

Superconductivity and Mottness: Exact Results

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Luke Yeo



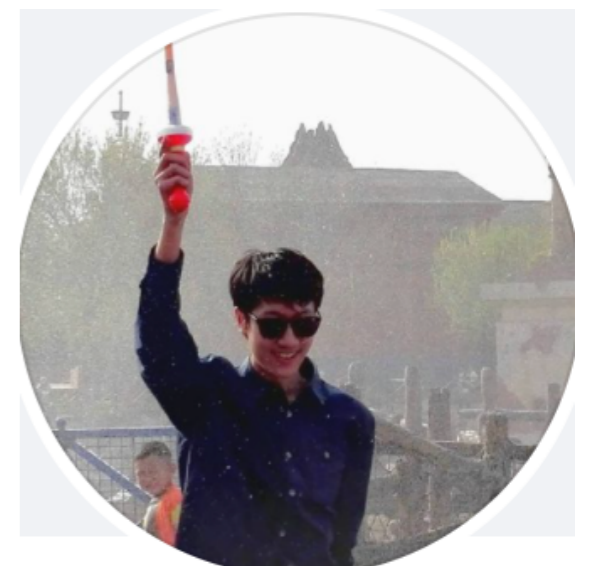
Edwin Huang



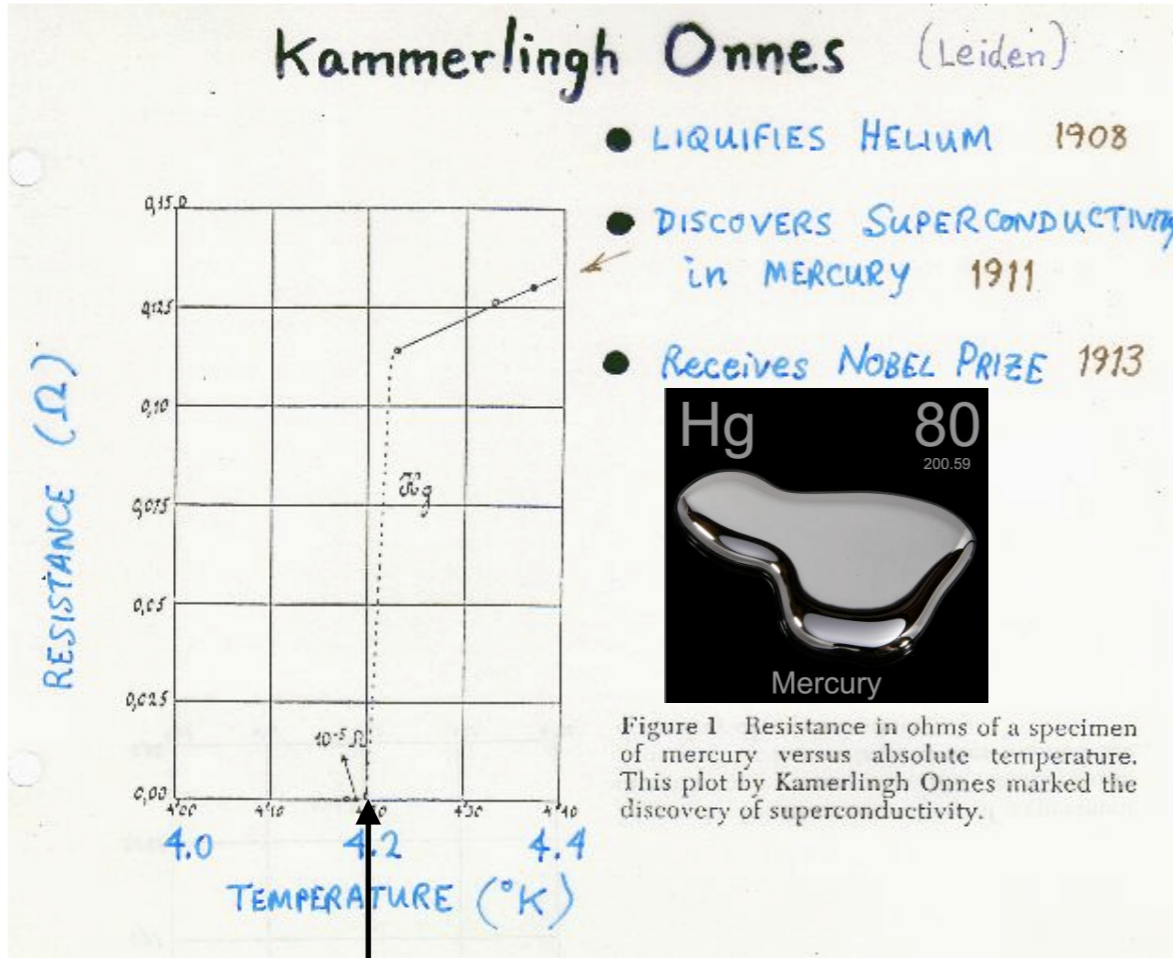
G. La Nave



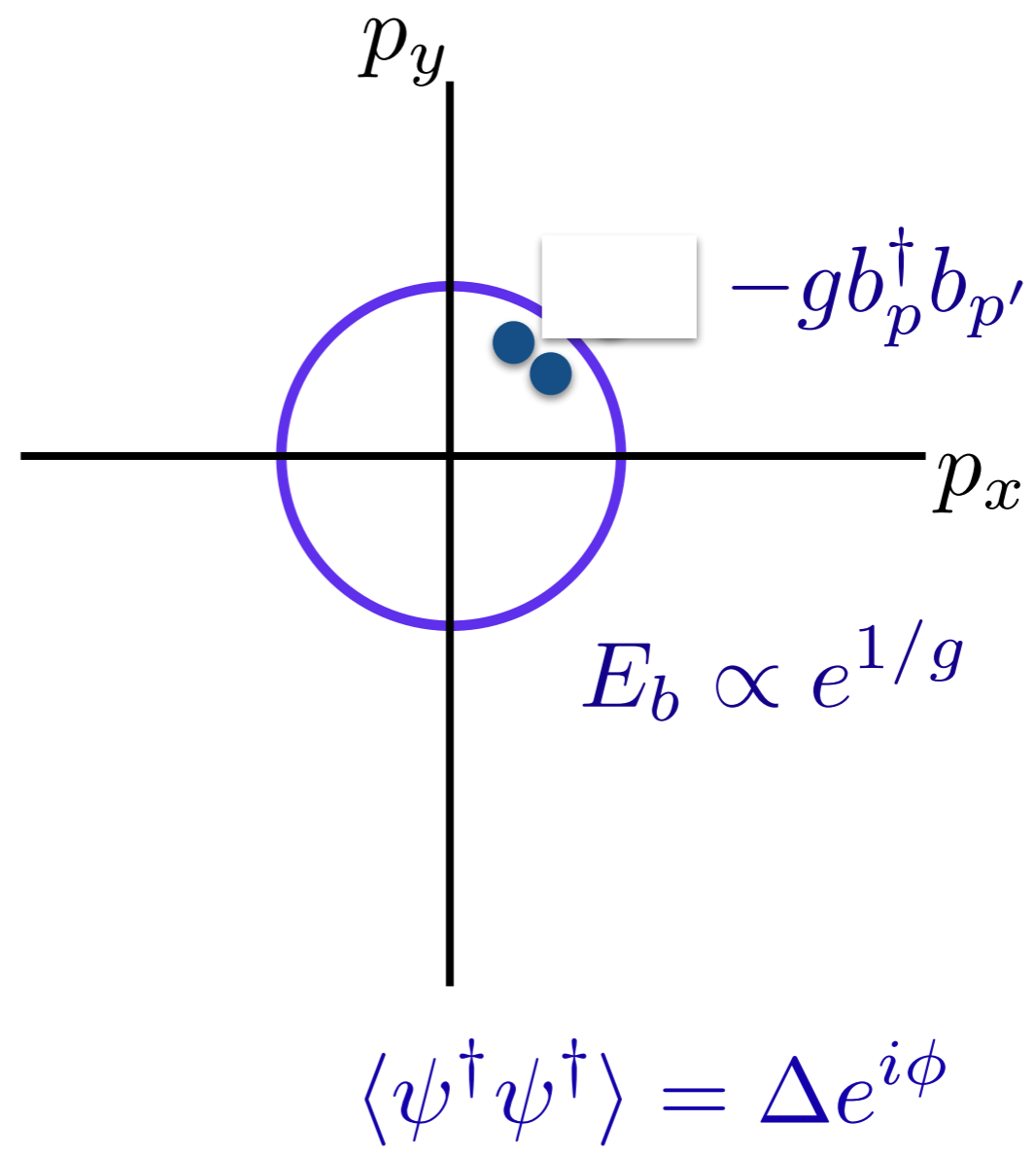
Jinchao Z.



