Superconductivity and Mottness: Exact Results

Nature Physics, vol.16, 1175-1180 (2020) with N&V by J. Zaanen

Luke Yeo

Edwin Huang
Cooper instability

\[ \langle \psi^\dagger \psi^\dagger \rangle = \Delta e^{i\phi} \]

\[ E_b \propto e^{1/g} \]

\[ -g b_p^\dagger b_{p'} \]

\[ T_c \]

Kammerlingh Onnes (Leiden)
- Liquifies helium 1908
- Discovers superconductivity in mercury 1911
- Receives Nobel Prize 1913

Figure 1. Resistance in ohms of a specimen of mercury versus absolute temperature. This plot by Kammerlingh Onnes marked the discovery of superconductivity.
superconductivity $\frac{2\Delta}{T_c} = 3.5$
Fermi gas

Fermi liquid

BCS superconductor

?
Is there physics beyond BCS?

fixed point beyond FL?
NiO insulates $d^8$? perhaps this costs energy

$U \gg t$

local real-space physics

$\mu = 0$

no change in size of Brillouin zone

$Y\ Ba\ Cu\ O$

Cuprate Superconductors
solve the Hubbard Model!!

Cooper instability??

Progress thus far?

DMFT  QMC  disputes
idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconductor, except that they coexist and mix with soft magnetic excitations no one knows how to describe very well.
Laughlin’s objection:

- Gap with no symmetry breaking not demonstrated!!
$\text{DetReG}(\omega = 0, p) = 0$ or $\neq 0 = \text{Mottness zeros}$
counting particles

is there a more efficient way?
Luttinger counting theorem

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

\[ n = 2 \sum_k \Theta(\Re G(k, \omega = 0)) \]

\[ \det G(\omega = 0, \vec{p}) = 0 \]

zero-crossing

counting poles (qp)
How do zeros obtain?

\[ \text{below gap} + \text{above gap} = 0 \]

\[ \text{DetReG}(k, \omega = 0) = 0 \] (single band)

strongly correlated gapped systems
Mottness

breakdown of particle concept

n=zeros+poles?

no propagation

zeros
Symmetry Breaking

\[ G_k(\omega) = \begin{pmatrix} \frac{1}{\omega - E_k^+} & 0 \\ 0 & \frac{1}{\omega - E_k^-} \end{pmatrix} \]

\[ \text{Det} G \neq 0 \]

\[ \mu \]

different band indices

no Mottness

Laughlin
Minimal model for Mottness?

Hubbard model
Fermi liquids

\[ H = \sum_{p, \sigma} (\epsilon(p) - \epsilon_F) n_{p\sigma} + \cdots \]

\((n_{p\uparrow}, n_{p\downarrow})\) conserved currents

\((c_{p\uparrow}, c_{p\downarrow}, \text{h.c.})\) 4 objects

\[ \text{SO}(4) \quad \text{proper rotations} \]

\[ \text{Det} M = 1 \]

\[ \text{SO}(4) \quad \text{improper rotations} \]

\[ \text{Det} M = -1 \]

\[ \text{Det} M = \pm 1 \implies Z_2 = O(4) \div SO(4) \]
\[ \epsilon(p) = \epsilon_F \]

Fermi Surface

\[ H = 0 \]

\[ \begin{align*}
  n_{p\uparrow} &\rightarrow -n_{p\uparrow} \\
  n_{p\downarrow} &\rightarrow n_{p\downarrow}
\end{align*} \]  \[ \mathbb{Z}_2 \]

at Fermi surface only
How to destroy Fermi liquids?

\[ H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F) n_{p\sigma} + U n_{p\uparrow} n_{p\downarrow} \]

- odd under $Z_2$
- scaling dimension $[n_{p\uparrow} n_{p\downarrow}] = -2$
- New fixed point!
- Hatsugai-Kohmoto model
- relevant interaction
- Hubbard not necessary!
Hatsugai-Kohmoto Model (1992)

\[
H_{\text{HK}} = -t \sum_{\langle j,l \rangle, \sigma} \left( c_{j\sigma}^\dagger c_{l\sigma} + h.c. \right) - \mu \sum_{j\sigma} c_{j\sigma}^\dagger c_{j\sigma}
\]

\[
c_{k\sigma} = \sum_{j} e^{ikj} c_{j\sigma}
\]

\[
H_{\text{HK}} = \sum_{k} H_{k} = \sum_{k} \left( \xi_{k}(n_{k\uparrow} + n_{k\downarrow}) + Un_{k\uparrow} n_{k\downarrow} \right).
\]

\[
\xi_{k} = \epsilon_{k} - \mu
\]
General HK Model

\[ \sum_k (\xi_k (n_{k\uparrow} + n_{k\downarrow}) + Un_{k\uparrow} n_{k'\downarrow}). \]

relevant perturbation

\[ [H_t, H_U] = 0 \]

