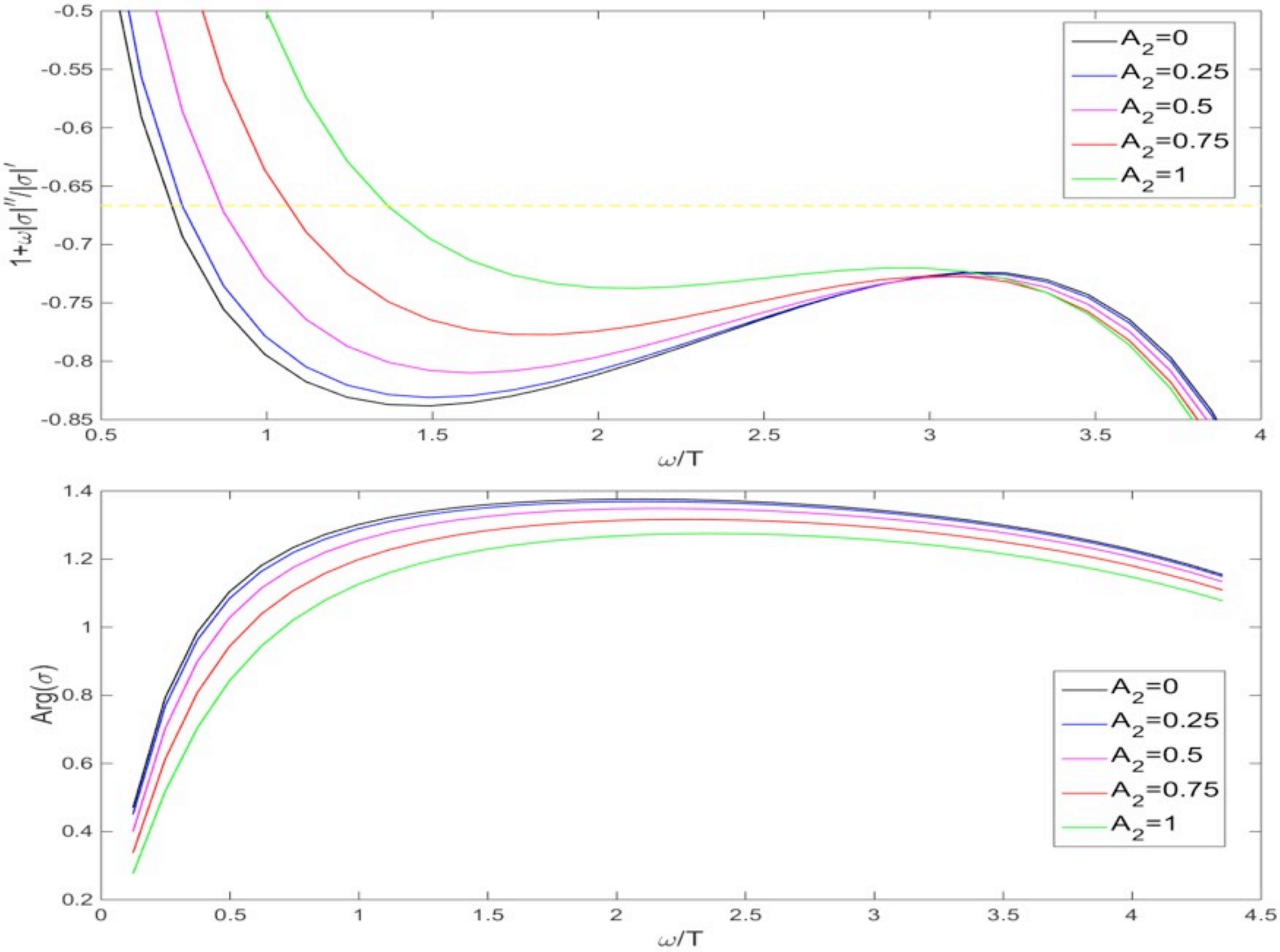

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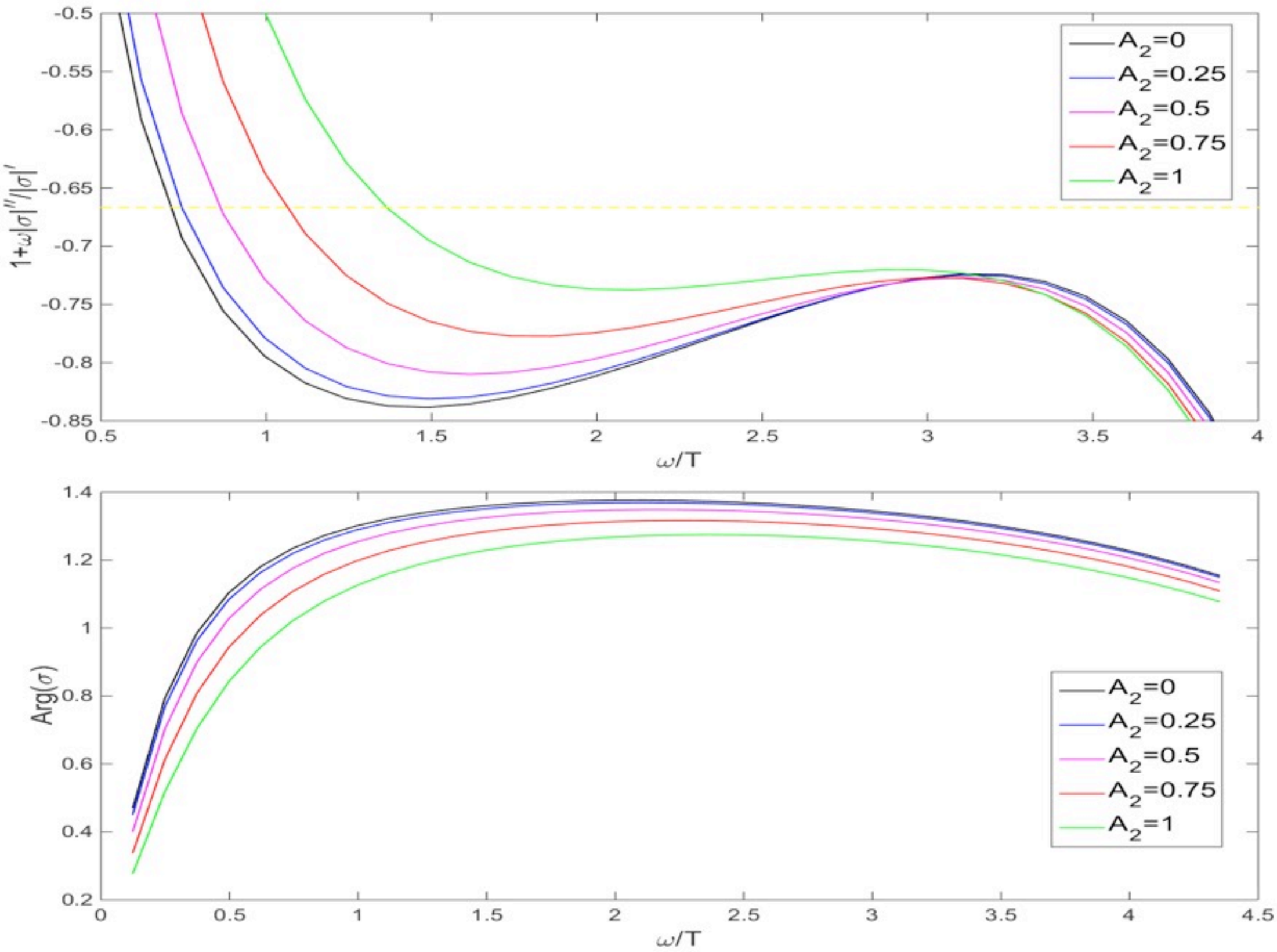

origin of power law?

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phenomenology

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scale-invariant propagators

$$\left(p^2\right)^{d_U - d/2}$$

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scale-invariant propagators

$$(p^2)^{d_U - d/2}$$

no well-defined mass

$$\mathcal{L}_{\text{eff}} = \int_0^\infty \mathcal{L}(x, m^2) dm^2$$

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scale-invariant propagators

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incoherent stuff (all energies)

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi + m^2 \phi^2$$

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$$\phi(x) \to \phi(x)$$

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no scale invariance

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theory with all possible mass!

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not particles

unparticles

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$$d_U - 2$$

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continuous mass