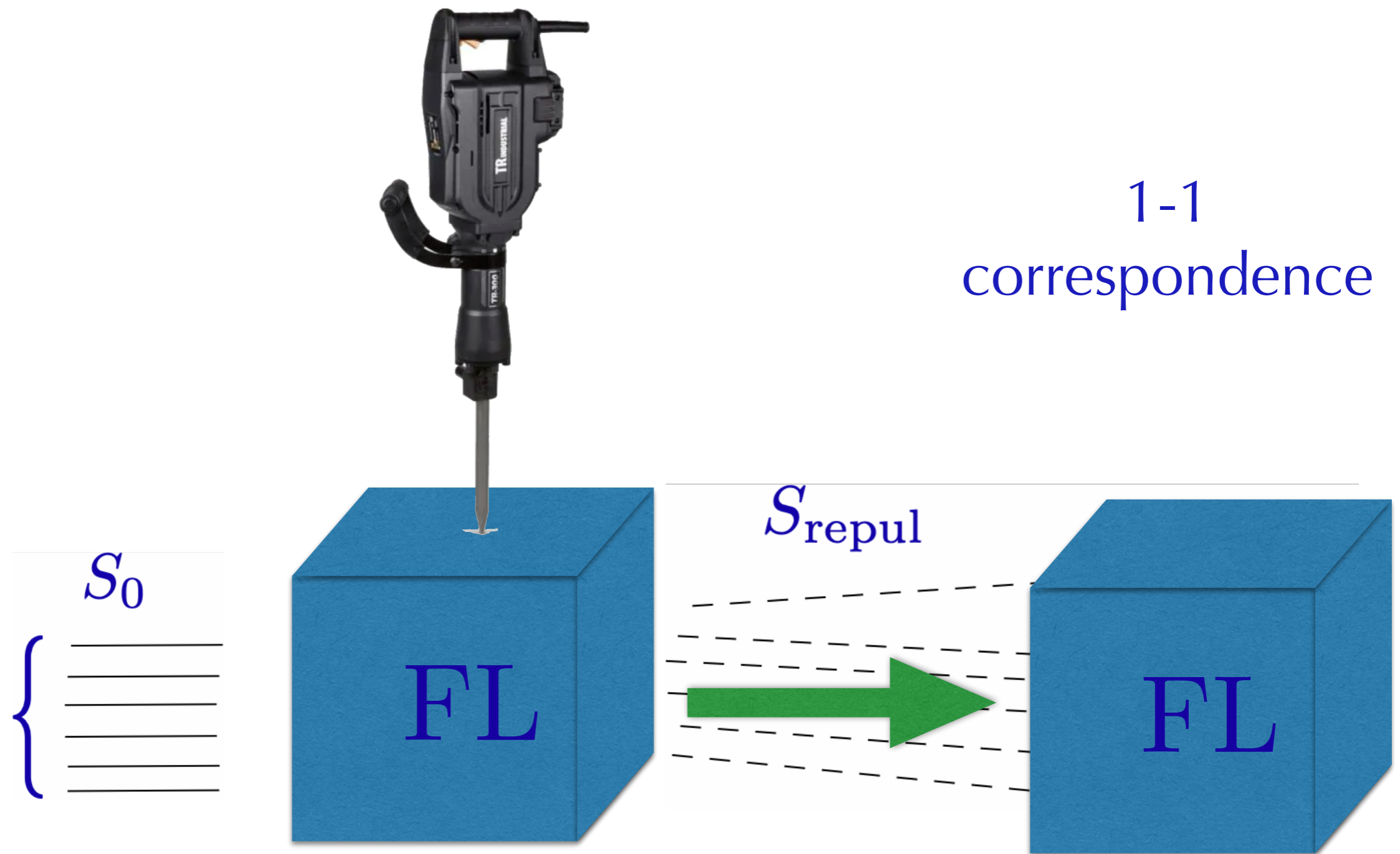
Cooper instability



T_c



1-1
correspondence



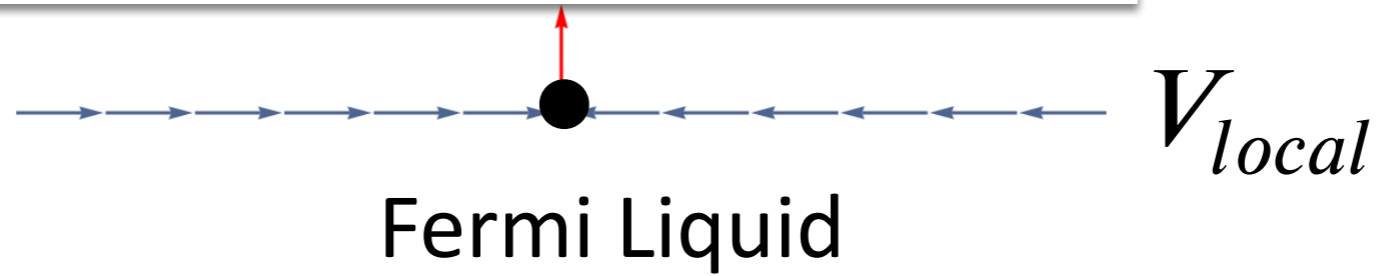
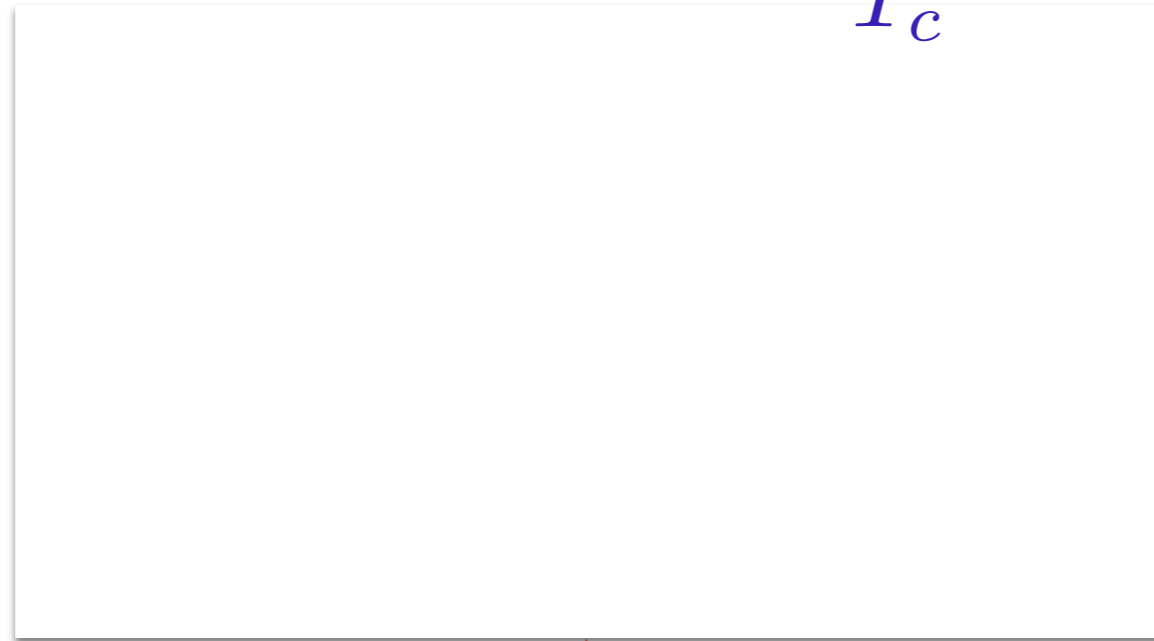
$$[S_{\text{repul}}] > 0$$

