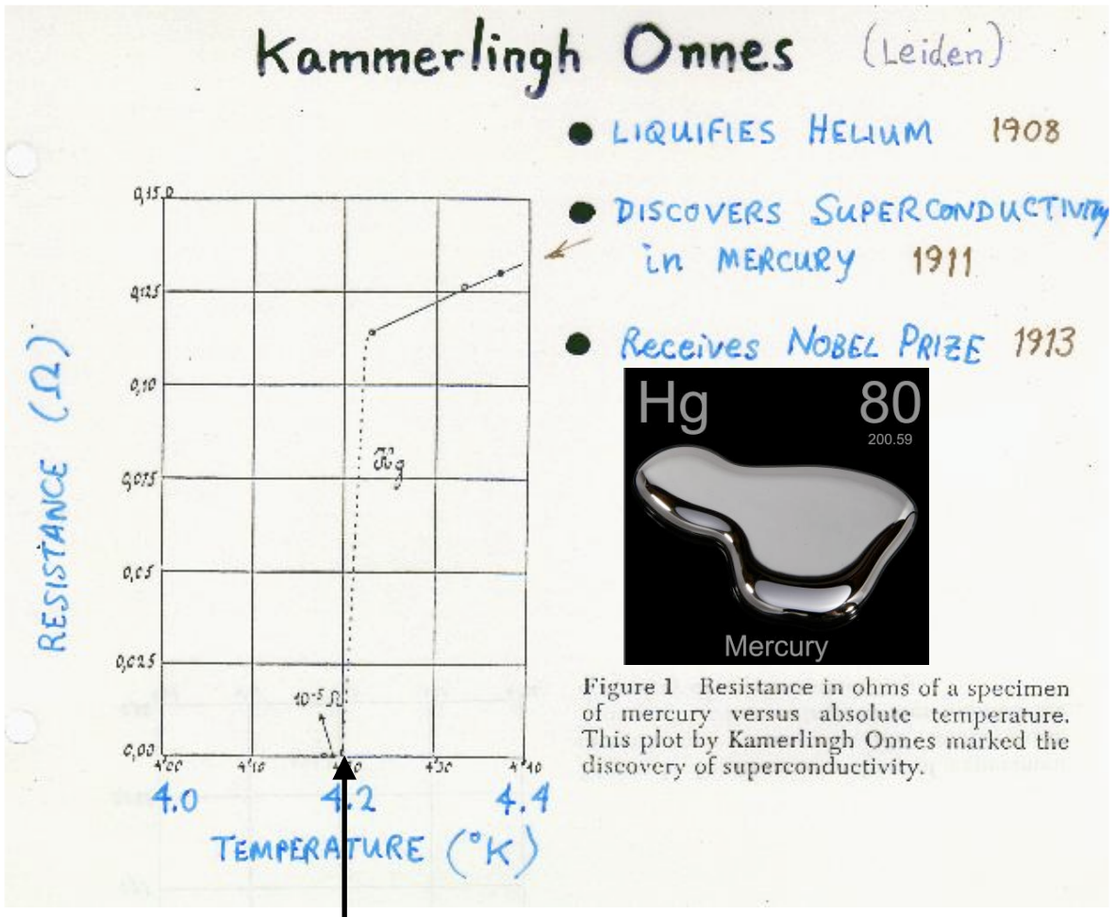
Superconductivity and Mottness: Exact Results

arXiv: 1912.01008
https://www.nature.com/articles/s41567-020-0988-4
with N&V by J. Zaanen

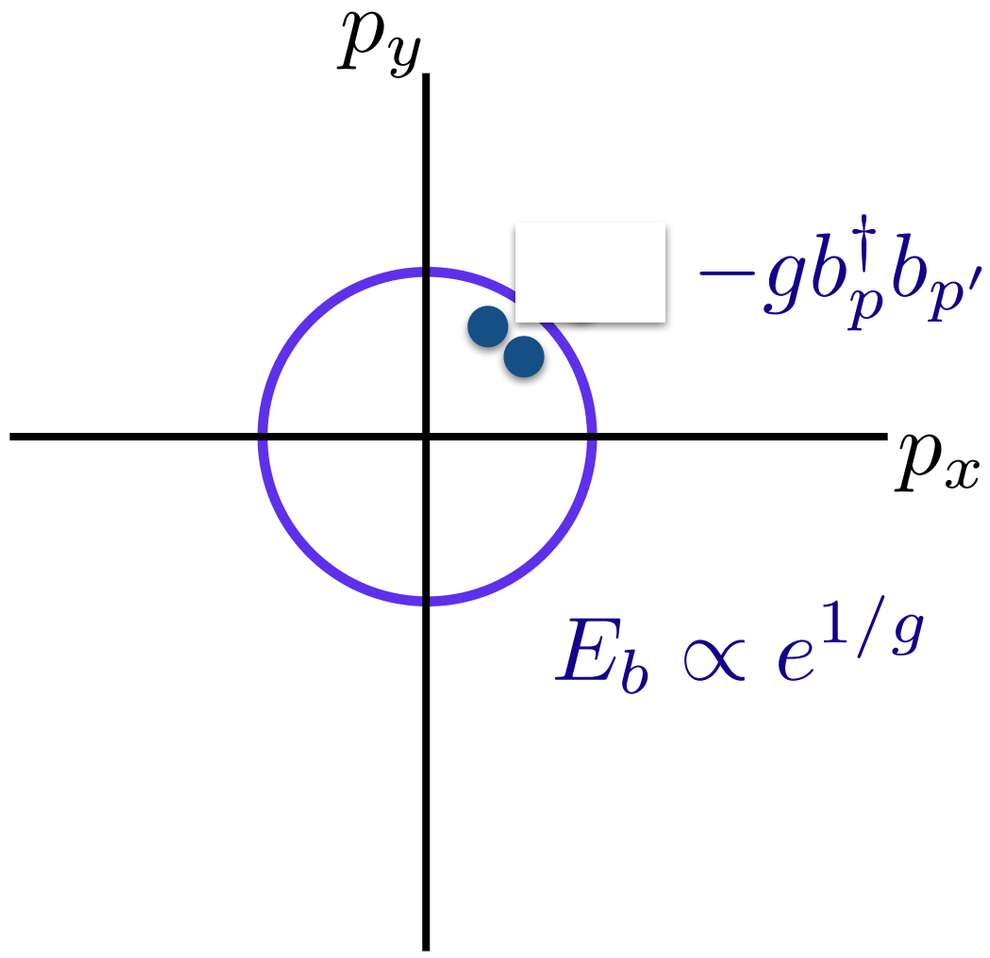
Luke Yeo

Edwin Huang

Cooper instability

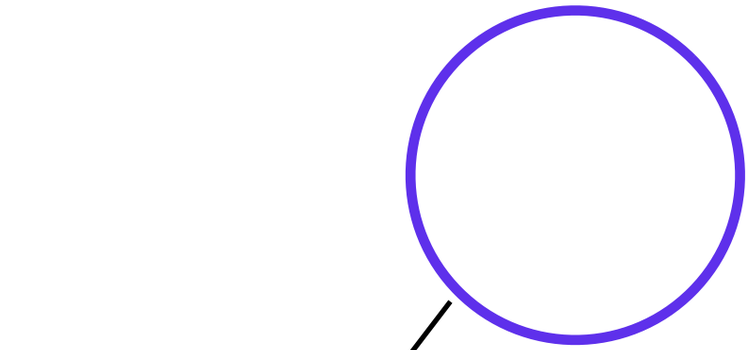


T_c



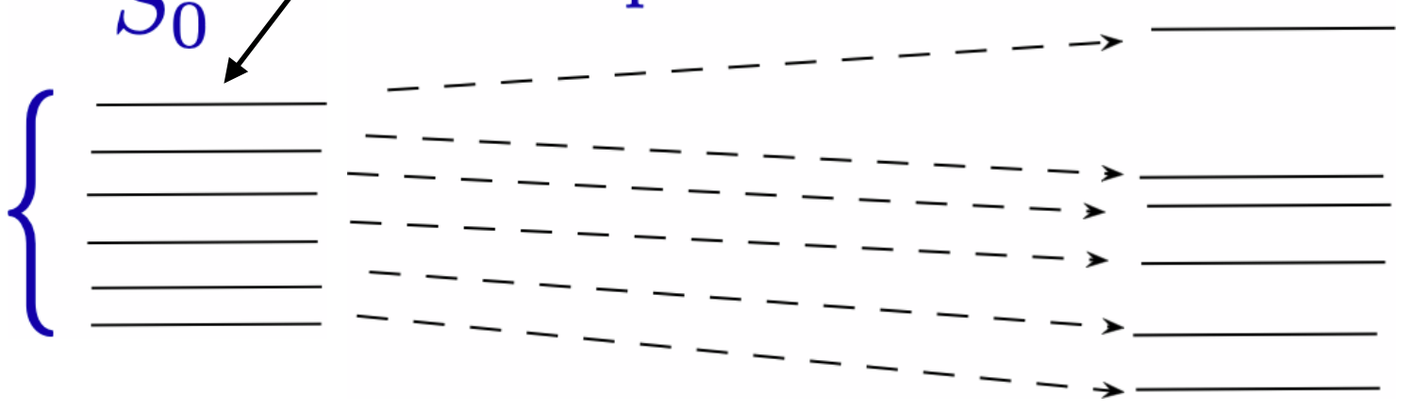
$$\langle \psi^\dagger \psi^\dagger \rangle = \Delta e^{i\phi}$$

$\mathrm{FL} \to \mathrm{BCS}$



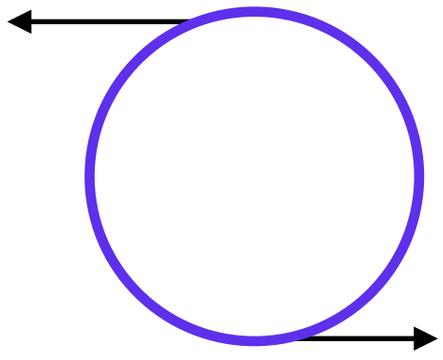
Fermi Surface

S_0 S_{repul}



1-1 correspondence

S_{repul} irrelevant

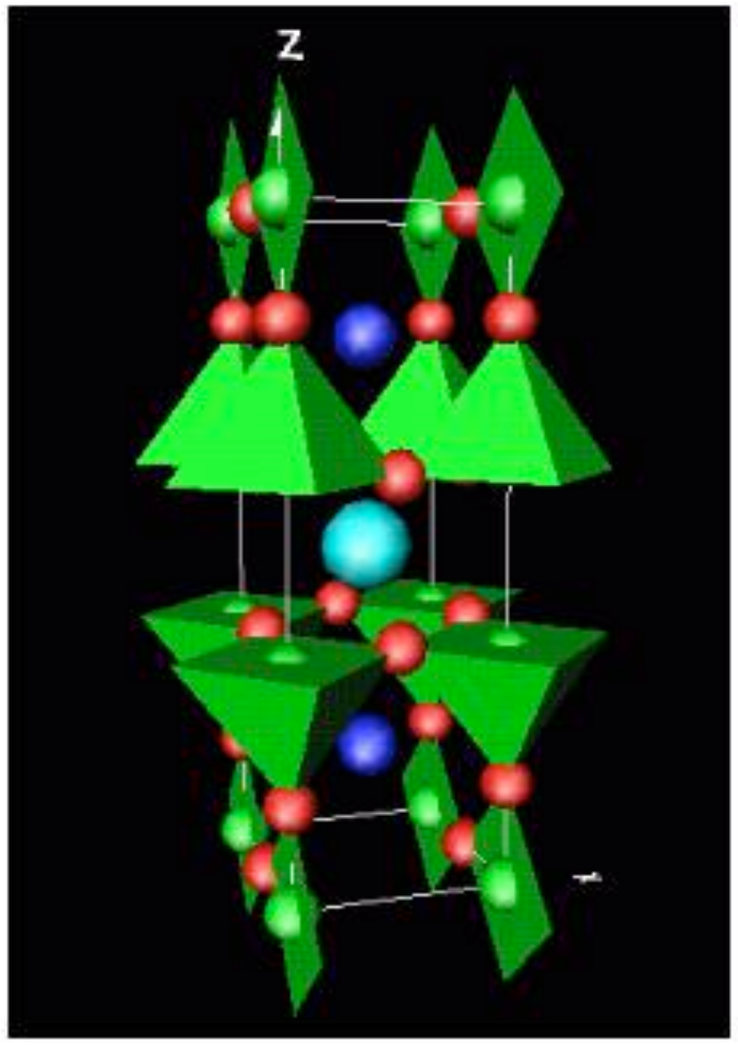


$+S_{\mathrm{pair}}$

superconductivity $\quad \dfrac{2\Delta}{T_c} = 3.5$ $\qquad \beta(g) = g^2$

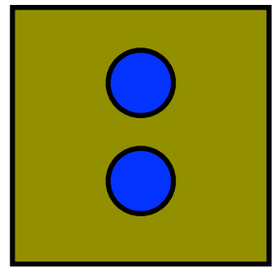
Is there physics
beyond
BCS?

fixed
point beyond
FL?