$\phi(x, m^2)$

flavors

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flavors

$e^2(m)$

Karch, 2005

multi-bands

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$\phi(x, m^2)$ ⟺ $e^2(m)$

flavors

multi-bands

Karch, 2005

use unparticle propagators
to calculate conductivity

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to calculate conductivity

assume Gaussian action

$$S = \int d^{d+1}p \; \phi_U^\dagger(p) i G^{-1}(p) \phi_U(p)$$

use unparticle propagators
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$$S = \int d^{d+1}p \; \phi_U^\dagger(p) i G^{-1}(p) \phi_U(p)$$

$$\phi_U(x) = \int_0^\infty dm^2 \; f(m^2) \phi(x, m^2)$$

use unparticle propagators
to calculate conductivity

assume Gaussian action

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$$G(p) \sim \frac{i}{(-p^2 + i\epsilon)^{\frac{d+1}{2} - d_U}}$$

gauge unparticles

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Wilson line

$$S = \int d^{d+1}x d^{d+1}y \; \phi_U^\dagger(x) F(x-y) W(x,y) \phi_U(y),$$

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vertices

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vertices

$$g\Gamma^\mu(p,q) = \frac{\delta^3 S}{\delta A^\mu(q)\delta\phi^\dagger(p+q)\delta\phi(p)}$$