irrelevant

$$\left(\frac{1}{L}\right)^\alpha, \quad \alpha > 0$$

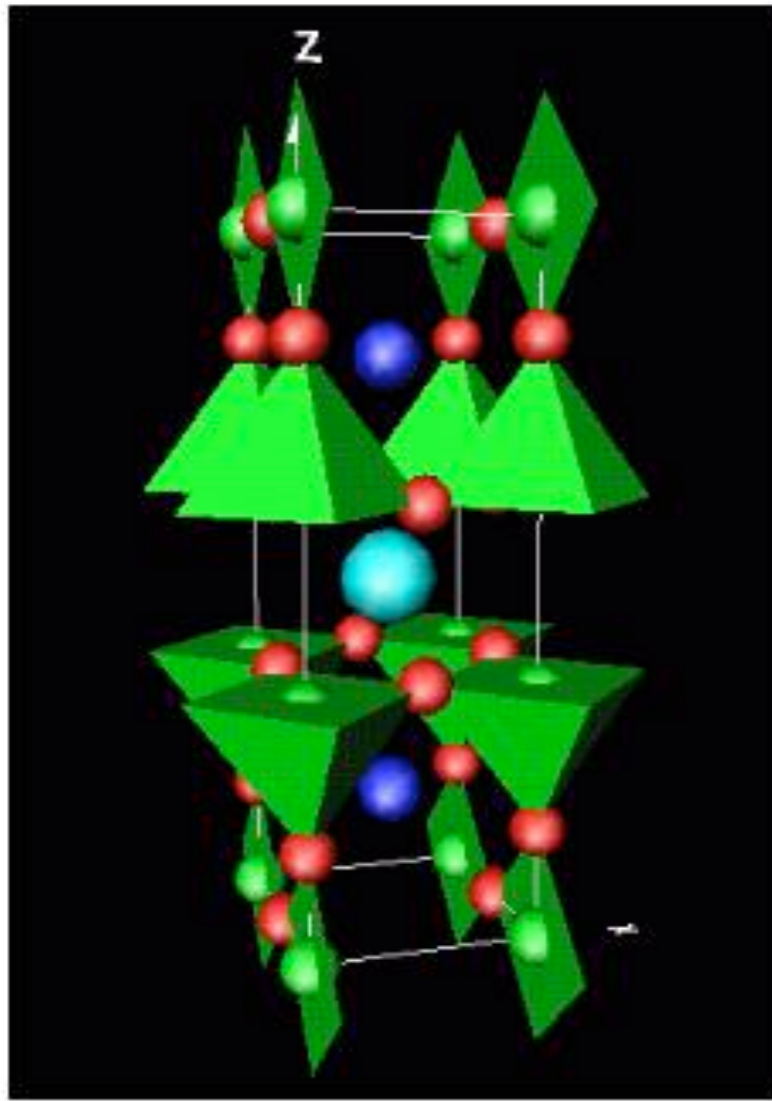
BCS Superconductor

$$\frac{2\Delta}{T_c} = 3.5$$



fixed
point beyond
FL?

quartic
interacting
theory?

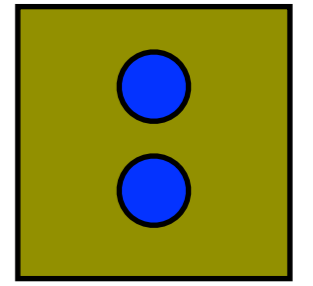


YBa₂Cu₃O₇
Cuprate Superconductors

NiO insulates

d^8 ?

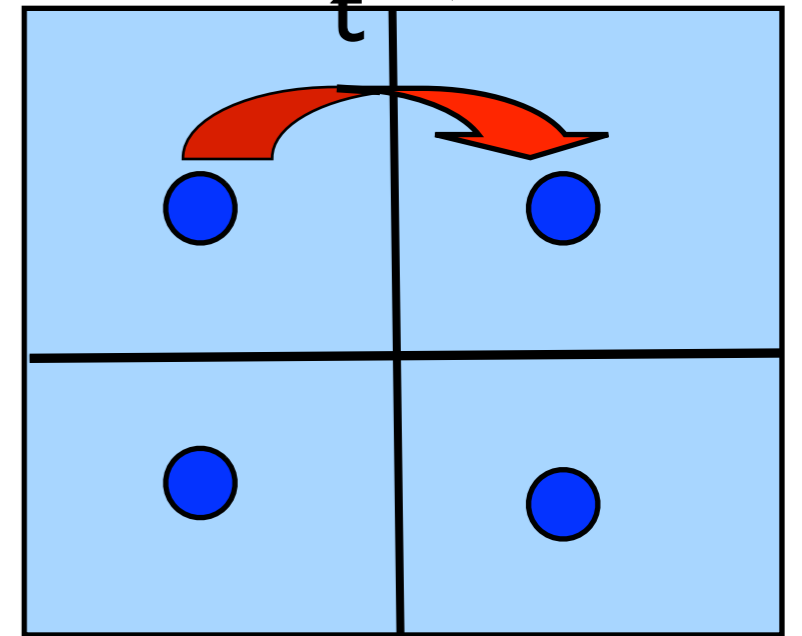
perhaps this costs energy



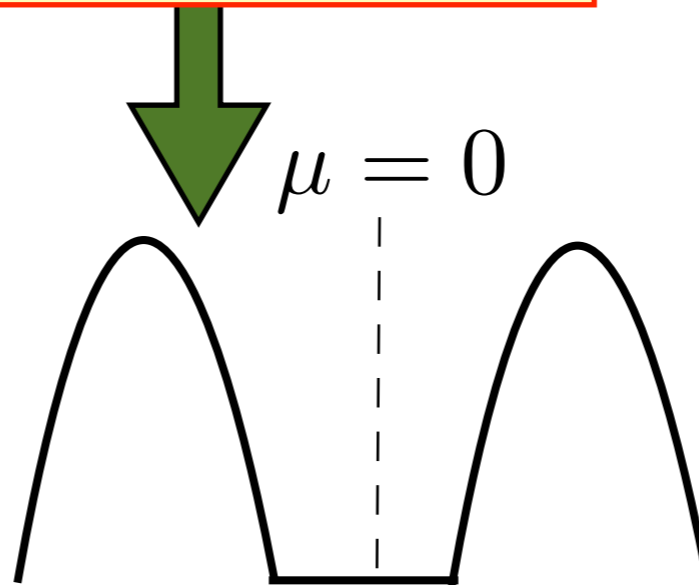
$$U \gg t$$



local real-space physics



no change in size of Brillouin zone



solve the Hubbard Model!!

Cooper instability??

Progress thus far?

DMFT

QMC

disputes

Sept. 1997

A Critique of Two Metals

R. B. Laughlin
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idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconductor, except that they coexist and mix with soft magnetic excitations no one knows how to describe very well.

Nov. 1997

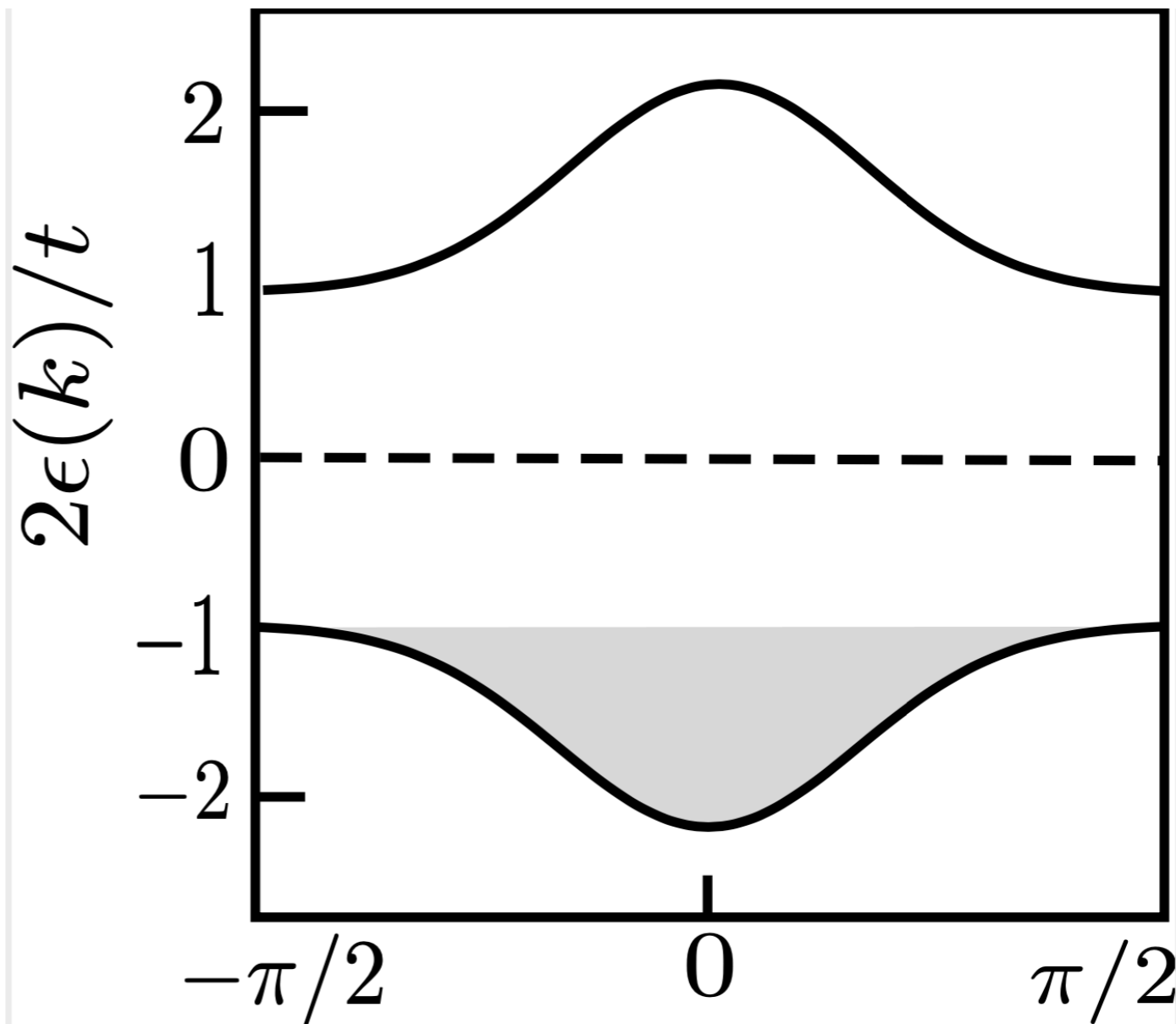
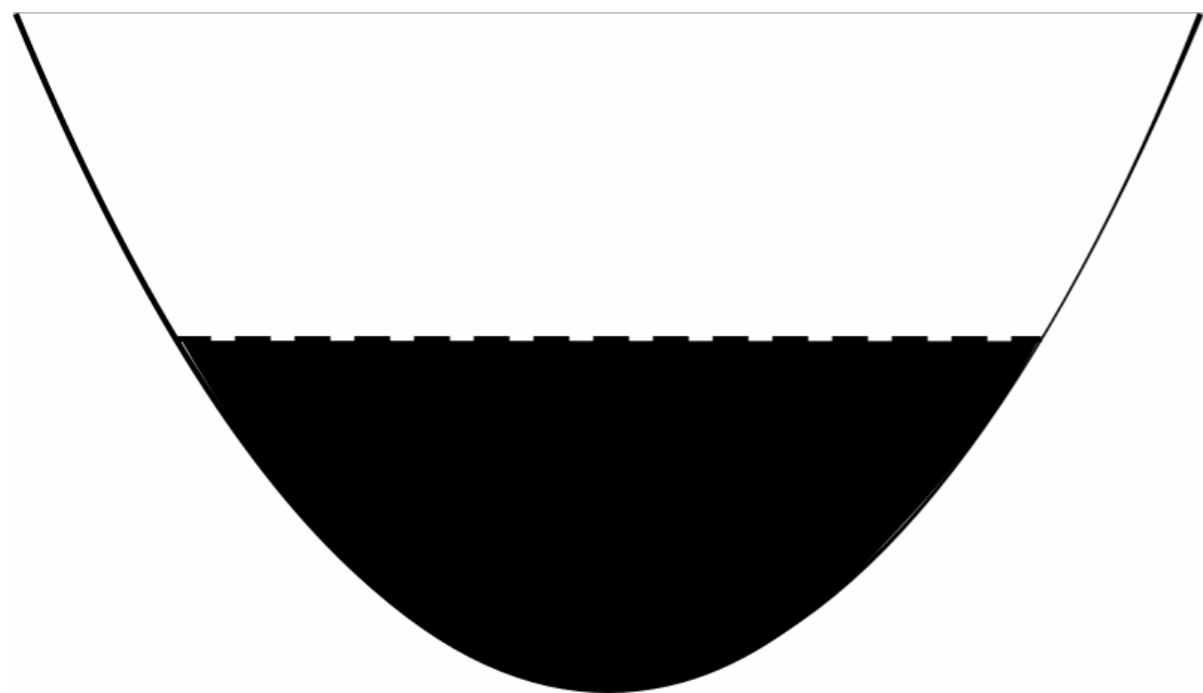
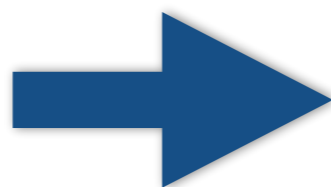
A Critique of “A Critique of Two Metals”