Solvable Mott transition

\[ G_{k\sigma}(i\omega_n \rightarrow z) = \]

lower Hubbard band

upper Hubbard band

\( \neq \frac{1}{z - \xi_k} \)
**Hubbard band operators**

\[ \zeta_{k\sigma}^\dagger = c_{k\sigma}^\dagger (1 - n_{k\bar{\sigma}}) \]

\[ G_{\sigma}^R(k, \omega) = \frac{1}{\omega + i0^+ - (\xi_k + U/2) - \frac{(U/2)^2}{\omega + i0^+ - (\xi_k + U/2)}} \]

- Fermi liquid
- QP
- \( \Sigma(k, \omega) \)
  - \( \omega = \xi_k + U/2 \)
  - \( \Re \Sigma = \Im \Sigma = \infty \)
  - zeros
Mott transition: composite excitations

\[ \Delta E = U - 4dt = U - W \]
counting charges

$$n_{\text{Lutt}} = \langle n \rangle_{\text{Lutt}} = 2\theta(\Re G(k, \omega = 0))$$

zeros $\neq$ particles

$U < W, \langle n \rangle = 1$

$U < W, \langle n \rangle < 1$
Why NFL?

FL

HK

Mott insulator

Non-Fermi liquid

\[ d \]

\[ U/W \]

\[ x = 1 - \langle n \rangle \]

\[ c_{k\uparrow} c_{k\downarrow} |G\rangle \]

\[ \zeta_{k\uparrow} \zeta_{k\downarrow} |G\rangle = 0 \]
Fermi liquids

Hubbard not necessary (universality class)
what does the HK model leave out??

\[ [H_t, H_U] \neq 0 \]

dynamical spectral weight transfer
Fermi gas

Fermi liquid

BCS superconductor

Mottness
Superconductivity?
Cooper Instability

\[ H = H_{HK} - gH_p \]

\[ |\psi\rangle = \sum_{k \in \Omega_0} \alpha_k b_k^\dagger |\text{GS}\rangle \]

\[ \langle n_{k\sigma} \rangle = 0 \]

\[ E_b = \langle \text{GS}|H|\text{GS} \rangle - \langle \psi|H|\psi \rangle \leq 0 \]

\[ 1 = -\frac{g}{L^d} \sum_{k \in \Omega_0} \frac{\langle 1 - n_{k\uparrow} - n_{-k\downarrow} \rangle}{E - 2\xi_k - U\langle n_{k\downarrow} + n_{-k\uparrow} \rangle} \]

\[ 1 = -g \int_{\mu}^{W/2} d\epsilon \frac{\rho(\epsilon)}{E - 2\epsilon + 2\mu}, \]
\[ E_b = -E \sim W \left(1 - \left(\frac{U}{W}\right)^2\right) e^{-\pi W \sqrt{1 - \left(\frac{U}{W}\right)^2}/g} \]
Pair Susceptibility

\[ \chi(i\nu_n) \equiv \frac{1}{L^d} \int_0^\beta d\tau e^{i\nu_n \tau} \langle T \Delta(\tau) \Delta^\dagger \rangle_g \]

\[ = \frac{\chi_0}{1 - g\chi_0} \]

\[ g\chi_0 = 1 \]

solve for \( T_c \)
\[ T_c = (W - U)^{4/5} \left( 1/5 \right) \frac{e^{\gamma}}{\pi} e^{-\frac{4}{5} \frac{W}{g}}. \]
variational wave function

$$|\psi_{BCS}\rangle = \prod_k (u_k + v_k b_k^\dagger |0\rangle)$$

$$|\psi_{BCS}\rangle = \prod_{k>0} (u_k^2 + v_k^2 b_k^\dagger b_{-k}^\dagger + u_k v_k (b_k^\dagger + b_{-k}^\dagger)) |0\rangle$$
\[ |\psi\rangle = \prod_{k > 0} \left( x_k + y_k b_k^\dagger b_{-k}^\dagger + \frac{z_k}{\sqrt{2}} (b_k^\dagger + b_{-k}^\dagger) \right) |0\rangle \]
three variational parameters