1-gauge

$$g^2\Gamma^{\mu\nu}(p,q_1,q_2) = \frac{\delta^4 S}{\delta A^\mu(q_1)\delta A^\nu(q_2)\delta\phi^\dagger(p+q_1+q_2)\delta\phi(p)}$$

2-gauge

use Ward-Takahashi identities to simplify vertices

use Ward-Takahashi identities to simplify vertices

$$-iq_\mu \Gamma^\mu(p,q) = G^{-1}(p+q) - G^{-1}(p)$$

$$q_{1\mu}\Gamma^{\mu\nu}(p,q_1,q_2) = \Gamma^\nu(p+q_1,q_2) - \Gamma^\nu(p,q_2)$$

use Ward-Takahashi identities to simplify vertices

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response function to an electric field

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response function to an electric field

$$K^{\mu\nu}_{-n,-n'}(-q,-q') = -\frac{(2\pi)^{2d}}{T^2}\mathcal{Z}^{-1}\left.\frac{\delta^2}{\delta A_{\mu,n}(q)\delta A_{\nu,n'}(q')}\right|_{A=0}\mathcal{Z}[A]$$

compute conductivity

$$\sigma^{\mu\nu}(i\omega_n) = \lim_{q\to 0} \frac{1}{\omega_n} K_n^{\mu\nu}(q)!$$

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no power law

$$\sigma(i\omega_n) = (\frac{d+1}{2} - d_U)\sigma_0(i\omega_n)$$

what went wrong?

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free field

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unparticle field has particle content

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free field

$$\phi_U(x) = \int_0 a^2 f(m^2) \quad x, m^2)$$

unparticle field has particle content

continuous mass taken seriously

$$S = \sum_{i=1}^{N} \int d\tau \int d^d x (|D_\mu \phi_i^2| + m_i^2 |\phi_i|^2)$$

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$$\propto \omega^\alpha \quad \alpha > 0 (\omega < 2M)$$

last attempt

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take experiments seriously

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$$\sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega\tau_i}$$

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continuous mass

$$\sigma(\omega) = \int_0^M \frac{\rho(m) e^2(m) \tau(m)}{m} \frac{1}{1 - i\omega\tau(m)} dm$$