Philip W. Anderson and G. Baskaran

Joseph Henry Laboratories of Physics

Princeton University, Princeton, NJ 08544

The fundamental argument is presented in the second paragraph: “Ten years of work by some of the best minds in theoretical physics have failed to produce any formal demonstration”...of the Mott insulating state. The statement would be ludicrous if it were not so influential. The proviso “at zero temperature” is added, because of course most Mott concern. It is the tragedy of Mott that although he almost certainly won his Nobel prize for the Mott insulator, Slater, who couldn’t think clearly about finite temperature, won the publicity battle.



No Mott Problem



—

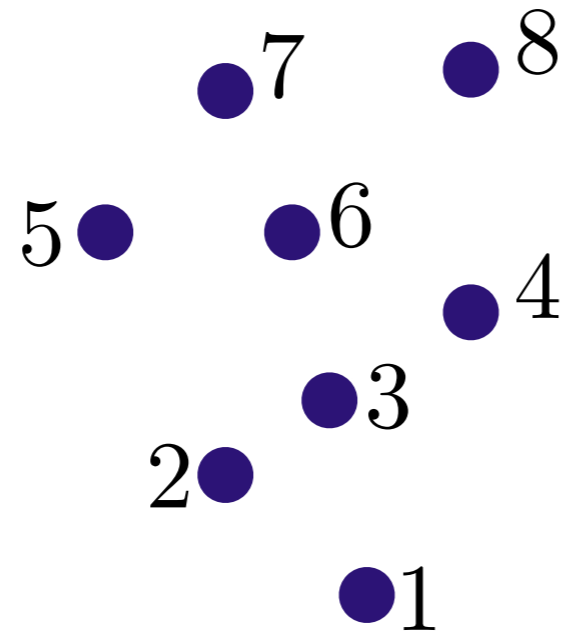


= Mottness

zeros

$= 0$ $\neq 0$
DetReG($\omega = 0, p$)

counting particles



is there a more efficient way?

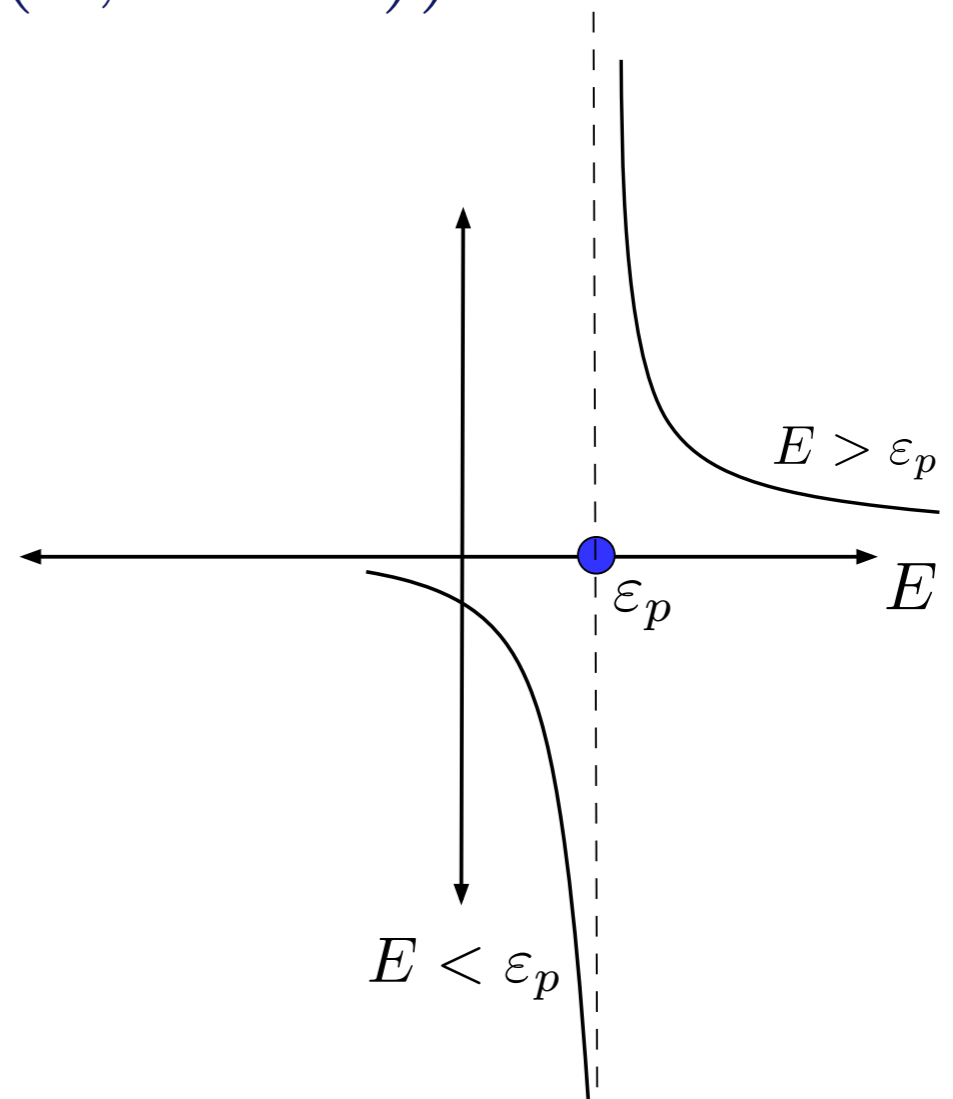
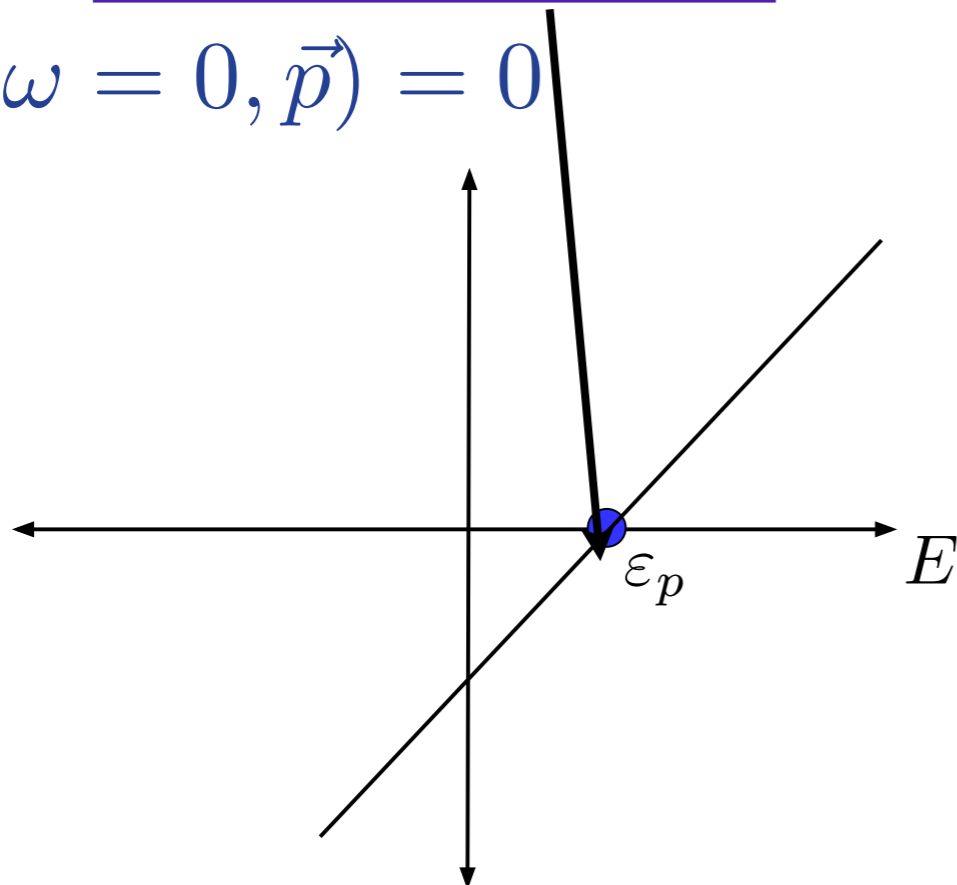
Luttinger counting theorem