\[ |x_k|^2 + |y_k|^2 + |z_k|^2 = 1 \]

gap equation

\[ 1 = \frac{g}{W} \sinh^{-1}\left(\frac{W - U}{2\Delta}\right) + \frac{g}{W} \sinh^{-1}\left(\frac{U}{2\Delta}\right) \]

\[ \Delta \ll U, W \]

\[ \Delta = (W - U)^{1/2} U^{1/2} e^{-\frac{W}{2g}} \]
\[
\Delta = (W - U)^{1/2}U^{1/2}e^{-\frac{W}{2g}}
\]

\[
T_c = (W - U)^{4/5}U^{1/5}\frac{e^\gamma}{\pi}e^{-\frac{4}{5}\frac{W}{g}}.
\]

\[
\lim_{g \to 0} \frac{\Delta}{T_c} \to \infty
\]

non-BCS superconductivity
excited states

\[ T_c \]

\( (e, h) \neq 0 \)

\[ n_p \approx e^{-\frac{\varepsilon_p}{k_BT}} \]
Bogoliubov excitations

\[ \gamma_{k\sigma} |\psi_{BCS}\rangle = 0 \]

\[ \gamma_{k\sigma} = u_{k\sigma} - \sigma V_{k\sigma} \zeta_{-k\bar{\sigma}}^\dagger \]

PYHons excitations

\[ \gamma^l_{k\sigma} \propto \sqrt{2} x_k \zeta_{k\sigma}^\dagger - \sigma z_k \zeta_{-k\bar{\sigma}} \]

\[ \gamma^u_{k\sigma} \propto z_k \eta_{k\sigma}^\dagger - \sigma \sqrt{2} y_k \eta_{-k\bar{\sigma}} \]
Excitation spectrum

\[ \gamma_{k\sigma}^{u/l} |\psi\rangle = 0 \]

\[ \langle \psi| \gamma_{k\sigma}^{u/l} H \gamma_{k\sigma}^{u/l} \rangle^\dagger |\psi\rangle = \langle \psi| H |\psi\rangle + E_{k}^{u/l} \]

\[ E_{k}^{u/l} = \sqrt{\xi_{k}^{u/l} + \Delta^2} \]

superconductivity affects both bands!
PYHon band
can we explain the color change?

REPORT

Superconductivity-Induced Transfer of In-Plane Spectral Weight in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$

H. J. A. Molegraaf$^1$, C. Presura$^1$, D. van der Marel$^{1,+}$, P. H. Kes$^2$, M. Li$^2$

* See all authors and affiliations

Science 22 Mar 2002:
Vol. 295, Issue 5563, pp. 2239-2241
DOI: 10.1126/science.1069947

\[ A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000\text{cm}^{-1} \]

\[ A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000\text{cm}^{-1} \]

\[ \frac{\Delta A_l}{A_l} \propto 3\% \]
Optical data are reported on a spectral weight transfer over a broad frequency range of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$, when this material became superconducting. Using spectroscopic ellipsometry, we observed the removal of a small amount of spectral weight in a broad frequency band from $10^4$ cm$^{-1}$ to at least $2 \times 10^4$ cm$^{-1}$, due to the onset of superconductivity. We observed a blue shift of the $ab$-plane plasma frequency when the material became superconducting, indicating that the spectral weight was transferred to the infrared range. Our observations are in agreement with models in which superconductivity is accompanied by an increased charge carrier spectral weight. The measured spectral weight transfer is large enough to account for the condensation energy in these compounds.
condensation energy: HK model

\[ W_L \]
why?

$$H = H_{HK} + H_p$$

$$[H_{HK}, H_p] \neq 0$$

dynamical spectral weight transfer
is this the general mechanism of the color change?
Superfluid Density
Mottness-induced suppression

\[ D_s \]

\[ U/t = 0 \]

- FL
- doped MI

\[ g = 1.5 \]
\[ g = 2.0 \]
\[ g = 3.0 \]
Mottness

$Z_2$

non-BCS superconductivity

violation of Luttinger

HM

HK

PYHons

non-BCS superconductivity