variable masses for everything

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

$$e(m) = e_0 \frac{m^b}{M^b}$$

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Karch, 2015

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perform integral

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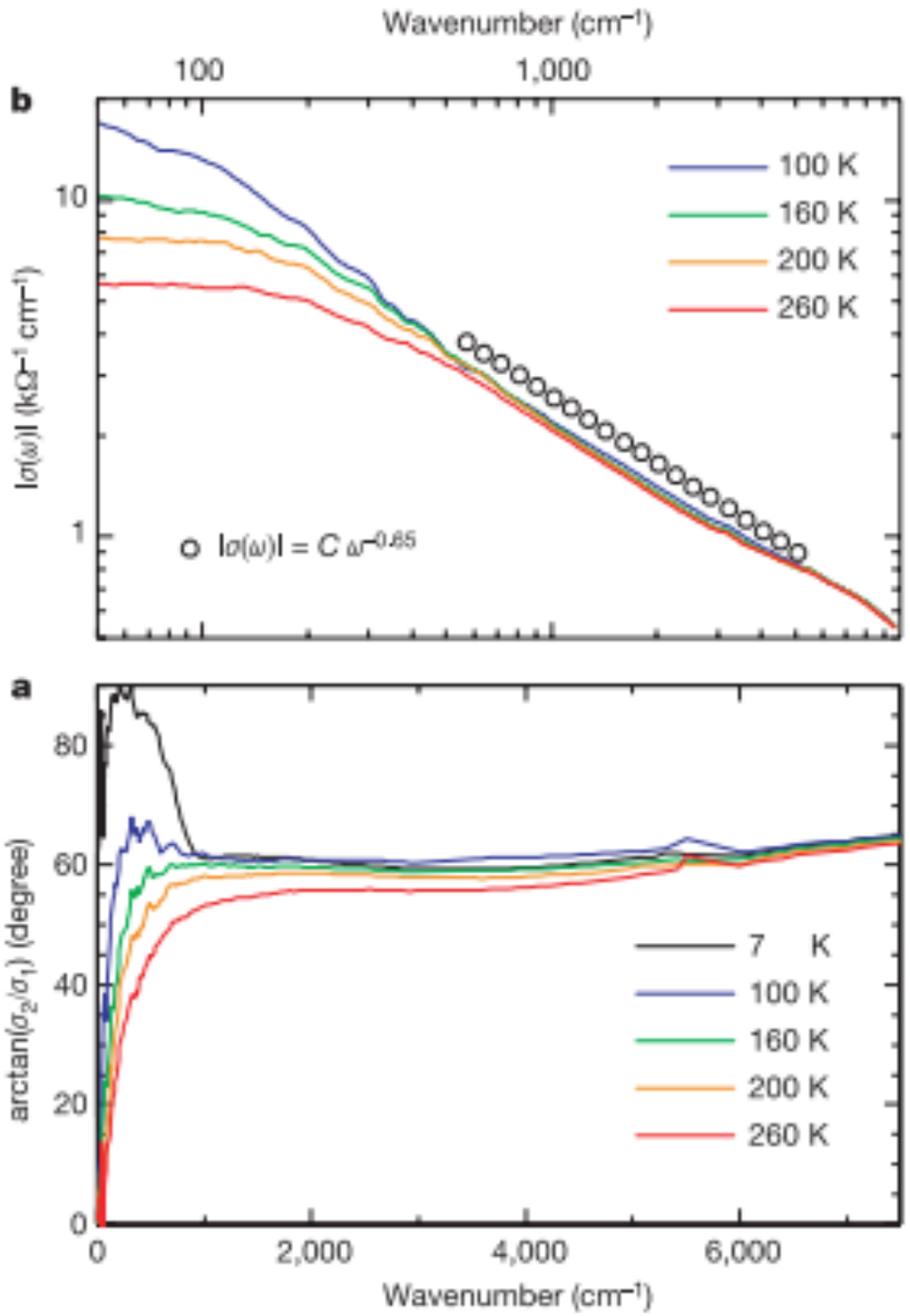
$$\tan\sigma = \sqrt{3}$$