$Y\,Ba_2Cu_3O_7$

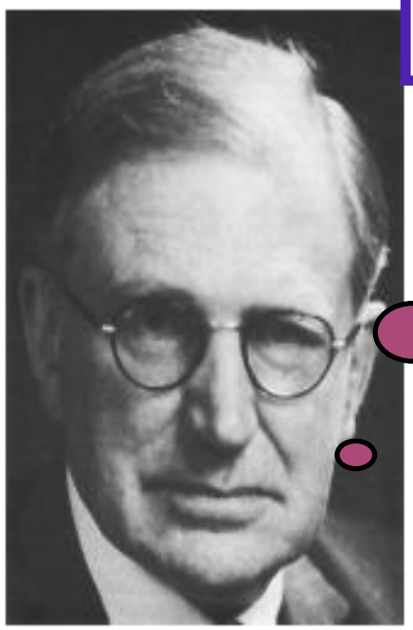
Cuprate Superconductors



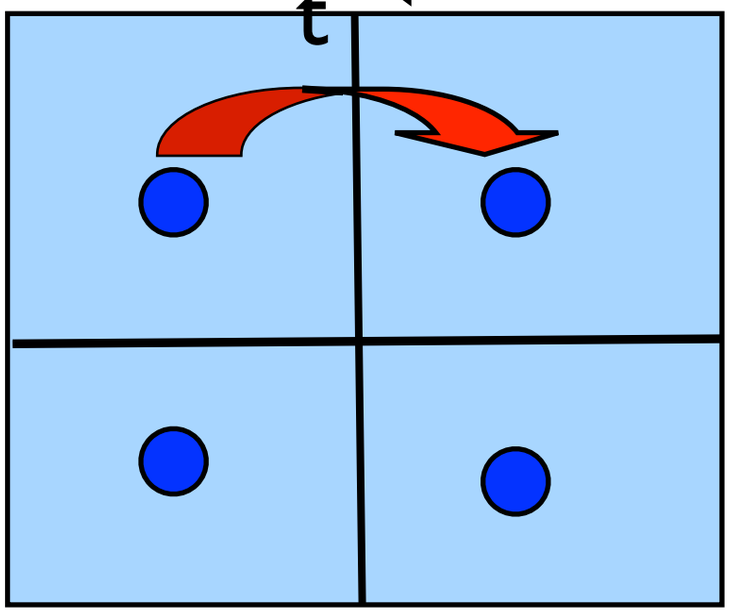
NiO insulates

d^8 ?

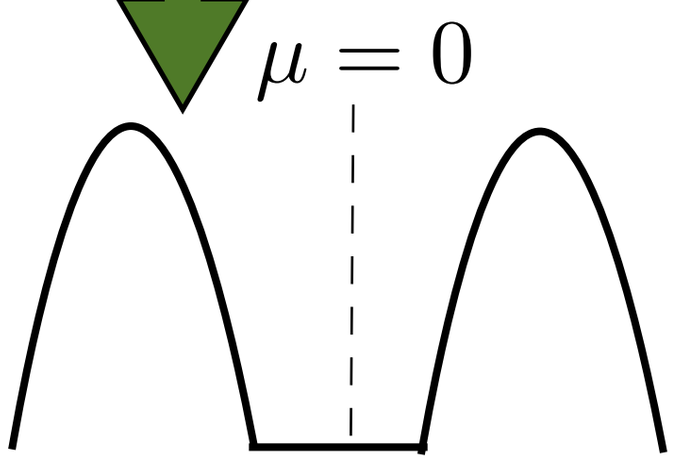
perhaps this costs energy



$U \gg t$



local real-space physics



no change in size of Brillouin zone

solve the Hubbard Model!!

Cooper instability??

Progress thus far?

DMFT

QMC

disputes

Sept. 1997

A Critique of Two Metals

R. B. Laughlin
Departrment of Physics
Stanford University
Stanford, California 94305

idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconductor, except that they coexist and mix with soft magnetic excitations no one knows how to describe very well.

Nov. 1997

A Critique of "A Critique of Two Metals"

Philip W. Anderson and G. Baskaran

Joseph Henry Laboratories of Physics

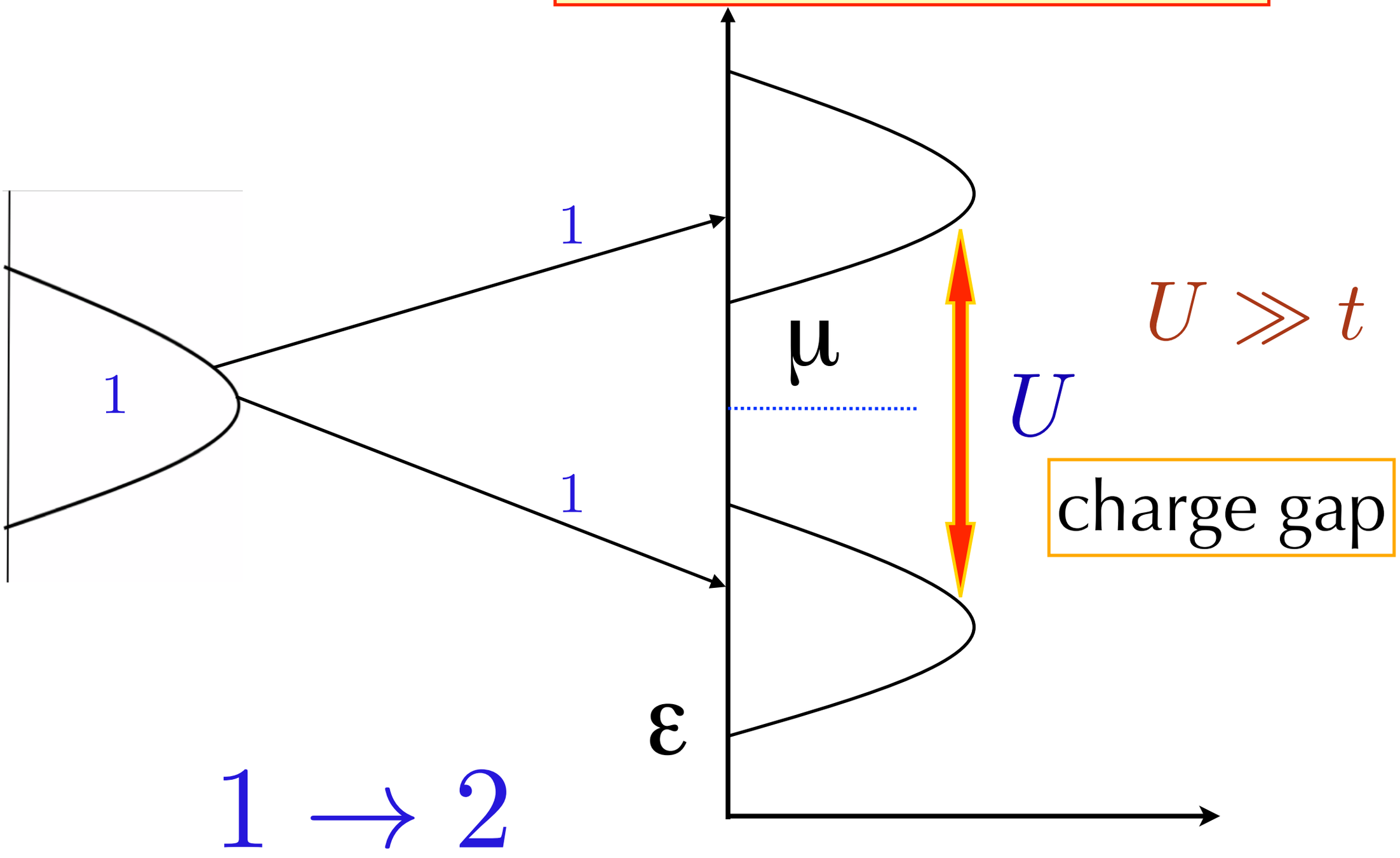
Princeton University, Princeton, NJ 08544

The fundamental argument is presented in the second paragraph: "Ten years of work by some of the best minds in theoretical physics have failed to produce any formal demonstration"...of the Mott insulating state. The statement would be ludicrous if it were not so influential. The proviso "at zero temperature" is added, because of course most Mott

concern. It is the tragedy of Mott that although he almost certainly won his Nobel prize for the Mott insulator, Slater, who couldn't think clearly about finite temperature, won the publicity battle.

Laughlin's objection:

gap with no symmetry breaking not demonstrated!!



—

= Mottness

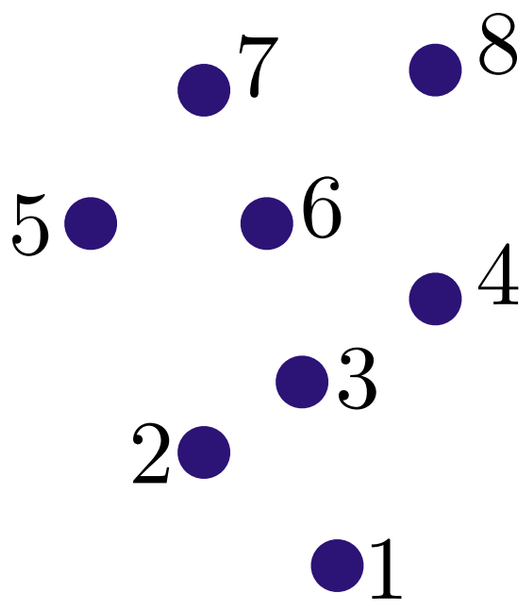
zeros

$= 0$

$\neq 0$

$\mathrm{DetReG}(\omega = 0, \mathrm{p})$

counting particles



is there a more efficient way?