$$G(E) = \frac{1}{E - \varepsilon_p}$$

$$n = 2 \sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

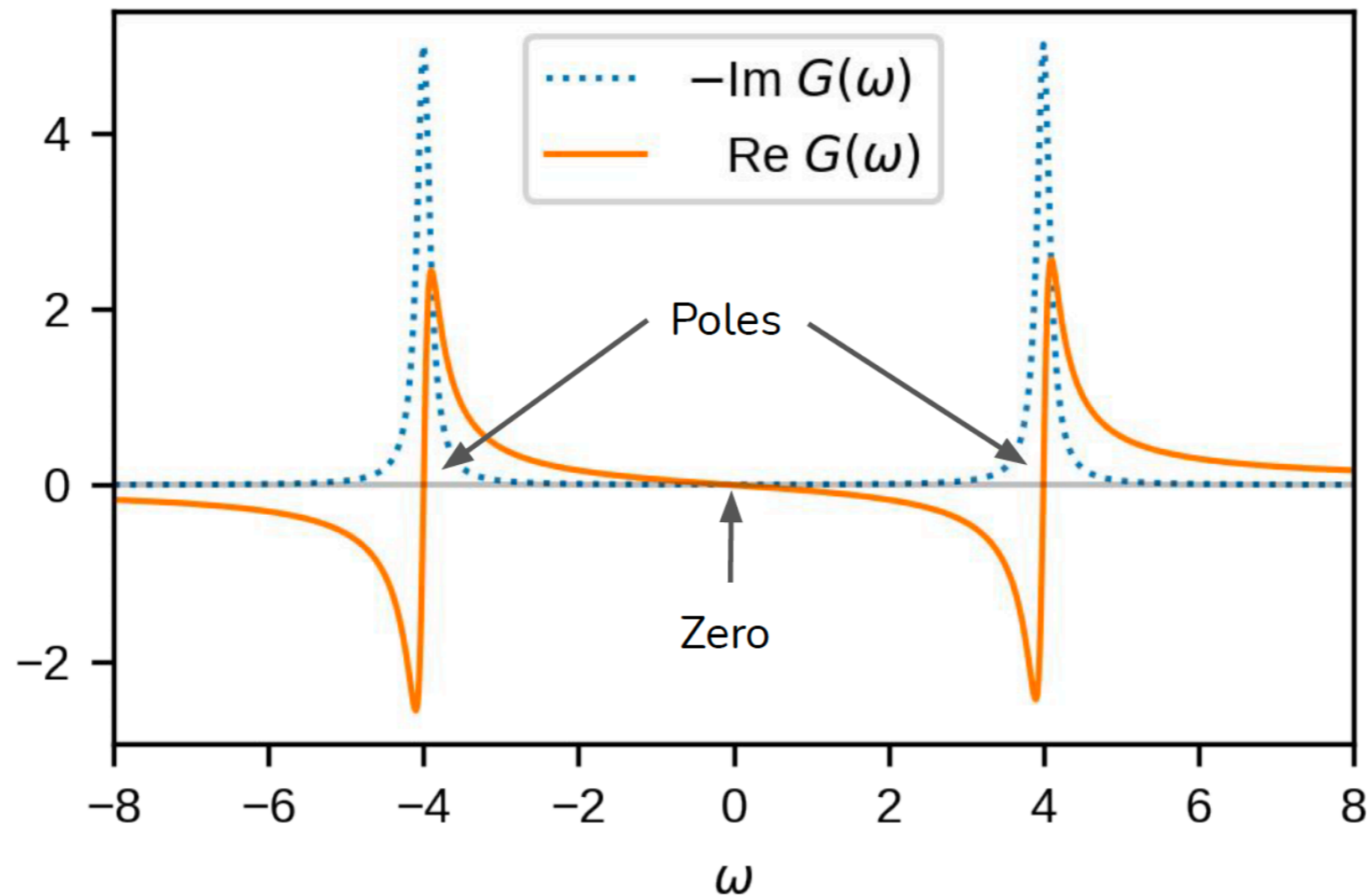
zero-crossing

$$\text{Det}G(\omega = 0, \vec{p}) = 0$$



counting poles (qp)

How do zeros obtain?



= below gap+above gap = 0

$\text{DetRe}G(k, \omega = 0) = 0$ (single band)

strongly correlated gapped systems

zeros

no propagation

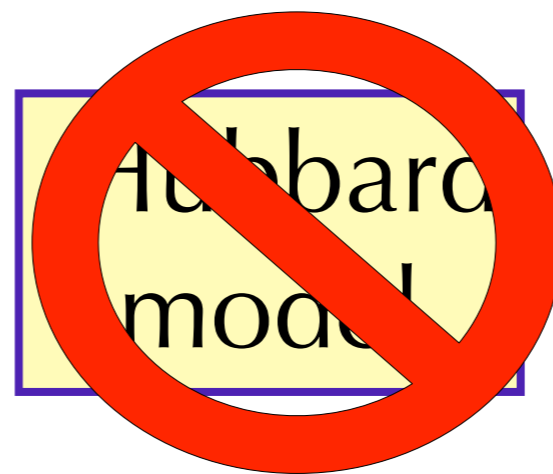


breakdown
of particle
concept

Mottness

$n = \text{zeros} + \text{poles}?$

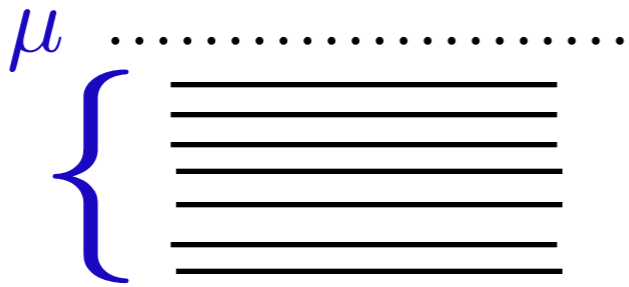
Minimal model
for Mottness?