$$60^\circ$$

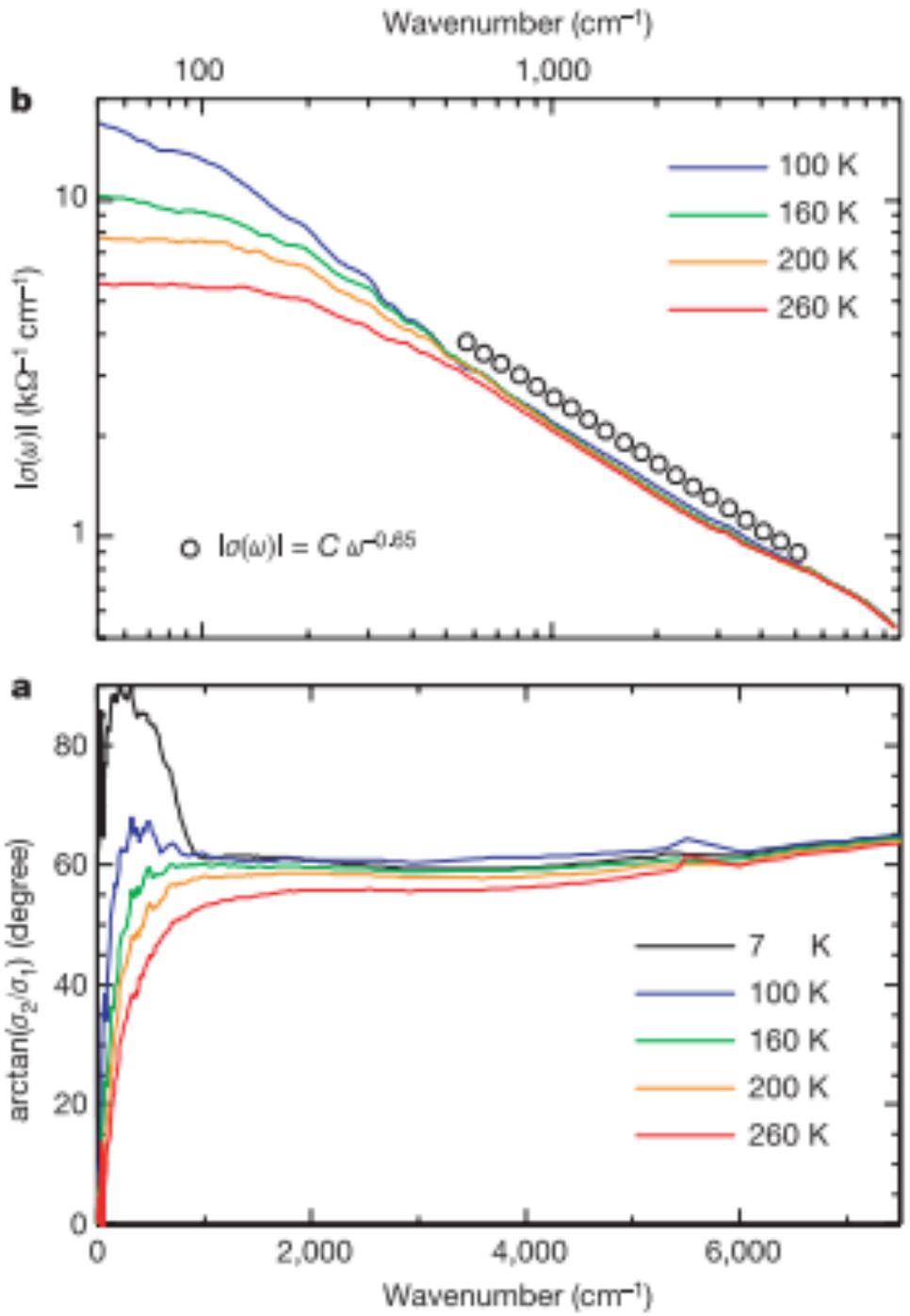
experiments



$$\sigma(\omega) = C\omega^{\gamma-2}e^{i\pi(1-\gamma/2)}$$

$$\gamma = 1.35$$

experiments



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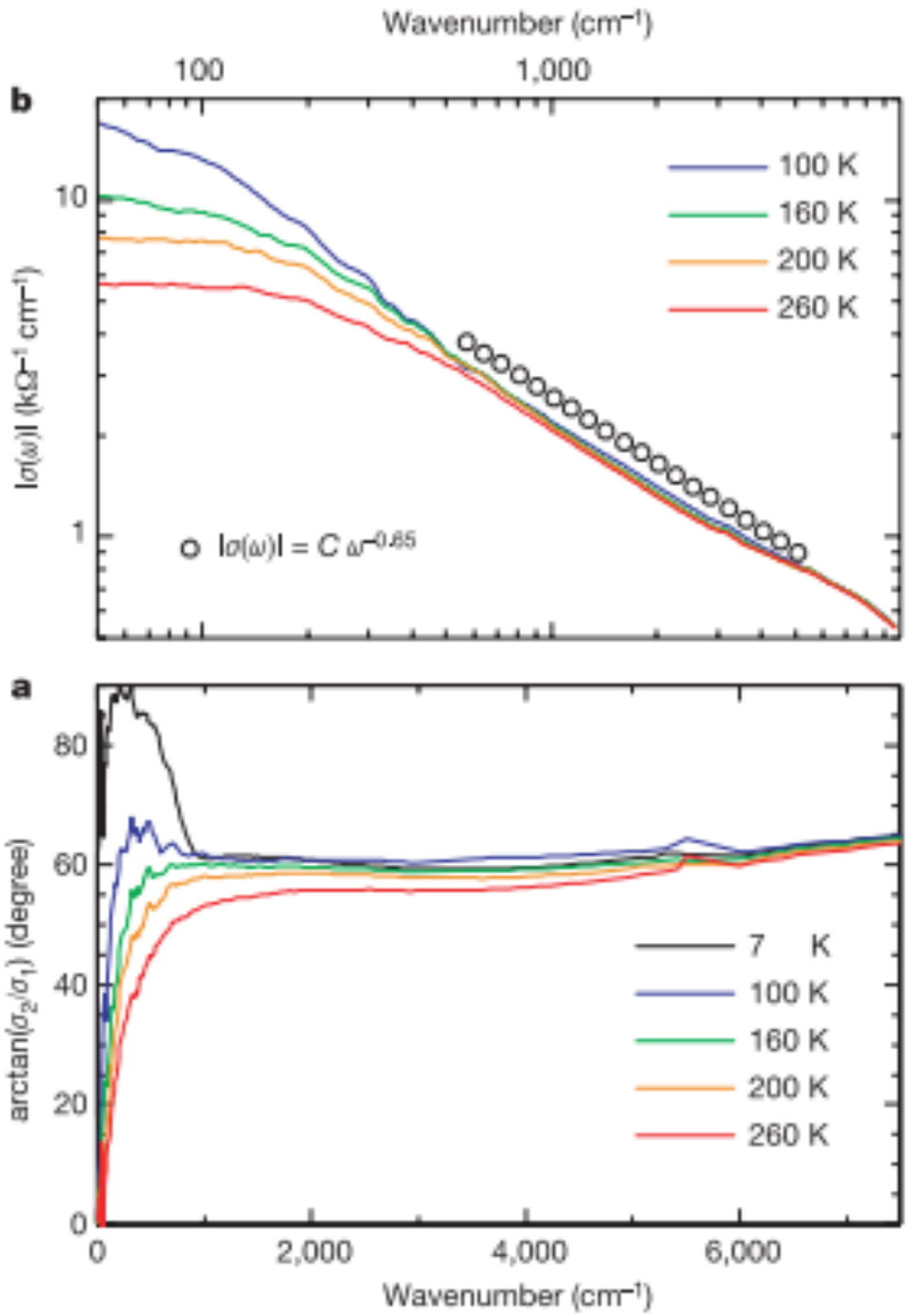