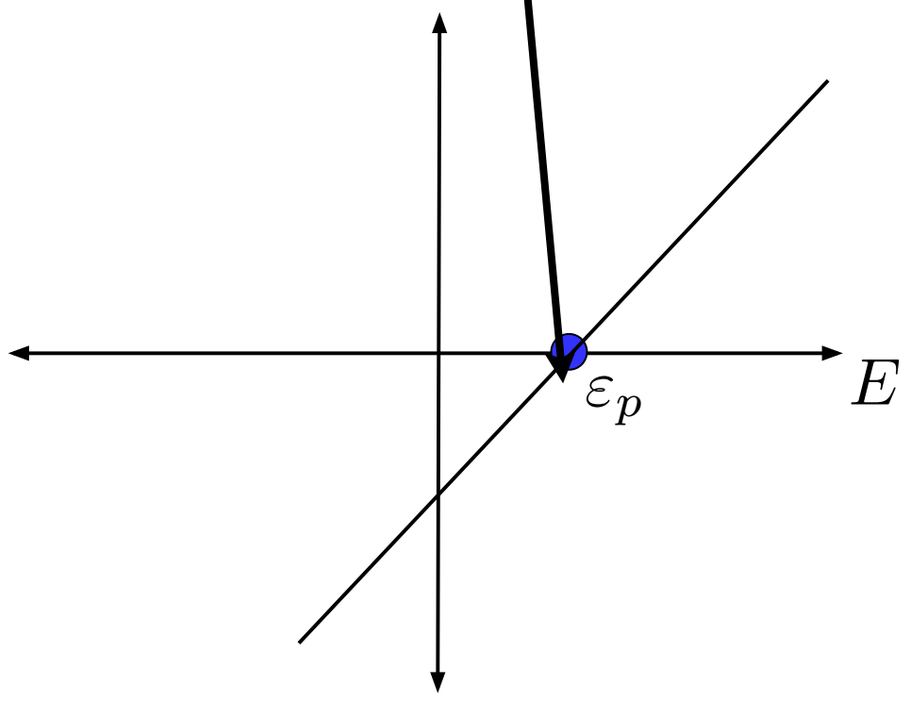
Luttinger counting theorem

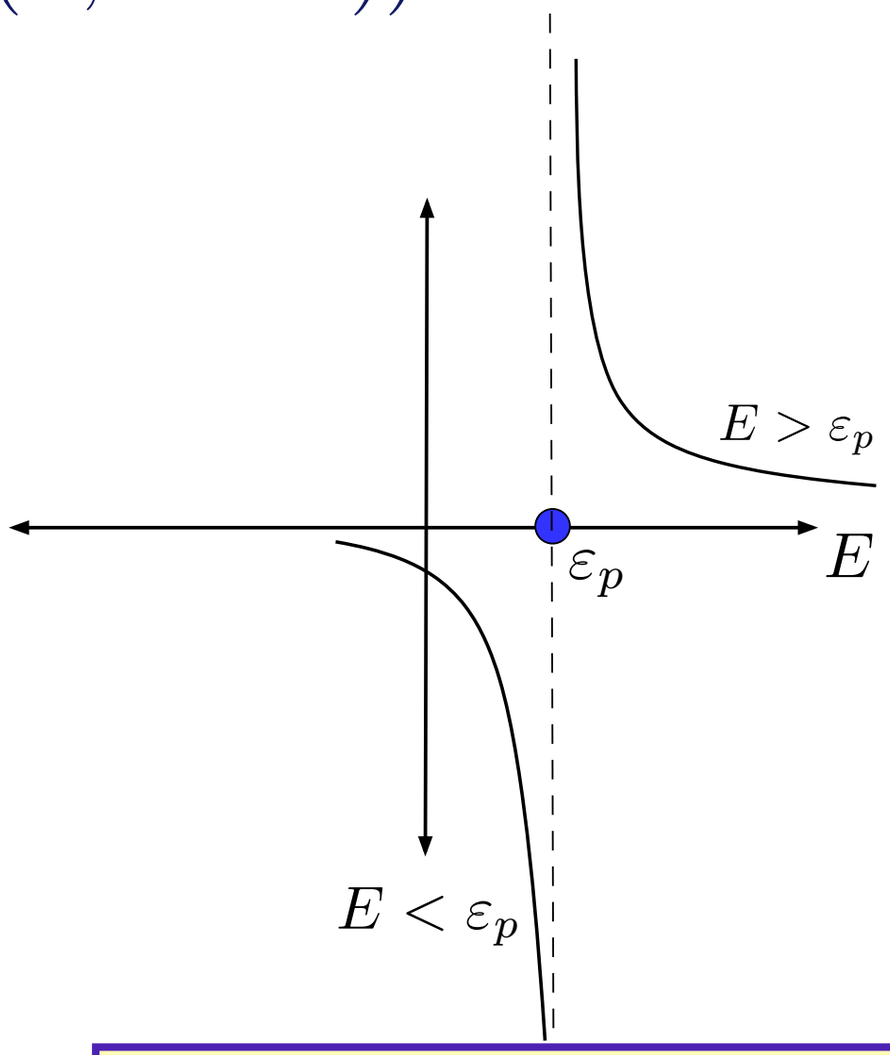
$$G(E) = \frac{1}{E - \varepsilon_p}$$

$$n = 2 \sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

zero-crossing

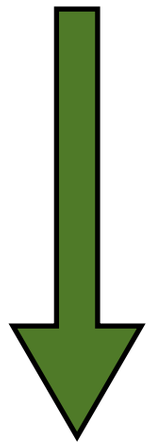
$\mathrm{Det} G(\omega = 0, \vec{p}) = 0$





counting poles (qp)

zeros



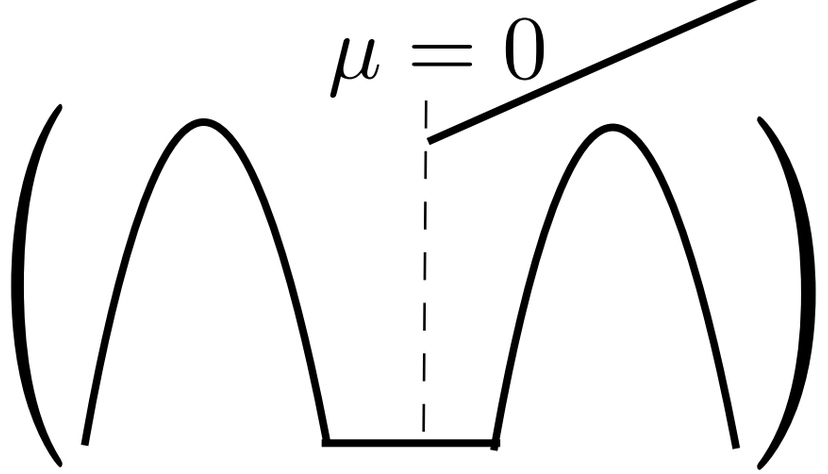
breakdown of particle concept

Mottness

How do zeros obtain?

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty} \left(\quad \right) d\omega$$



Kramers -Kronig

$$\omega$$

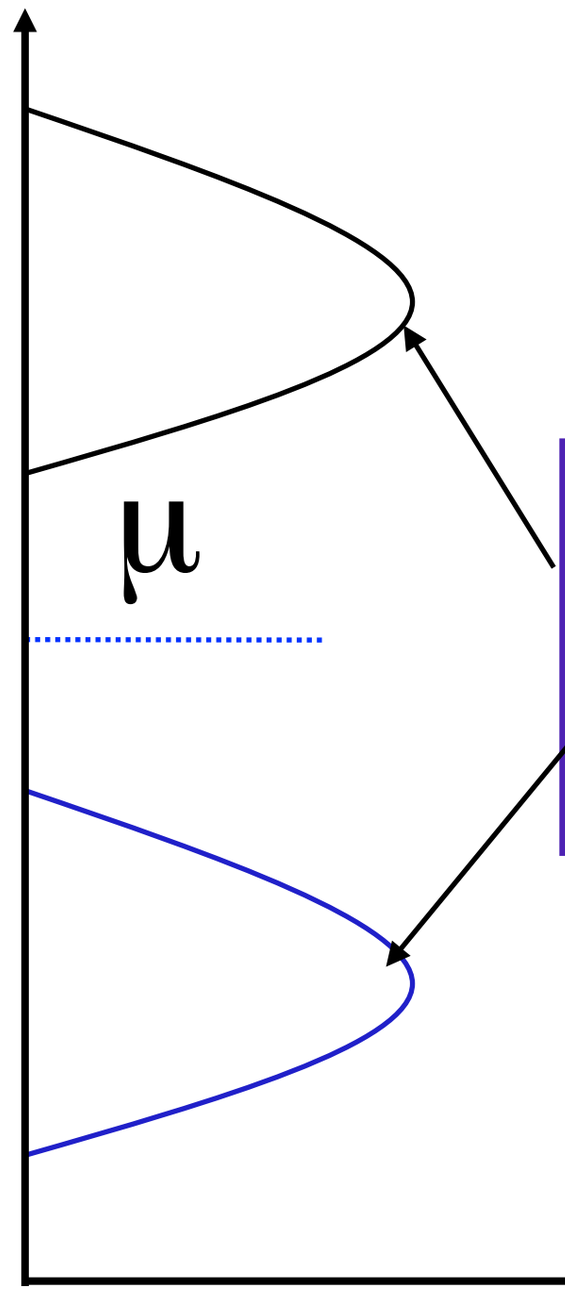
= below gap+above gap = 0

$$\mathrm{Det}G(\mathbf{k}, \omega = \mathbf{0}) = \mathbf{0}$$ (single band)

strongly correlated gapped systems

Symmetry Breaking

$$G_k(\omega) = \begin{pmatrix} \dfrac{1}{\omega - E_k^+} & 0 \\ 0 & \dfrac{1}{\omega - E_k^-} \end{pmatrix}$$



different band indices

$$\mathrm{Det}G \neq 0$$

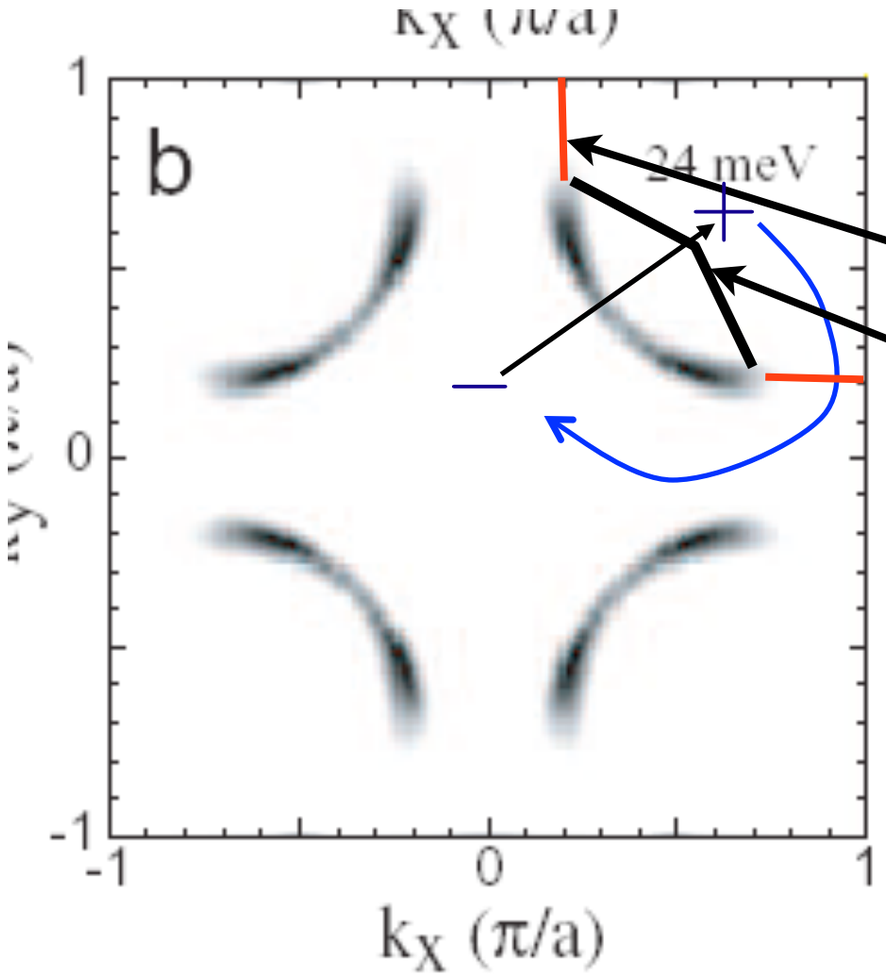
no Mottness

are zeros
important?

in a metal?