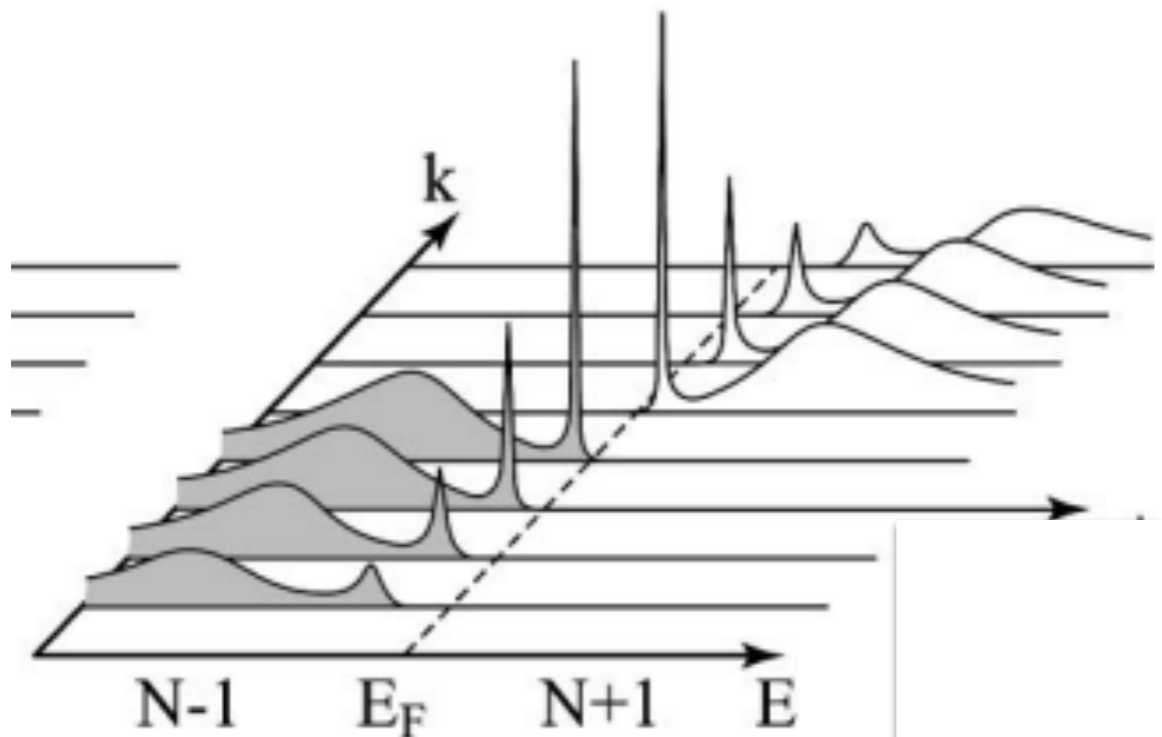
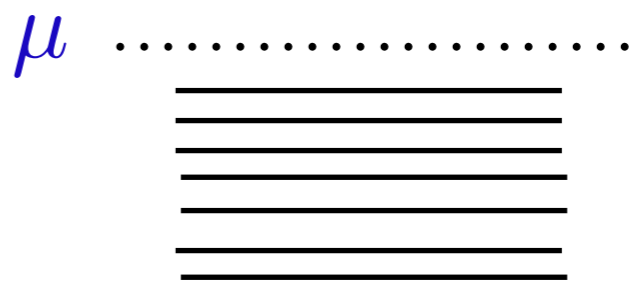
Fermi liquids

NFL

doubly occupied



Is single occupancy below chemical potential possible?

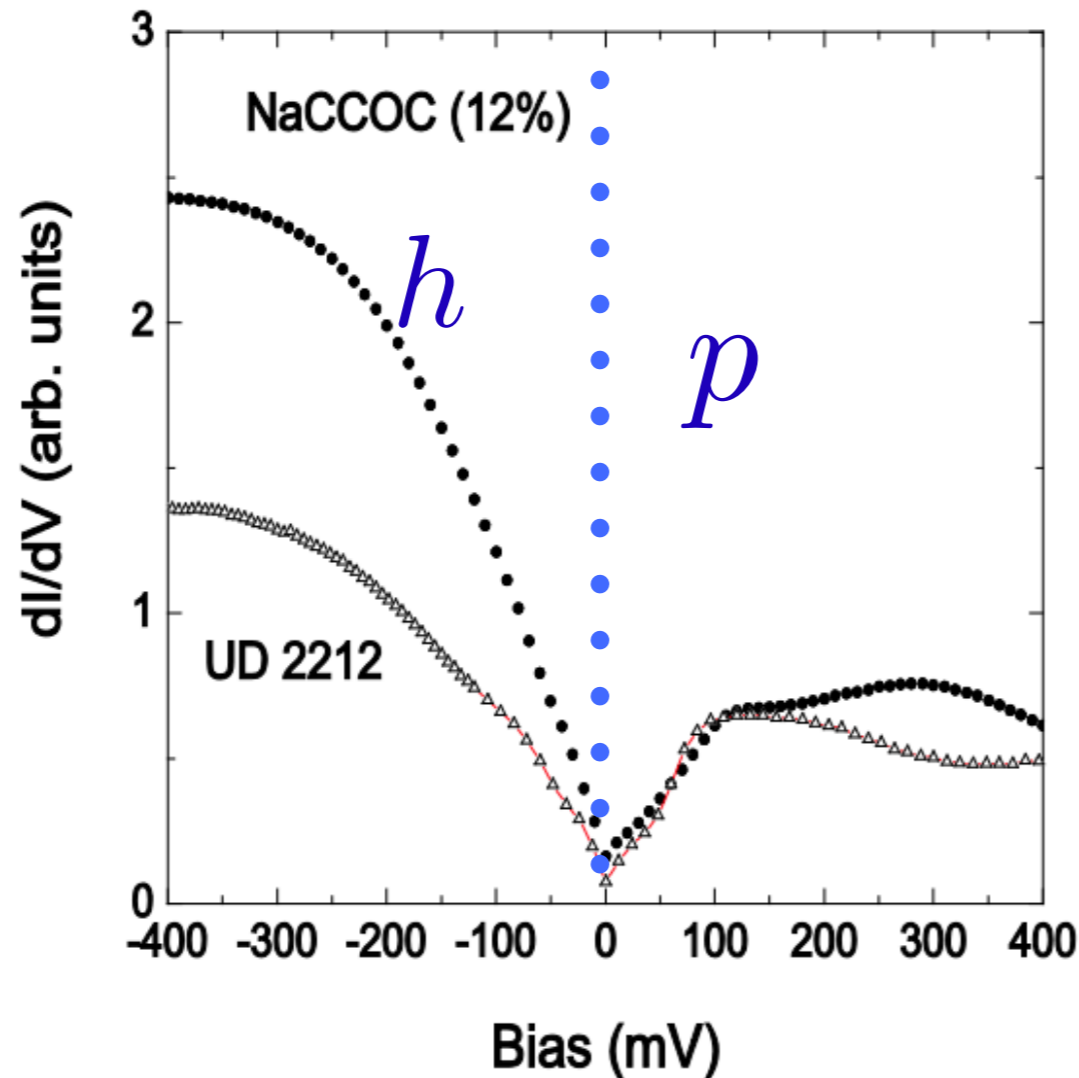


Fermi-liquid system

p-h symmetry

with time-reversal symmetry intact?

p-h asymmetry



LAST WORDS ON THE CUPRATES

P W Anderson, Princeton University

theory. I remain baffled by the almost universal refusal of theorists to confront this evident fact of hole-particle asymmetry head on. Its meaning is that the first step of any

single
occupancy

?

particle-hole
asymmetry



Anderson
Haldane
2000

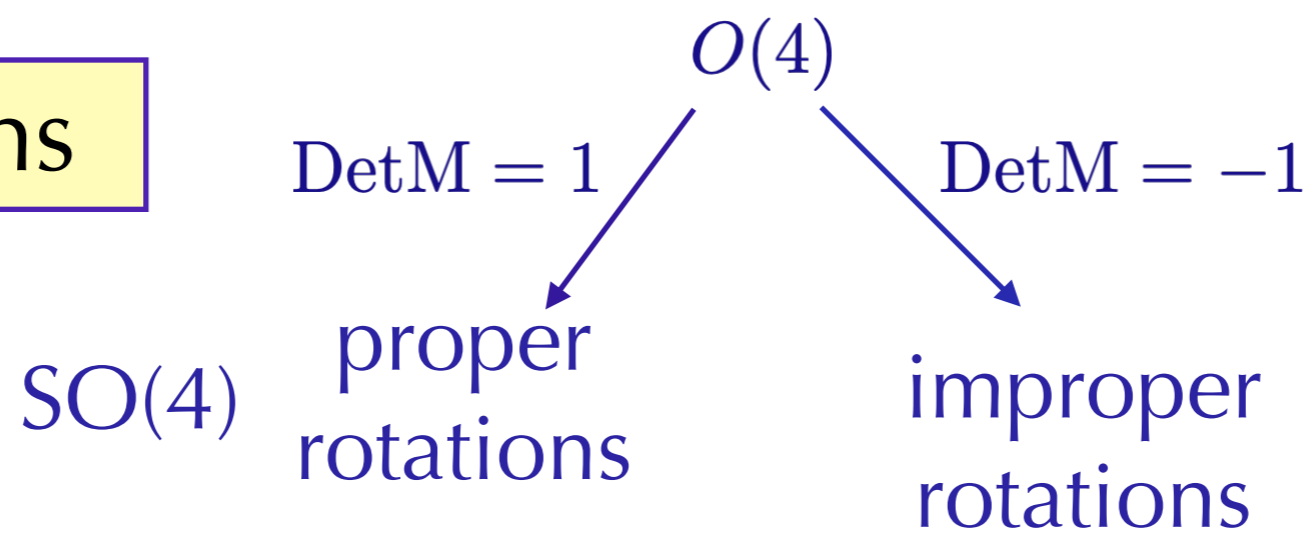
3 citations

Fermi liquids

$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F) n_{p\sigma} + \dots \rightarrow 0$$