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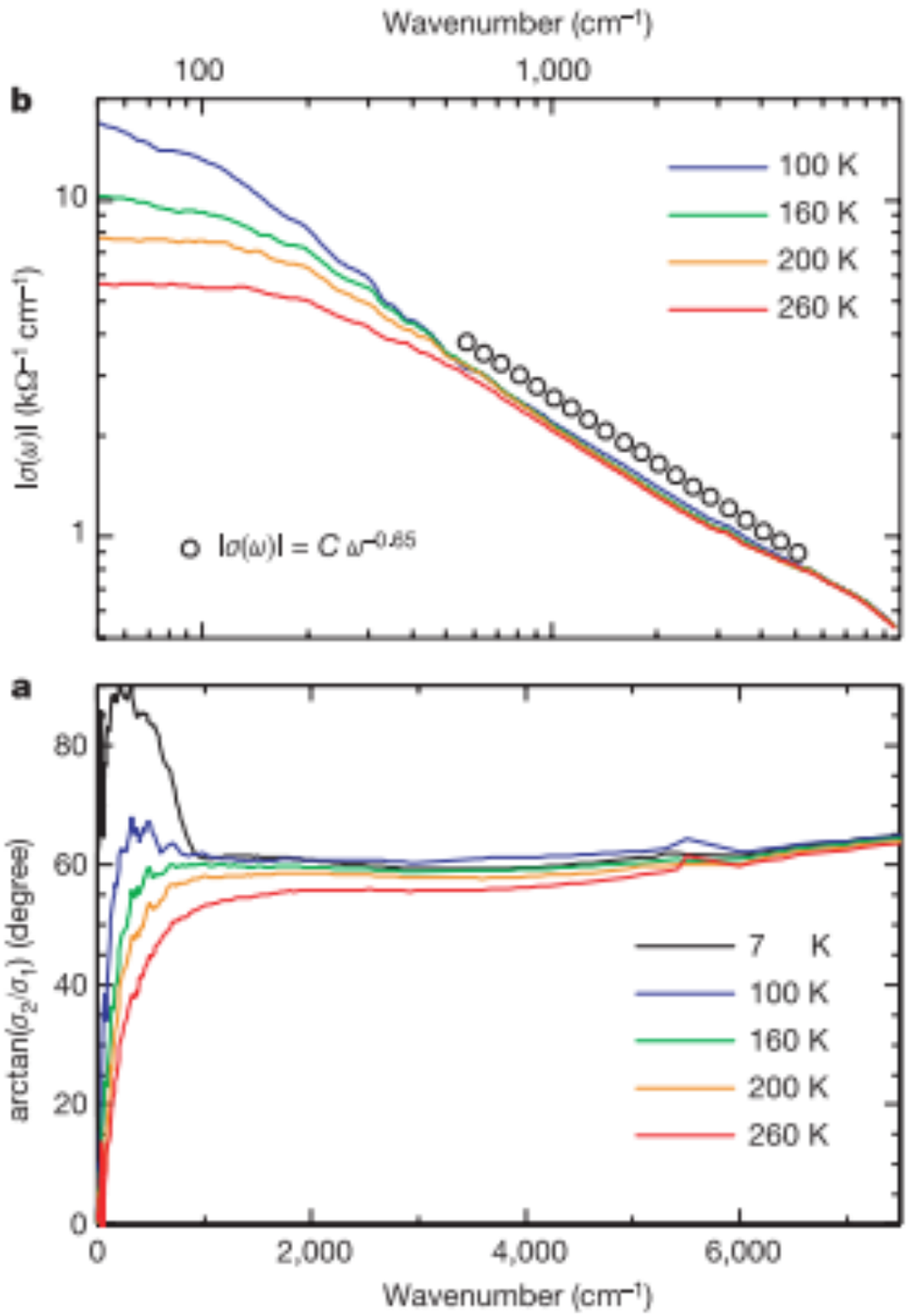

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victory!!