Fermi Arcs

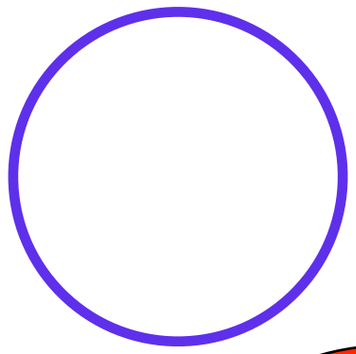
Re G
Changes
Sign across
An arc



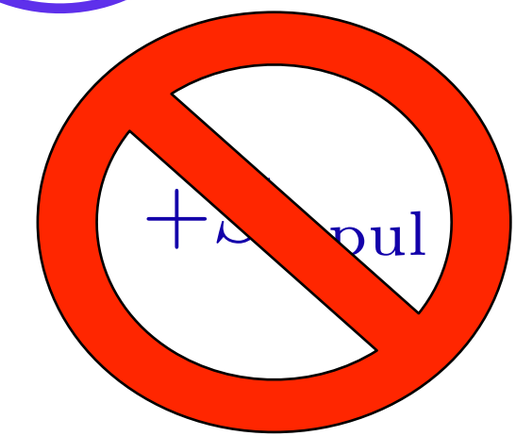
Must cross
A zero line
(DetG=0)!!!

Fermi arcs necessarily imply zeros exist.

Where's Mottness??



Fermi Surface



irrelevant

What went wrong?

simplest model with zeros

atomic limit of Hubbard model

$$H_{t=0} = -\mu \sum_{i\sigma} n_{i\sigma} + \sum_{i} U n_{i\uparrow} n_{i\downarrow}$$

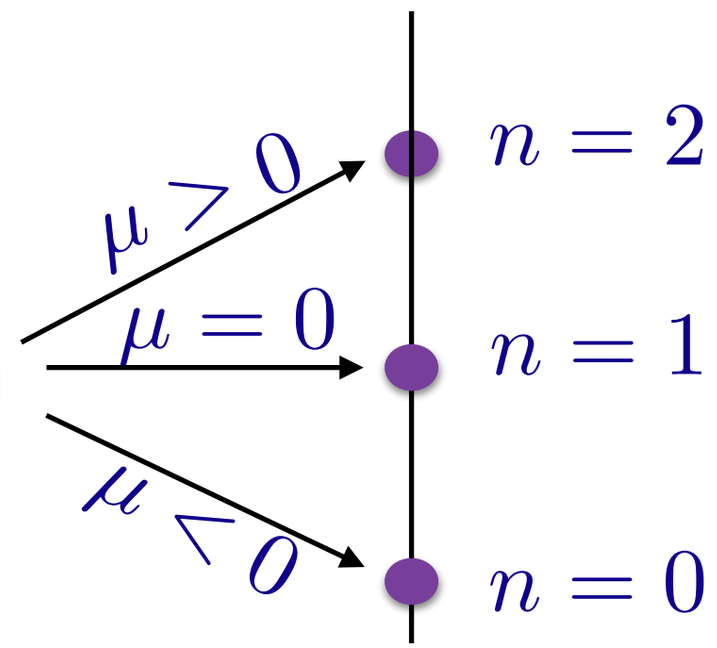
Exact Green Function

$$G_i^R(\omega) = \frac{1+x}{\omega - \mu + \frac{U}{2}} + \frac{1-x}{\omega - \mu - \frac{U}{2}},$$

$x = 0$

$$G_i^R(\omega = \mu = 0) = 0$$

$$n = 2\Theta\left(\frac{-\mu}{\mu^2 - U^2/4}\right)$$



LSR violation

Can the same physics be retained in the presence of a kinetic energy?

Mott insulator in momentum space???

Hatsugai-Khomoto Model (1992)

$$H_{\mathrm{HK}} = -t \sum_{\langle j,l \rangle,\sigma} \left(c^{\dagger}_{j\sigma} c_{l\sigma} + h.c. \right) - \mu \sum_{j\sigma} c^{\dagger}_{j\sigma} c_{j\sigma}$$

$$c_{k\sigma} = \sum_{j} e^{ikj} c_{j\sigma}$$

$$H_{\mathrm{HK}} = \sum_{k} H_k = \sum_{k} \left(\xi_k (n_{k\uparrow} + n_{k\downarrow}) + U n_{k\uparrow} n_{k\downarrow} \right).$$

$$\xi_k = \epsilon_k - \mu$$

relevant interaction

General HK Model

$$\sum_k \left(\xi_k(n_{k\uparrow} + n_{k\downarrow}) + U n_{k\uparrow}\, n_{k'\downarrow}\right) .$$

relevant perturbation

$$[H_t, H_U] = 0$$

Solvable Mott transition

$$G_{k\sigma}(i\omega_n \to z) = \frac{1 - \langle n_{k\bar{\sigma}} \rangle}{z - \xi_k} + \frac{\langle n_{k\bar{\sigma}} \rangle}{z - (\xi_k + U)}$$

lower Hubbard band

upper Hubbard band

Hubbard band operators

$$c^\dagger_{k\sigma}$$

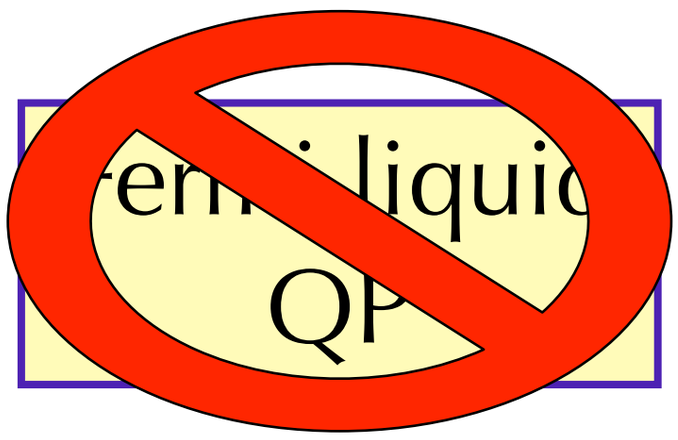
$$\zeta_{k\sigma} = c^\dagger_{k\sigma}(1 - n_{k\bar{\sigma}})$$

$$+$$

$$\eta_{k\sigma} = c^\dagger_{k\sigma} n_{k\bar{\sigma}}$$

$$\langle n_{k\sigma} \rangle = \frac{1}{2}$$

$$G^R_\sigma(k,\omega) = \frac{1}{\omega + i0^+ - (\xi_k + U/2) - \dfrac{(U/2)^2}{\omega + \underbrace{i0^+ - (\xi_k + U/2)}_{\Sigma(k,\omega)}}}$$