$(n_{p\uparrow}, n_{p\downarrow})$ conserved currents

$(c_{p\uparrow}, c_{p\downarrow}, \text{h.c.})$ 4 objects



$$\text{Det}M = \pm 1 \implies Z_2 = O(4) \div SO(4)$$

Improper Rotations

Majorana basis

$$\begin{pmatrix} c_{p\uparrow} \\ c_{p\uparrow}^\dagger \\ c_{p\downarrow} \\ c_{p\downarrow}^\dagger \end{pmatrix} \longrightarrow \begin{pmatrix} c_{p\uparrow} + c_{p\uparrow}^\dagger \\ i(c_{p\uparrow} - c_{p\uparrow}^\dagger) \\ c_{p\downarrow} + c_{p\downarrow}^\dagger \\ i(c_{p\downarrow} - c_{p\downarrow}^\dagger) \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} c_{p\uparrow} + c_{p\uparrow}^\dagger \\ i(c_{p\uparrow} - c_{p\uparrow}^\dagger) \\ c_{p\downarrow} + c_{p\downarrow}^\dagger \\ i(c_{p\downarrow} - c_{p\downarrow}^\dagger) \end{pmatrix} \longrightarrow c_{p\downarrow} \rightarrow c_{p\downarrow}^\dagger$$

p-h transformation

$$\epsilon(p) = \epsilon_F$$

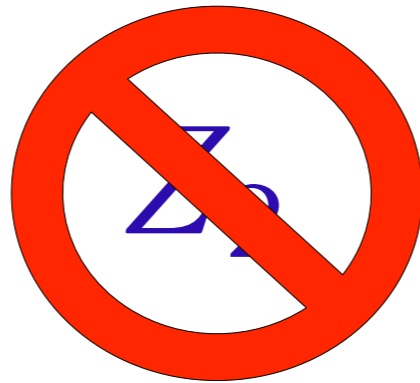
Fermi
Surface

$$H = 0$$



$$\left. \begin{array}{l} n_{p\uparrow} \rightarrow 1 - n_{p\uparrow} \\ n_{p\downarrow} \rightarrow n_{p\downarrow} \end{array} \right\} \mathbb{Z}_2 \text{ at Fermi surface only}$$

How to destroy Fermi liquids?



$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F) n_{p\sigma} + U n_{p\uparrow} n_{p\downarrow}$$

odd
under Z_2

scaling dimension

$$[n_{p\uparrow} n_{p\downarrow}] = -2$$

relevant
interaction

New fixed point!

Hatsugai-Kohmoto or
Baskaran model

Hubbard
not
necessary!

General HK Model

$$\sum_k (\xi_k (n_{k\uparrow} + n_{k\downarrow}) + U n_{k\uparrow} n_{k\downarrow})$$

Solvable Mott transition: $U > W$

$$G_{k\sigma}(i\omega_n \rightarrow z) = \frac{1 - \langle n_{k\bar{\sigma}} \rangle}{z - \xi_k} + \frac{\langle n_{k\bar{\sigma}} \rangle}{z - (\xi_k + U)} \neq \frac{1}{z - \omega_k}$$