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are anomalous dimensions necessary

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$$c = 1$$

$$a + 2b = 2/3$$

$$\Rightarrow$$

$$b = 0$$

$$a = 2/3$$

No

but they are possible!

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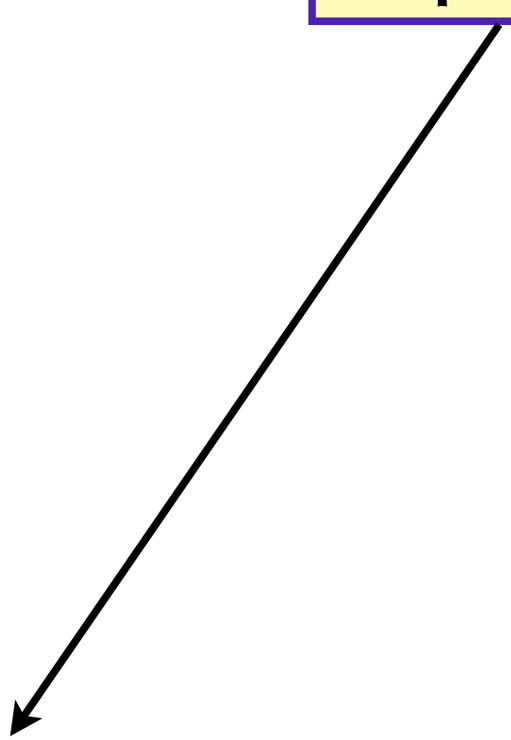