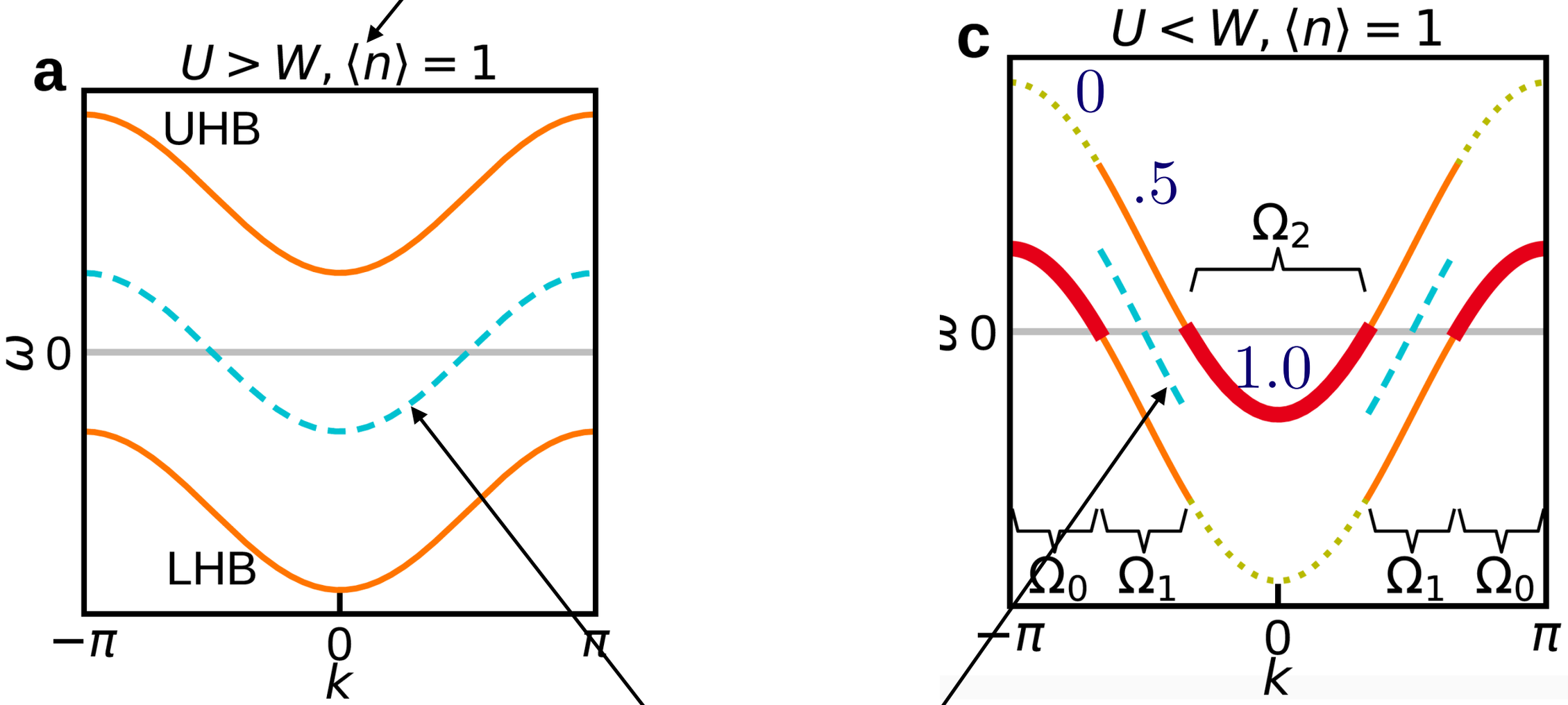


$$\omega = \xi_k + U/2$$

$$\Re\Sigma = \Im\Sigma = \infty$$

Mott transition: composite excitations

$$\Delta E = U - 4dt = U - W$$



insulator

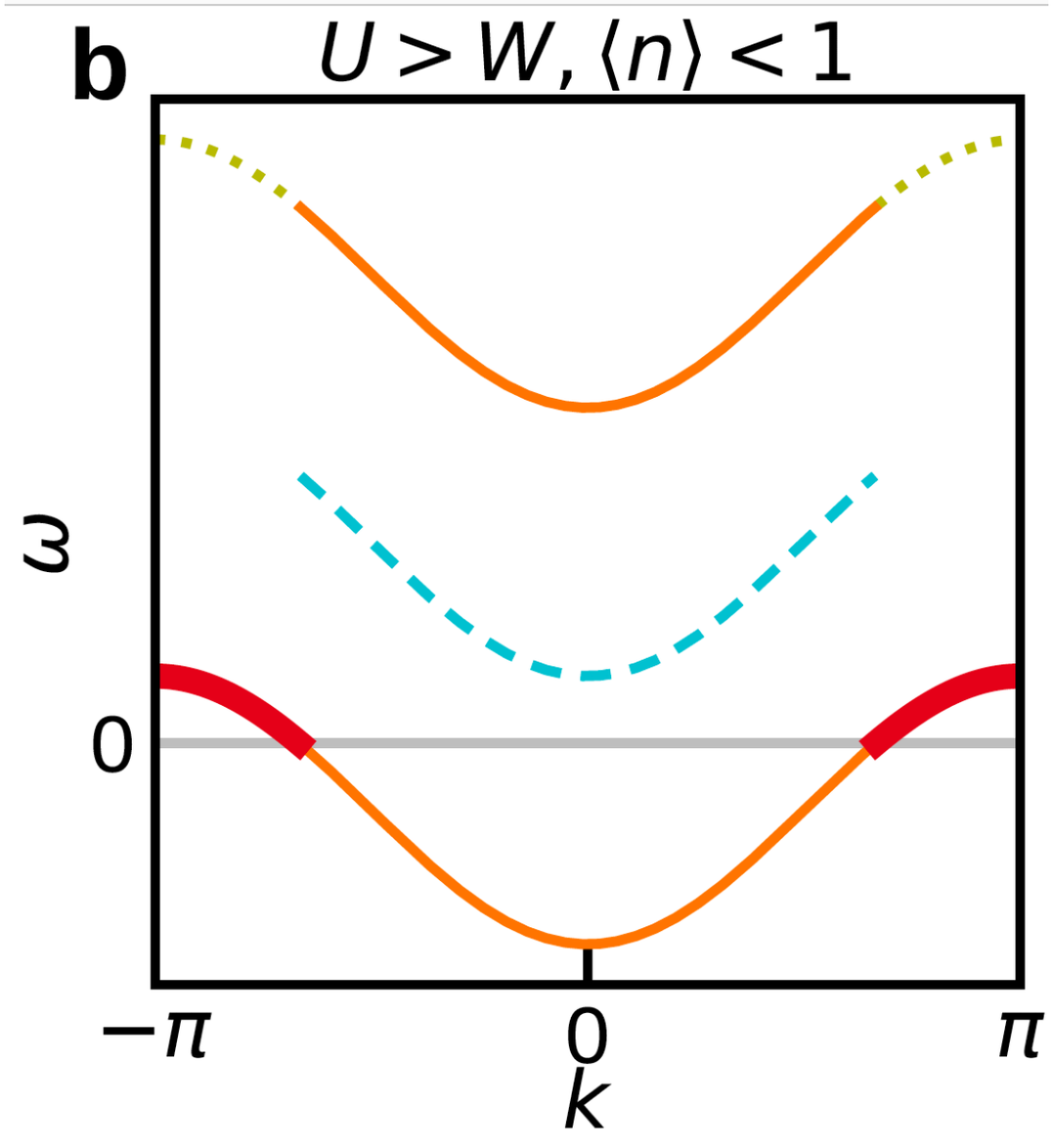
Luttinger Surface

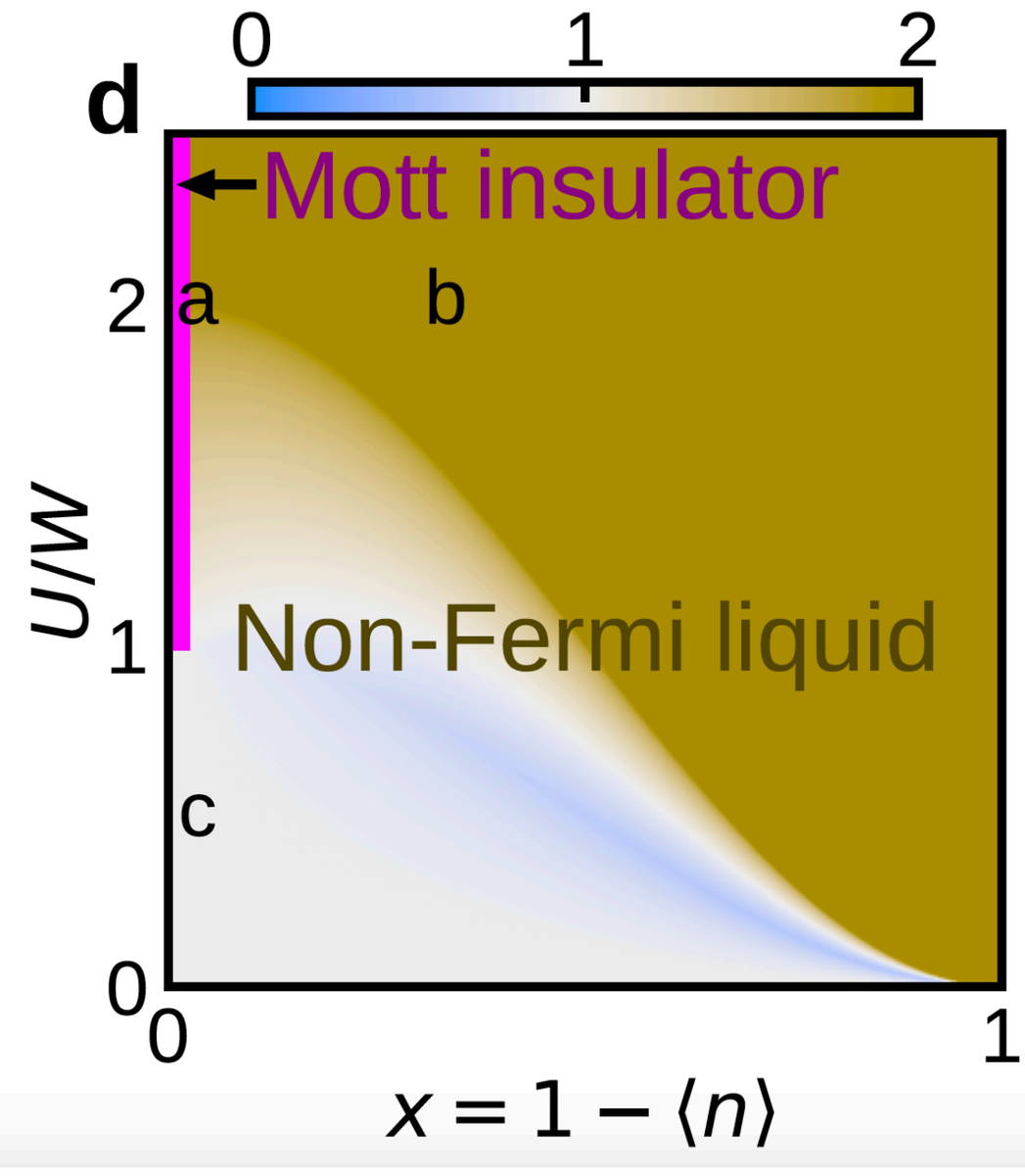
metal

$$\mathsf{FL}+\ Un_{k\uparrow}n_{k'\downarrow}\ = M.I.$$

metal is degenerate:
SU(2) invariance

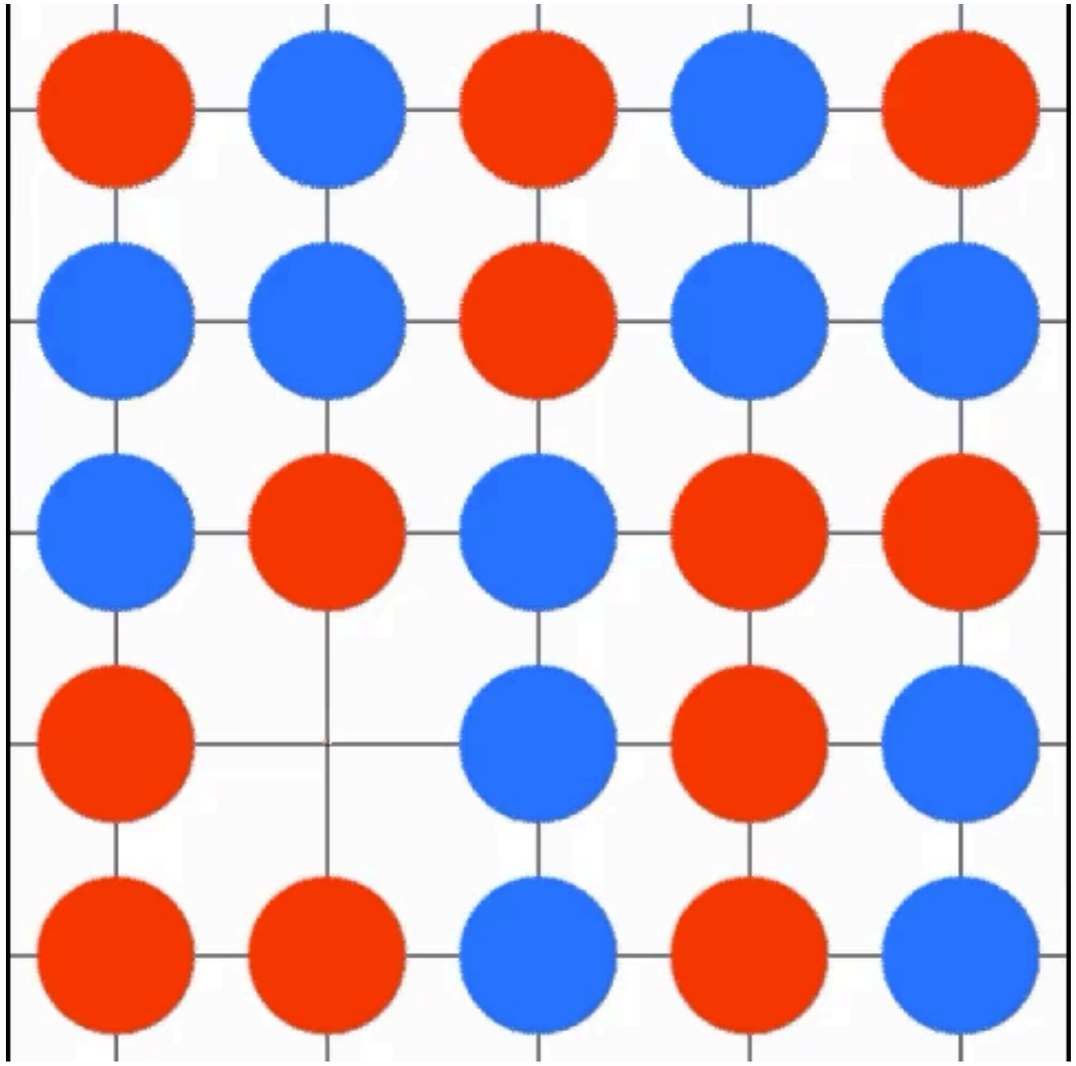
$$|\Psi_G;\{\sigma_k\}\rangle = \prod_{k\in\Omega_1} c^\dagger_{k\sigma_k} \prod_{k\in\Omega_2} c^\dagger_{k\uparrow} c^\dagger_{k\downarrow}|0\rangle,$$





what does the HK model leave out??

$$[H_t, H_U] \neq 0$$



dynamical spectral weight transfer

Superdconductivity?

C. Setty
2019

ILS+ Pair fluctuations= SYK

Is this true in general?

Cooper Instability

$$H = H_{\mathrm{HK}} - gH_p$$

$$|\psi\rangle = \sum_{k\in\Omega_0} \alpha_k b_k^\dagger |\mathrm{GS}\rangle$$

$\langle n_{k\sigma}\rangle = 0$

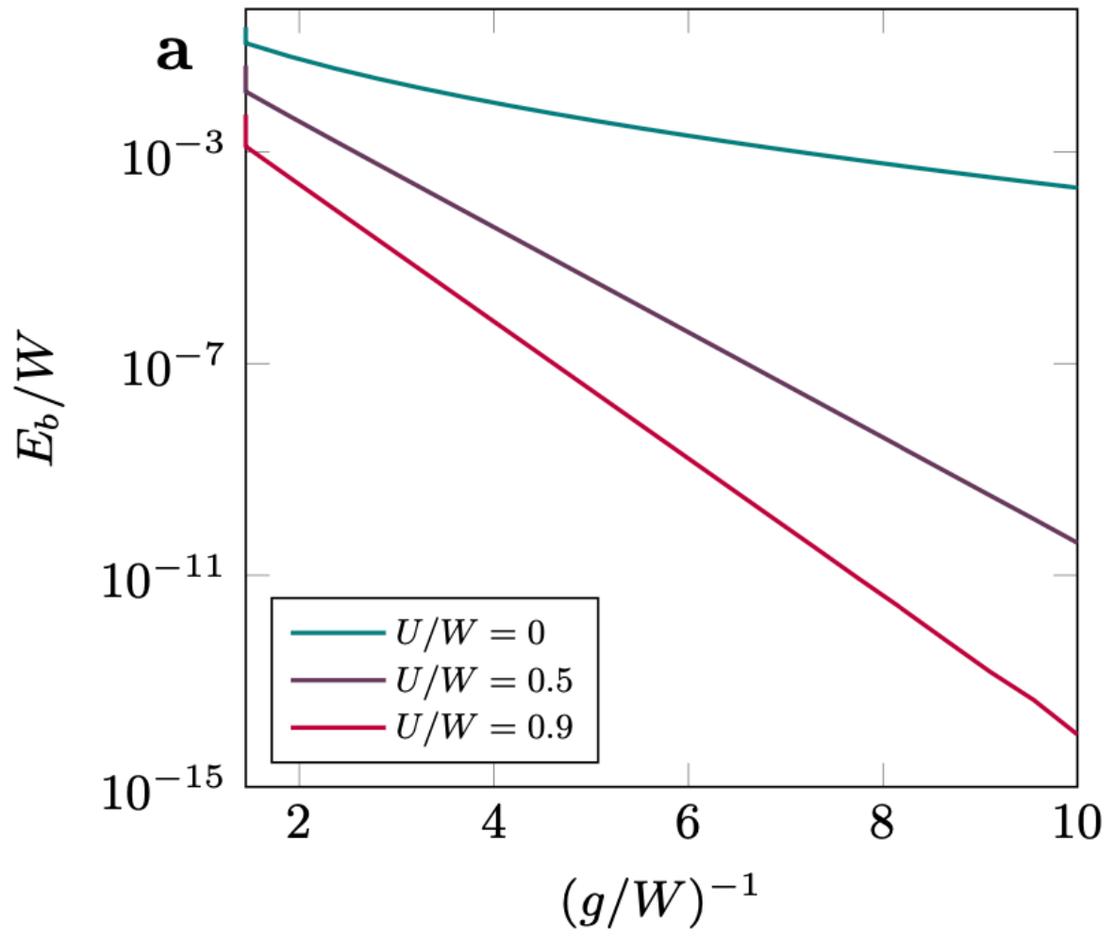
$$E_b = \langle \mathrm{GS}|\mathrm{H}|\mathrm{GS}\rangle - \langle\psi|\mathrm{H}|\psi\rangle \leqslant 0$$

$$1 = -\frac{g}{L^d}\sum_{k\in\Omega_0} \frac{\langle 1 - n_{k\uparrow} + n_{-k\downarrow}\rangle}{E - 2\xi_k - U\langle n_{k\downarrow} + n_{-k\uparrow}\rangle}$$

$$1 = -g\int_{\mu}^{W/2} d\epsilon \frac{\rho(\epsilon)}{E - 2\epsilon + 2\mu},$$

Cooper Instability

$$E_b = -E \sim W(1-(U/W)^2)e^{-\pi W\sqrt{1-(U/W)^2}/g}$$



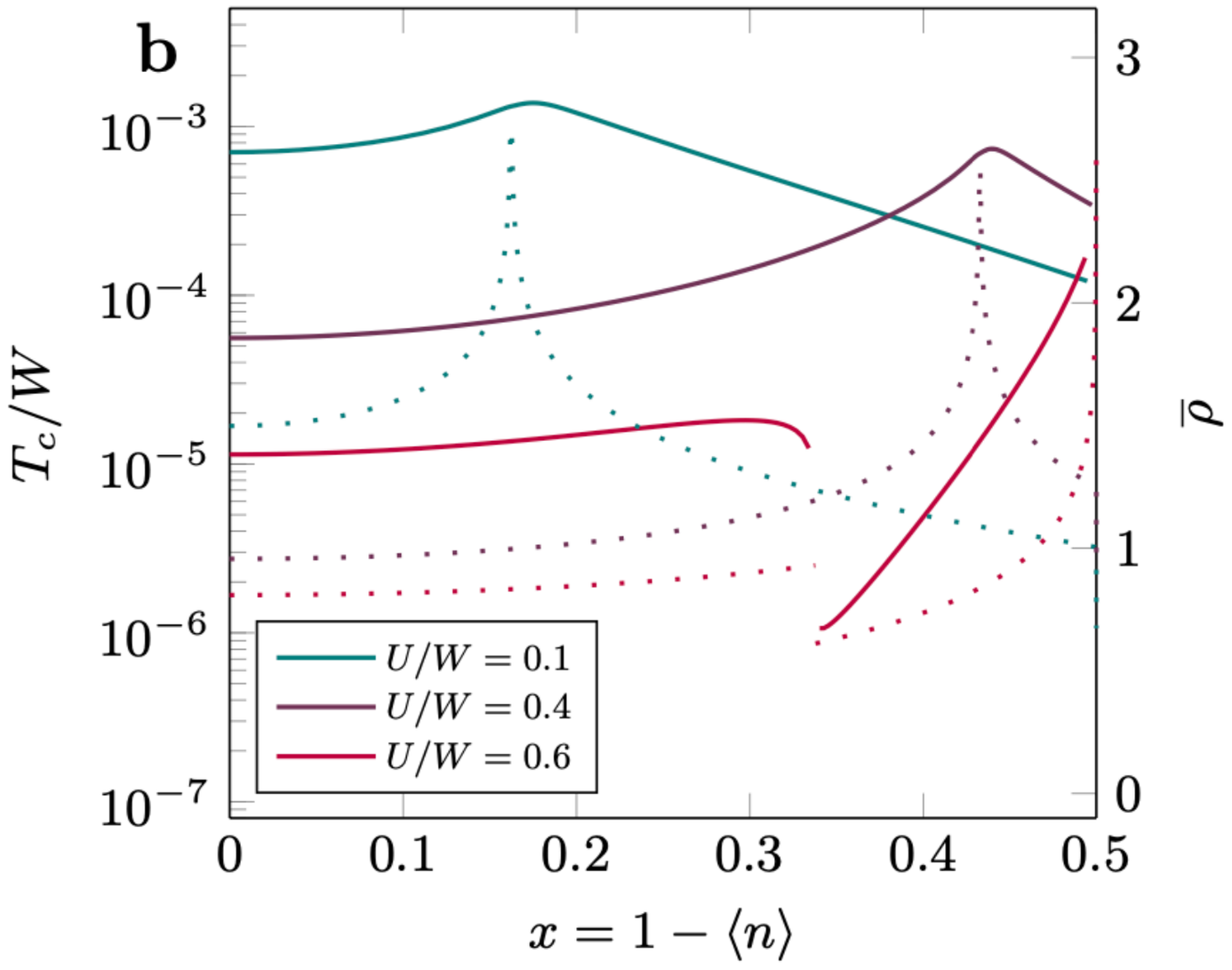
Pair Susceptibility

$$\chi(i\nu_n) \equiv \frac{1}{L^d}\int_0^\beta d\tau e^{i\nu_n\tau}\langle T\Delta(\tau)\Delta^\dagger\rangle_g$$

$$= \frac{\chi_0}{1-g\chi_0}$$

$$\sum_k G_{-k\downarrow}(\tau)G_{k\uparrow}(\tau)$$

solve for T_c



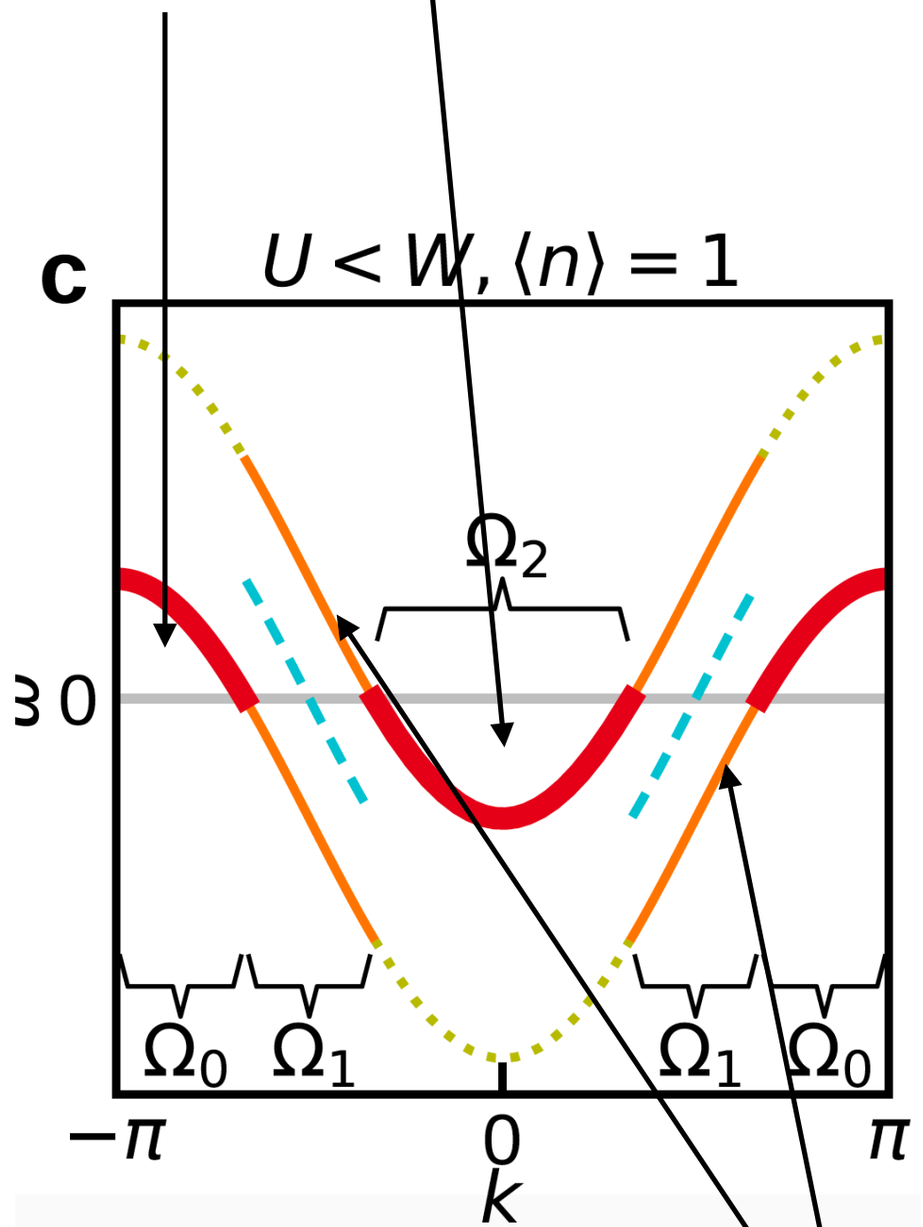
$$T_c = (W-U)^{4/5} U^{1/5} \frac{e^{\gamma}}{\pi} e^{-\frac{4}{5}\frac{W}{g}}.$$

variational wave function

$$|\psi_{\rm BCS}\rangle = \prod_k (u_k + v_k b_k^\dagger |0\rangle$$

$$|\psi_{\rm BCS}\rangle = \prod_{k>0} (u_k^2 + v_k^2 b_k^\dagger b^\dagger{}_{-k} + u_k v_k (b_k^\dagger + b^\dagger{}_{-k}))|0\rangle$$

$$|\psi\rangle = \prod_{k>0} \left(x_k + y_k b_k^\dagger b_{-k}^\dagger + \frac{z_k}{\sqrt{2}} (b_k^\dagger + b_{-k}^\dagger) \right) |0\rangle$$