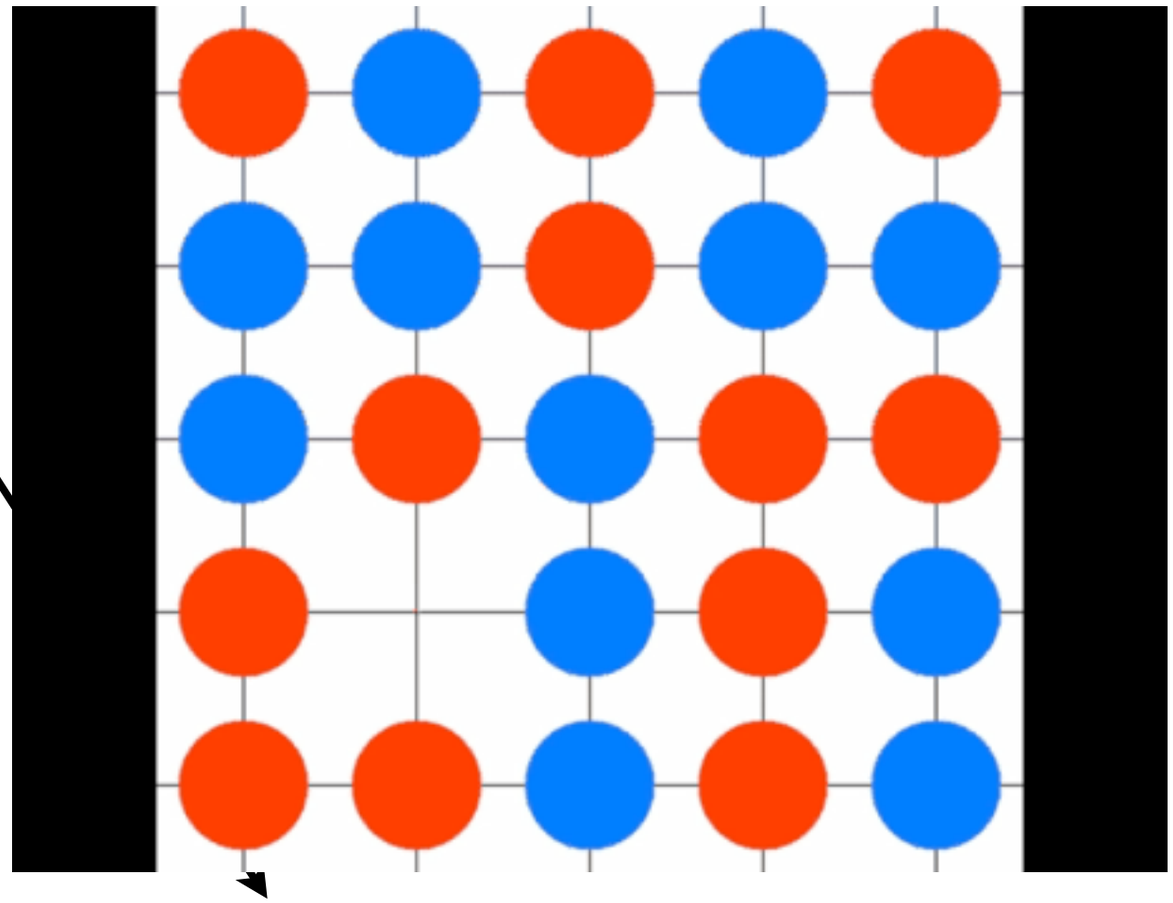
continuous mass

but they are possible!

continuous mass

charge non-conservation

but they are possible!



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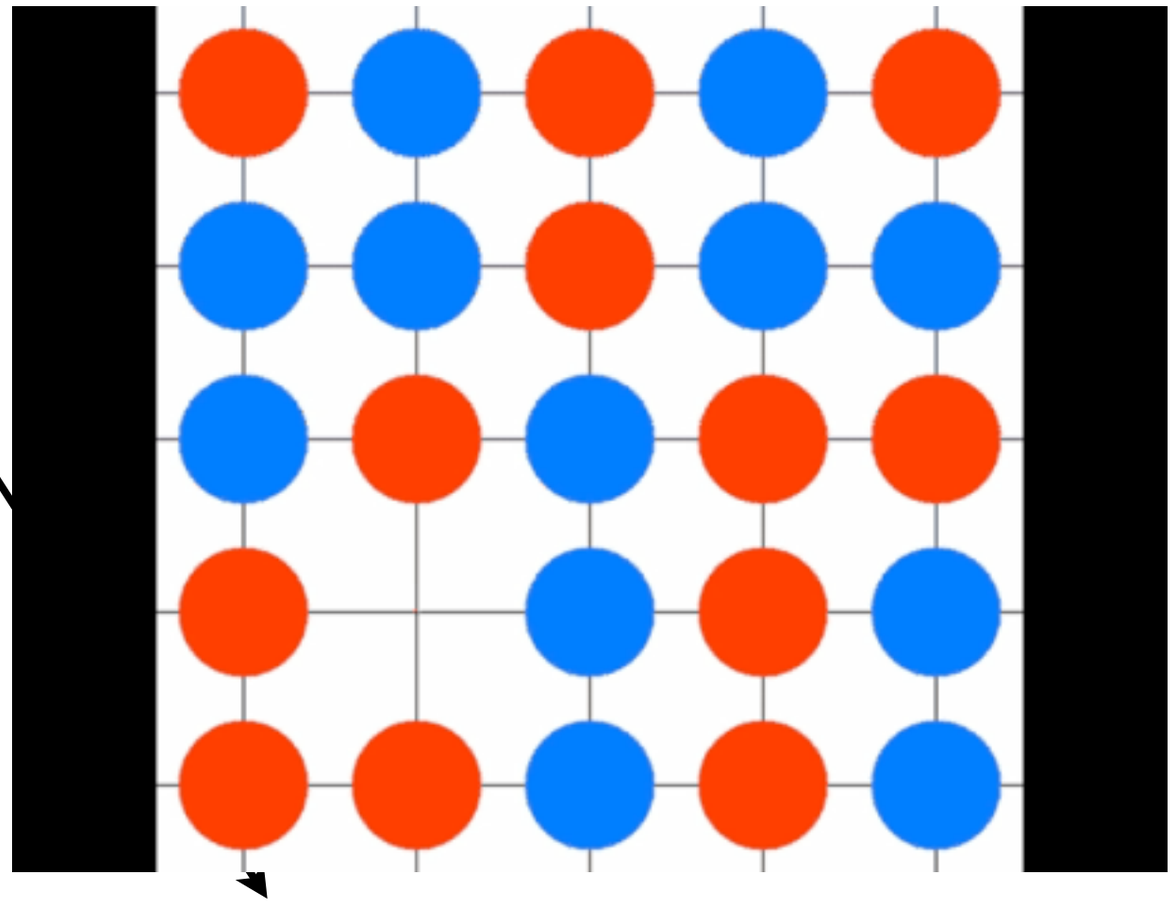

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unparticles