HK generalization

$$|\psi\rangle = \prod_{k>0} \left(x_k + y_k b_k^\dagger b_{-k}^\dagger + \frac{z_k}{\sqrt{2}} (b_k^\dagger + b_{-k}^\dagger) \right) |0\rangle$$

three variational parameters

$$|x_k|^2 + |y_k|^2 + |z_k|^2 = 1$$

gap equation

$$1 = \frac{g}{W}\sinh^{-1}(\frac{W-U}{2\Delta}) + \frac{g}{W}\sinh^{-1}(\frac{U}{2\Delta})$$

$\Delta \ll U, W$

$$\Delta = (W-U)^{1/2}U^{1/2}e^{-\frac{W}{2g}}$$

gap/T_c ratio

$$\Delta = (W-U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

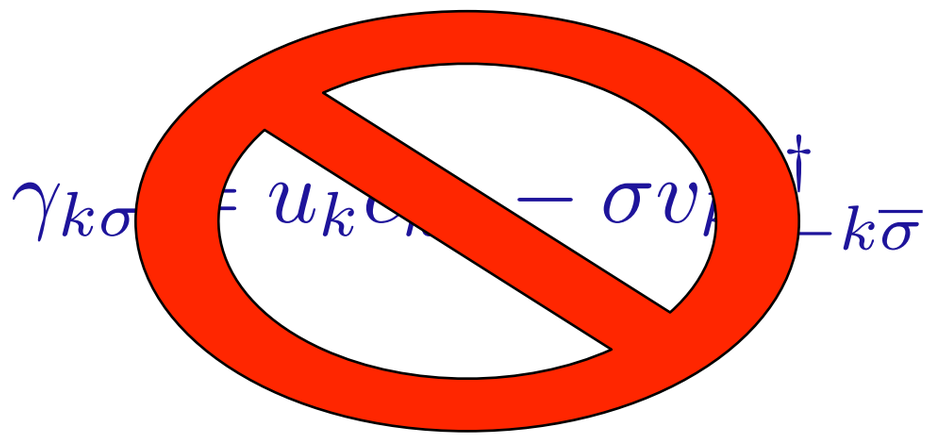
$$T_c = (W-U)^{4/5} U^{1/5} \frac{e^{\gamma}}{\pi} e^{-\frac{4}{5}\frac{W}{g}} .$$

$$\lim_{g \to 0} \frac{\Delta}{T_c} \to \infty$$

non-BCS superconductivity

Bogoliubov excitations

$$\gamma_{k\sigma}|\psi_{\mathrm{BCS}}\rangle = 0$$



PYHons excitations

$$\gamma^{l}_{k\sigma} \propto \sqrt{2}x_k\zeta^{\dagger}_{k\sigma} - \sigma z_k\zeta_{-k\bar{\sigma}}$$

$$\gamma^{u}_{k\sigma} \propto z_k\eta^{\dagger}_{k\sigma} - \sigma\sqrt{2}y_k\eta_{-k\bar{\sigma}}$$

Excitation spectrum

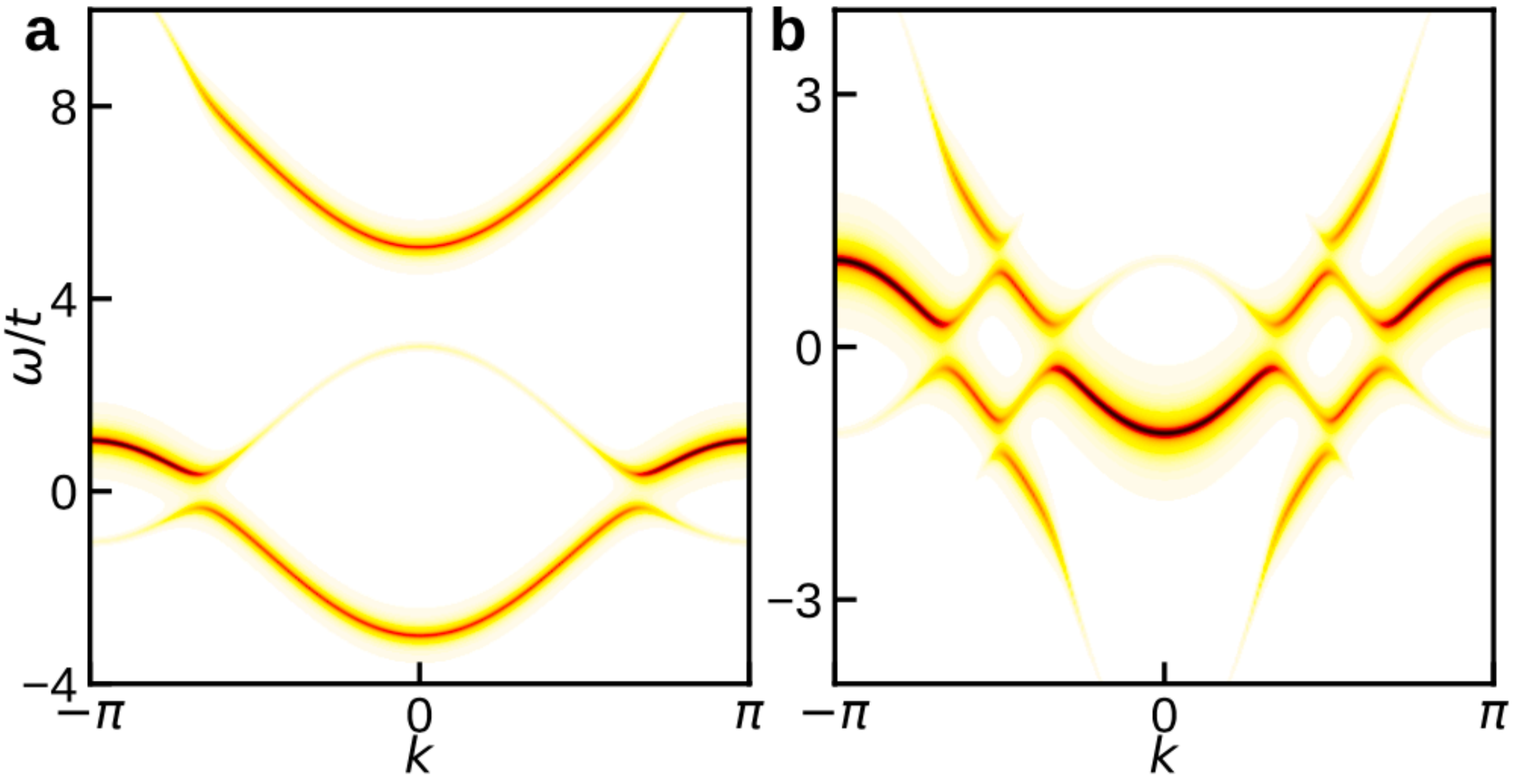
$$\gamma_{k\sigma}^{u/l}|\psi\rangle = 0$$

$$\langle\psi|\gamma_{k\sigma}^{u/l} H \gamma_{k\sigma}^{u/l})^{\dagger}|\psi\rangle = \langle\psi|H|\psi\rangle + E_k^{u/l}$$

$$E_k^{u/l} = \sqrt{{\xi_k^{u/l}}^2 + \Delta^2}$$

superconductivity affects both bands!

spectral function: superconducting state



Doped MI

half-filled metal

BCS

HK

a

b

3

0

−3

ω/t

$-\pi$

0

π

k

PYHon band

can we explain the color change?

REPORT

Superconductivity-Induced Transfer of In-Plane Spectral Weight in $Bi_2Sr_2CaCu_2O_{8+\delta}$

H. J. A. Molegraaf[1], C. Presura[1], D. van der Marel[1,*], P. H. Kes[2], M. Li[2]
+ See all authors and affiliations

Science 22 Mar 2002:
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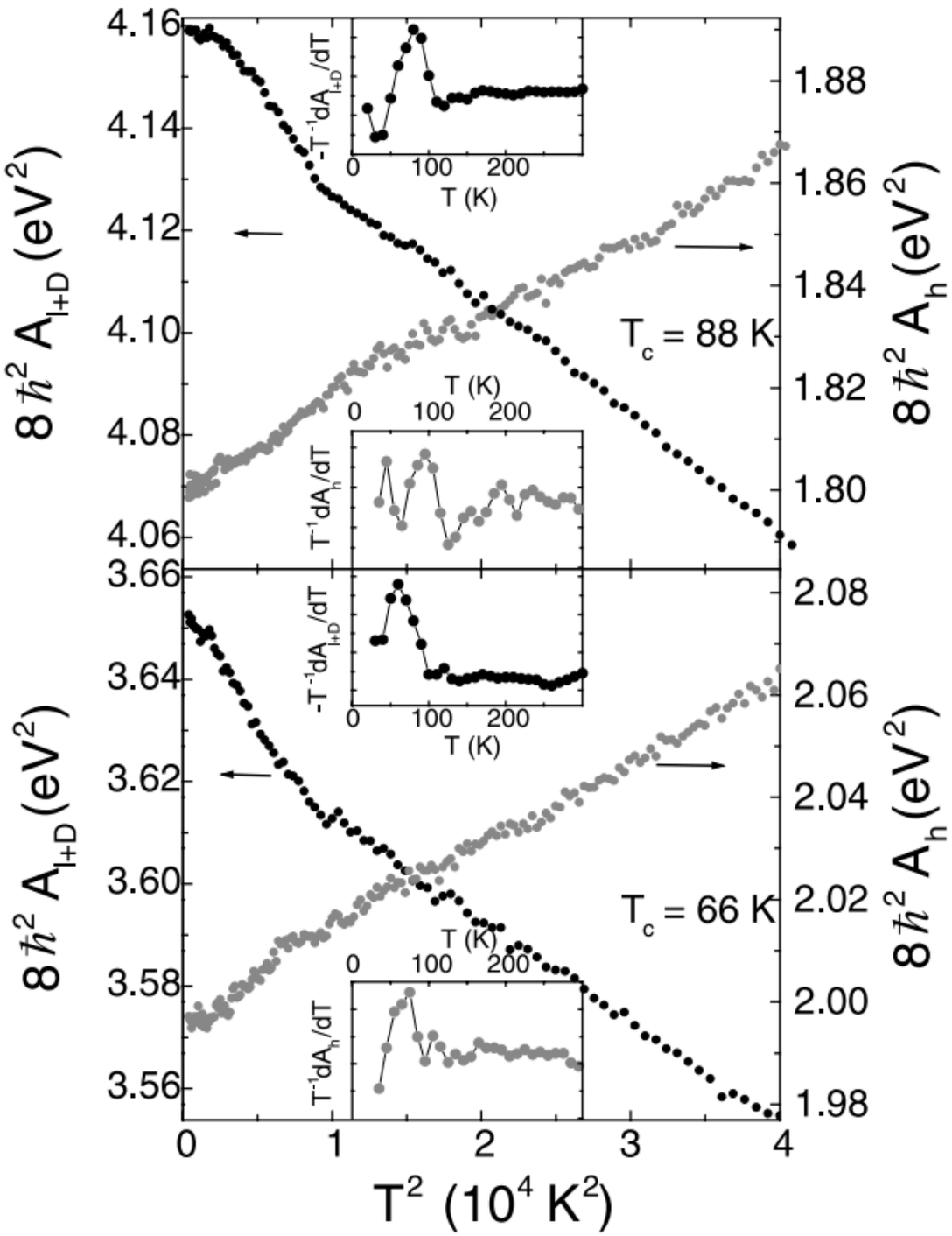
DOI: 10.1126/science.1069947

$$A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

$$A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

$$\frac{\Delta A_l}{A_l} \propto 3\%$$

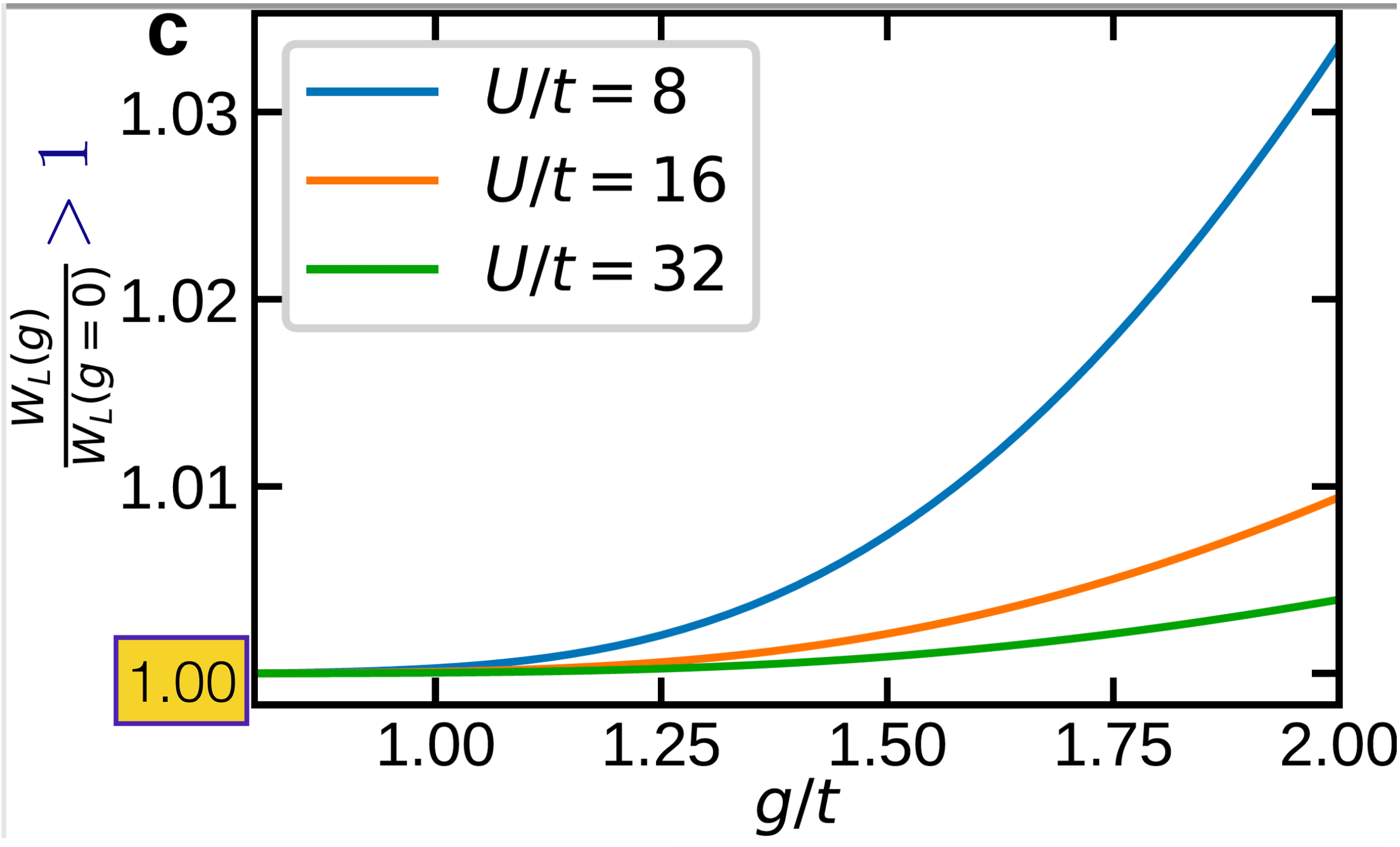


condensation energy

Optical data are reported on a spectral weight transfer over a broad frequency range of $Bi_2Sr_2CaCu_2O_{8+\delta}$, when this material became superconducting. Using spectroscopic ellipsometry, we observed the removal of a small amount of spectral weight in a broad frequency band from 10^4 cm^{-1} to at least 2×10^4 cm^{-1}, due to the onset of superconductivity. We observed a blue shift of the *ab*-plane plasma frequency when the material became superconducting, indicating that the spectral weight was transferred to the infrared range. Our observations are in agreement with models in which superconductivity is accompanied by an increased charge carrier spectral weight. The measured spectral weight transfer is large enough to account for the condensation energy in these compounds.

UV-IR mixing

condensation energy:HK model



$\Delta W_L \propto O(2-3\%)$

as in experiments

why?

$$H = H_{\mathrm{HK}} + H_p$$

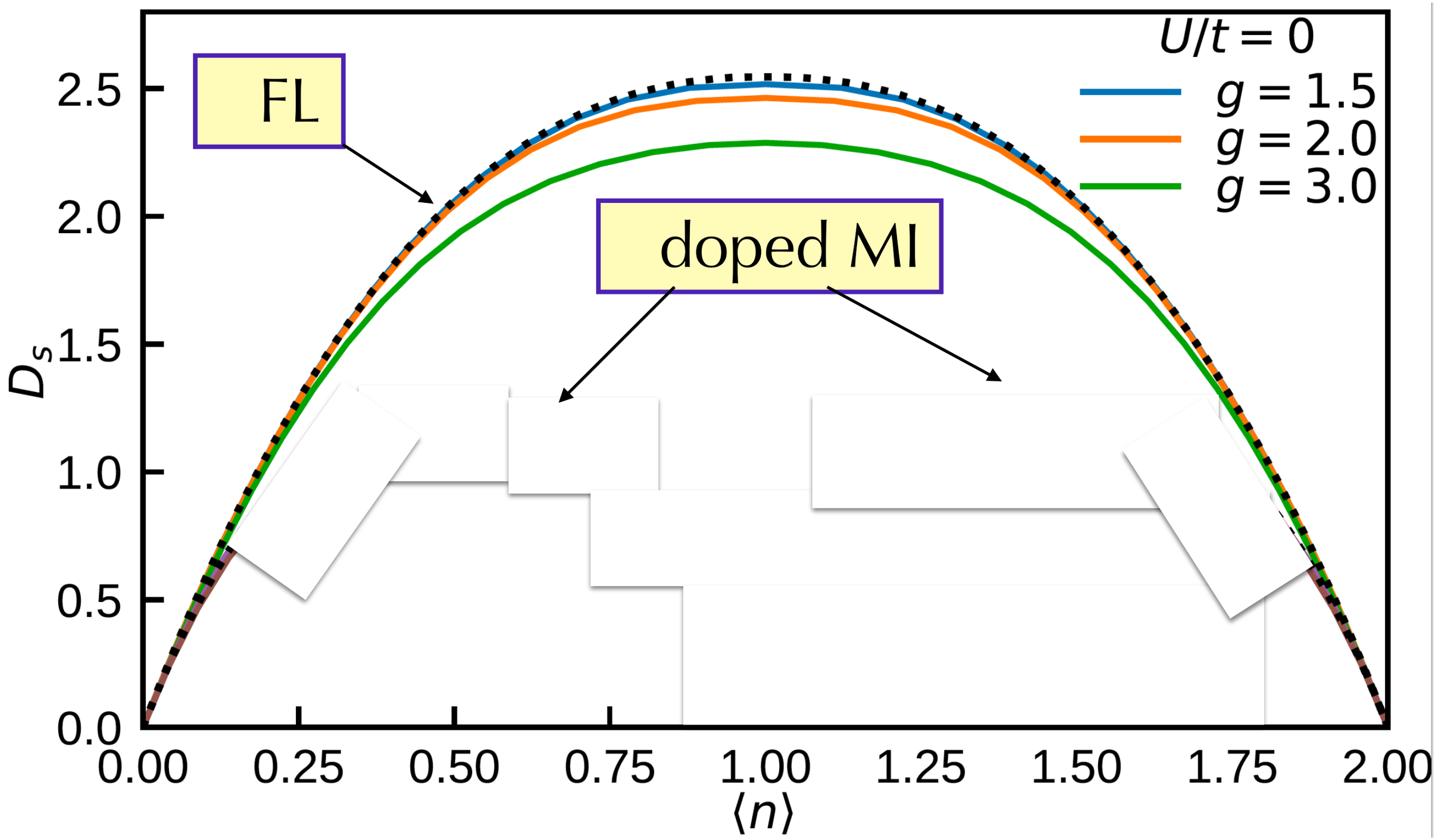
$$[H_{\mathrm{HK}}, H_p] \neq 0$$

dynamical
spectral weight
transfer

is this the general mechanism of the color change?

Superfluid Density

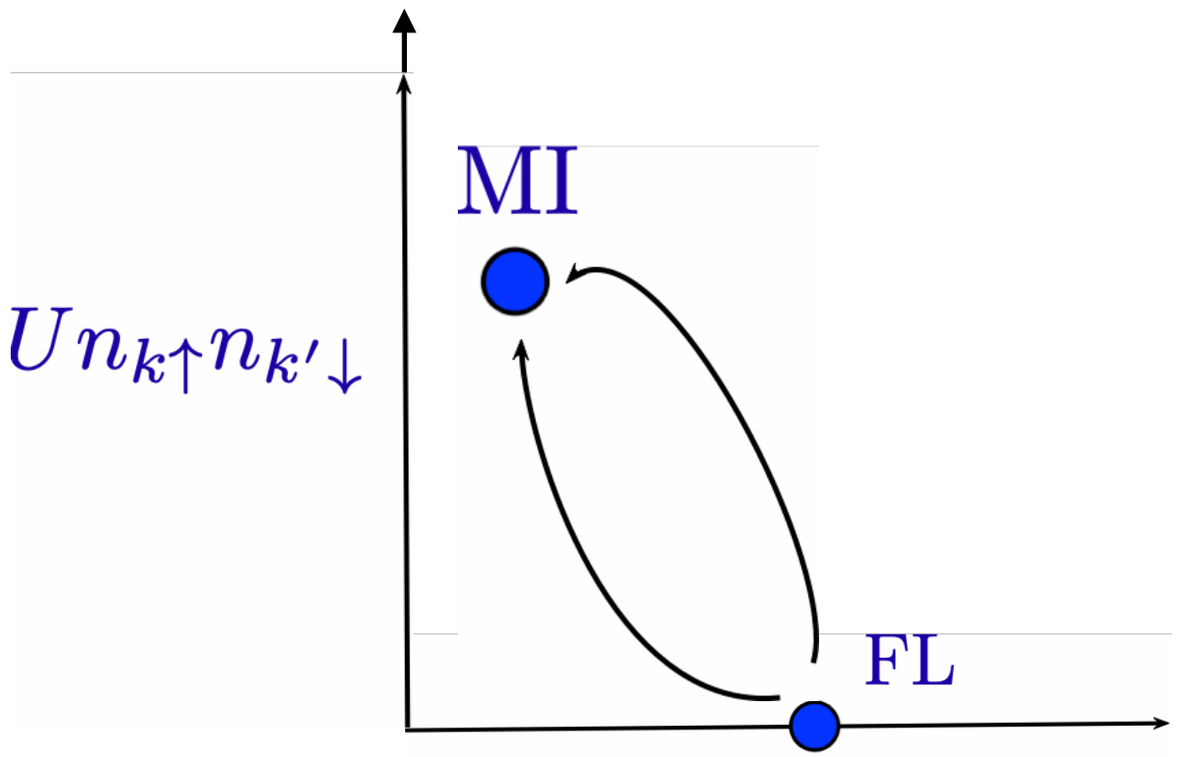
Mottness-induced suppression



Mott gap

$[H_t, H_V] = 0$

Mottness in momentum space



$\gamma^l_{k\sigma} \propto \sqrt{2}x_k\zeta^\dagger_{k\sigma} - \sigma z_k \zeta_{-k\bar{\sigma}}$

$V(p_1, p_2, p_3, p_4)$

$\gamma^u_{k\sigma} \propto z_k\eta^\dagger_{k\sigma} - \sigma\sqrt{2}y_k\eta_{-k\bar{\sigma}}$

PYHons

$[H_{\mathrm{HK}}, H_p] \neq 0$

$\frac{\Delta}{T_c} \propto \infty$

non-BCS superconductivity

$\frac{W_L(g)}{W_L(g=0)} > 1$