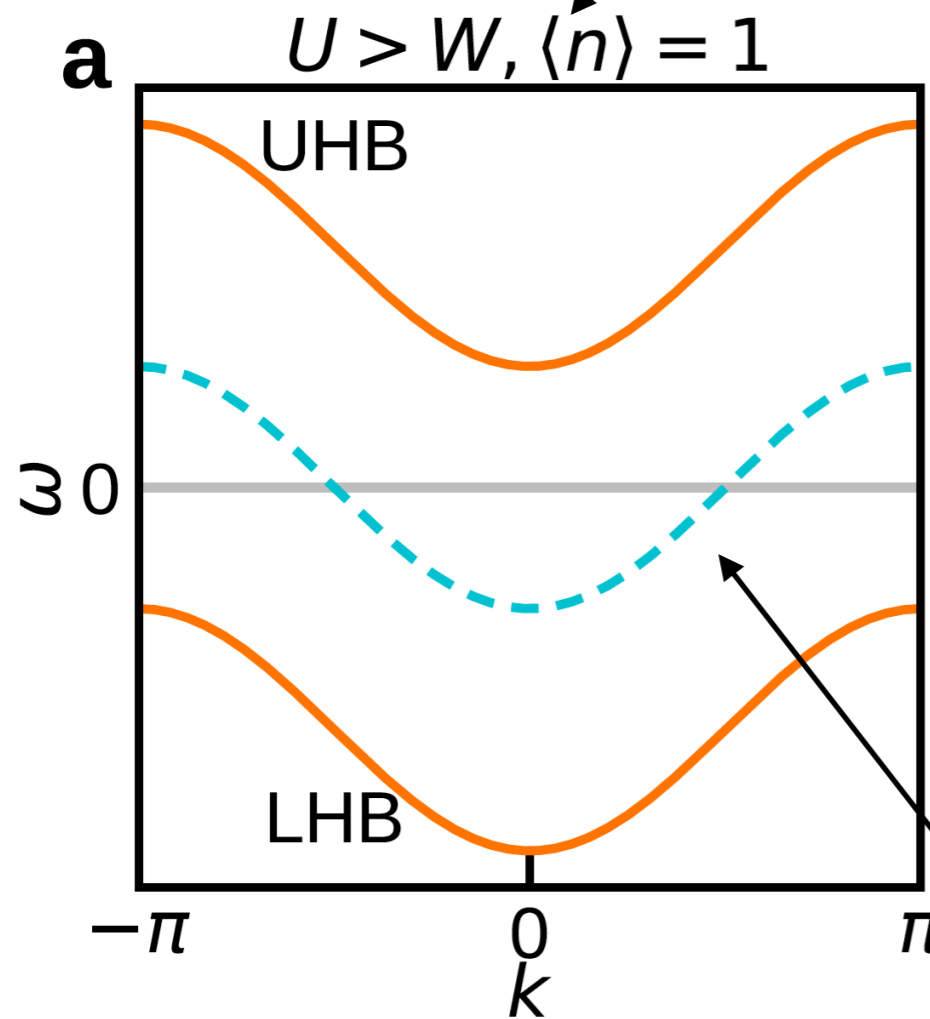
lower Hubbard band

upper Hubbard band

zeros

Mott transition: composite excitations

$$\Delta E = U - 4dt = U - W$$

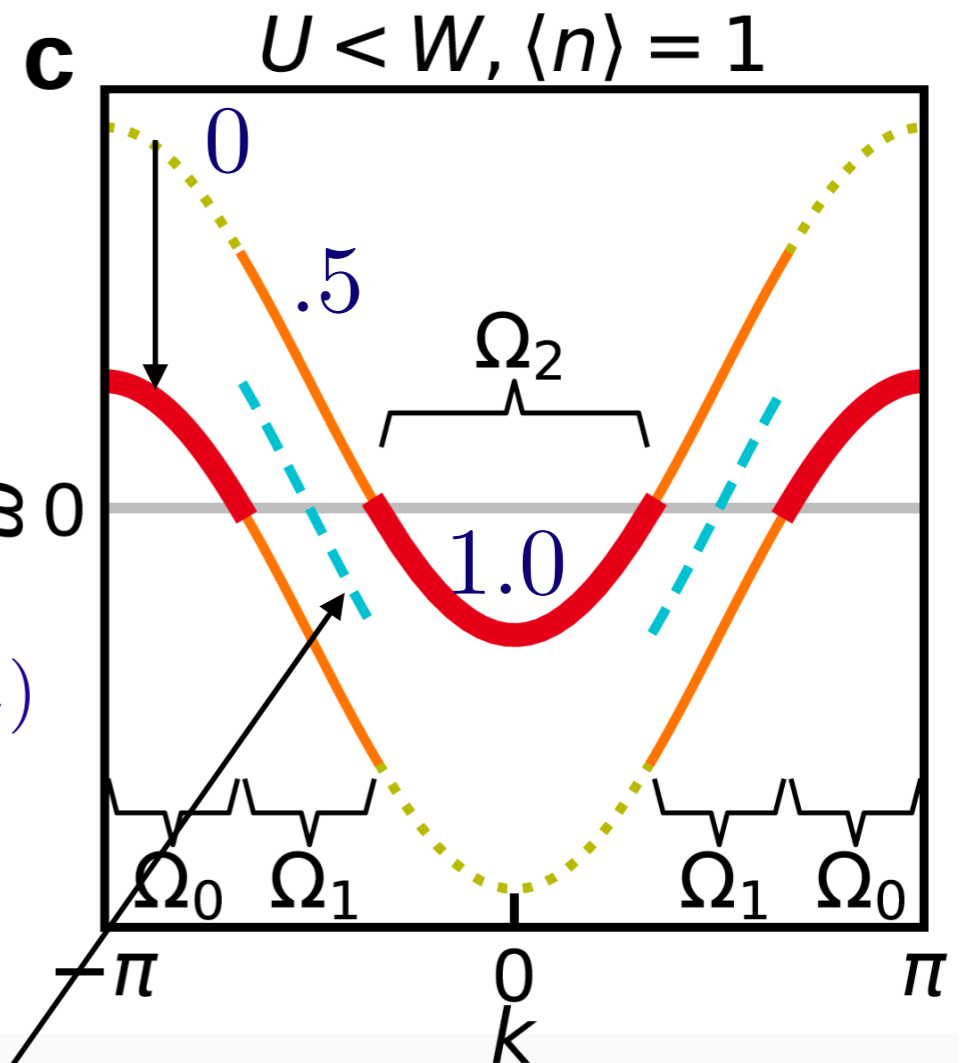


insulator

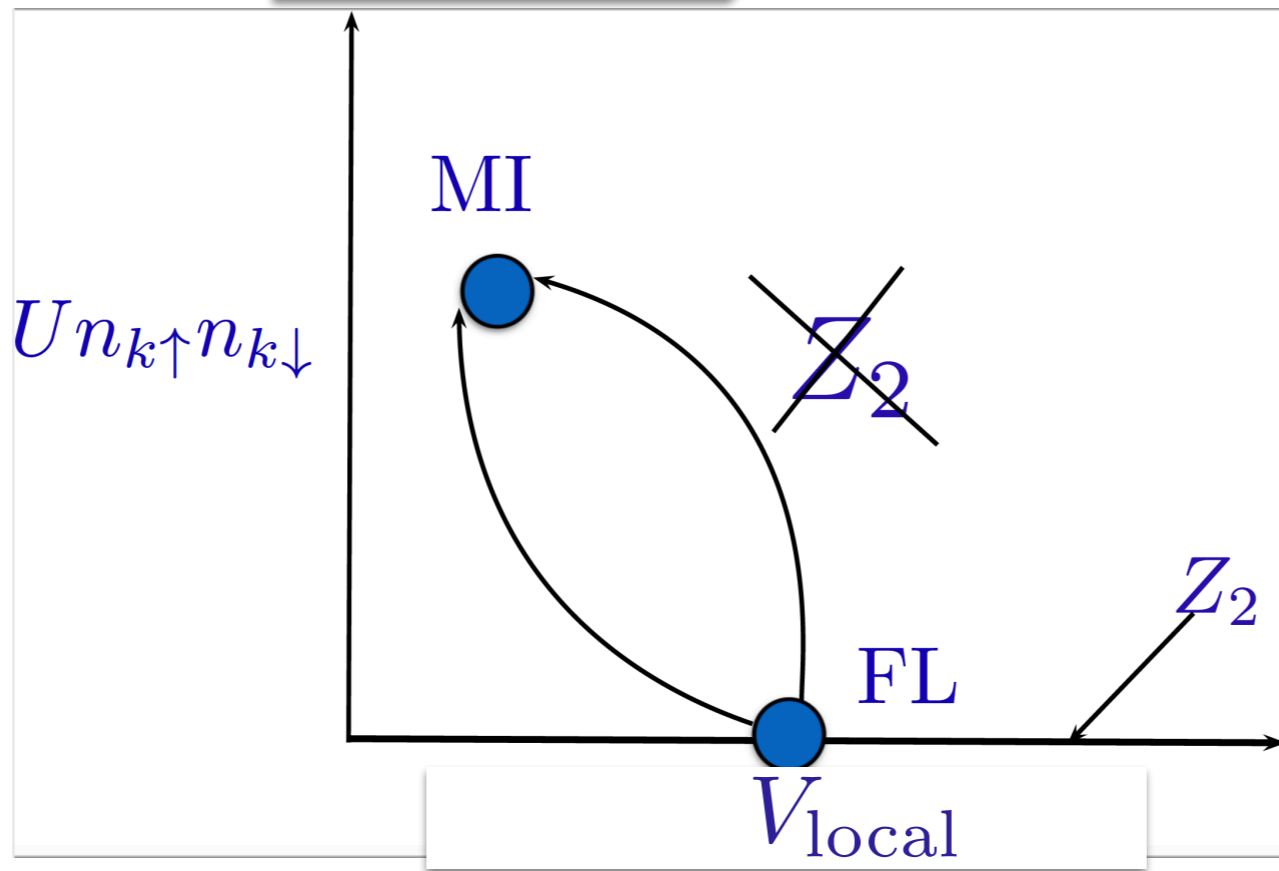
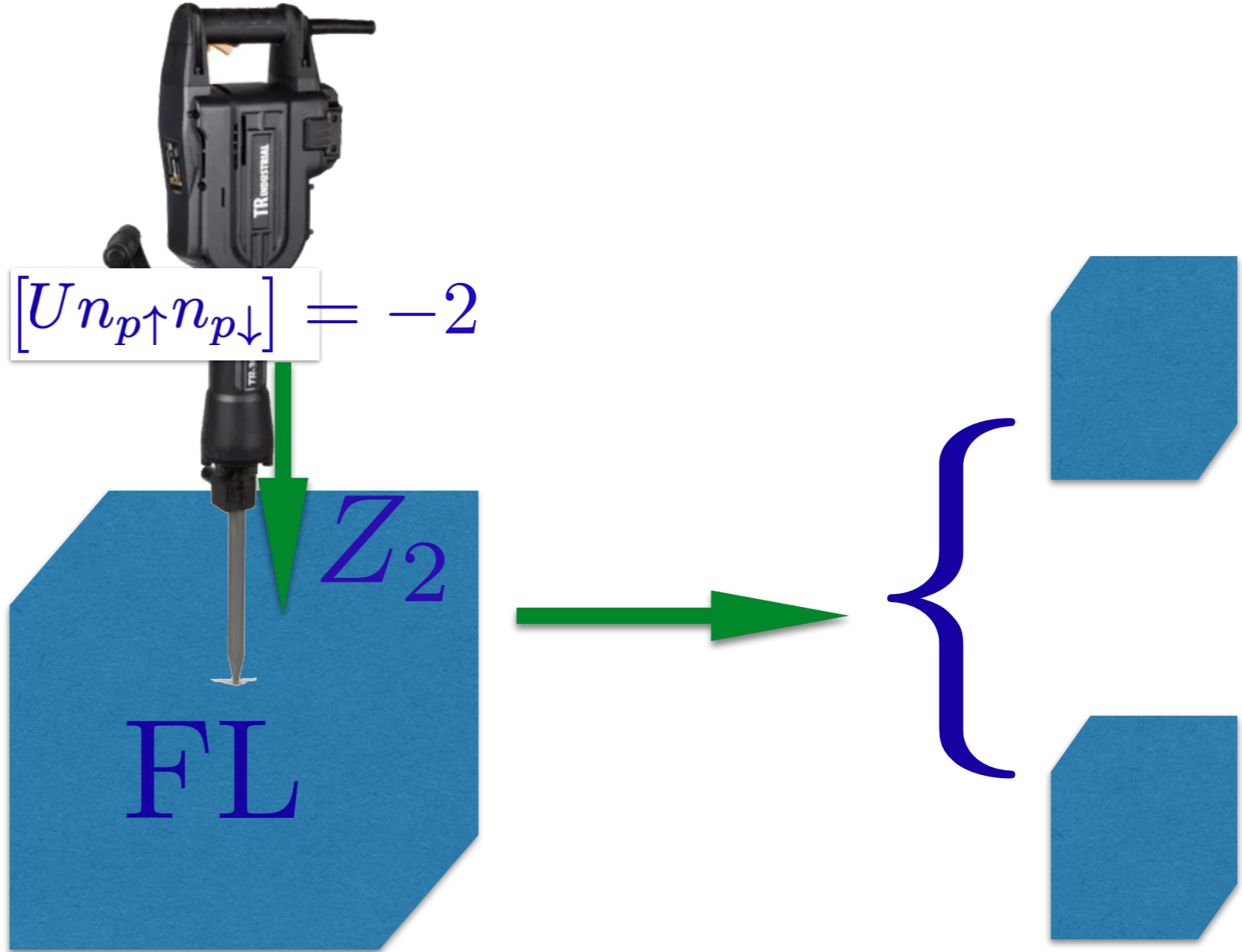
$$\eta_{k\sigma}^\dagger = c_{k\sigma}^\dagger n_{k\bar{\sigma}}$$

$$\zeta_{k\sigma}^\dagger = c_{k\sigma}^\dagger (1 - n_{k\bar{\sigma}})$$

Luttinger Surface



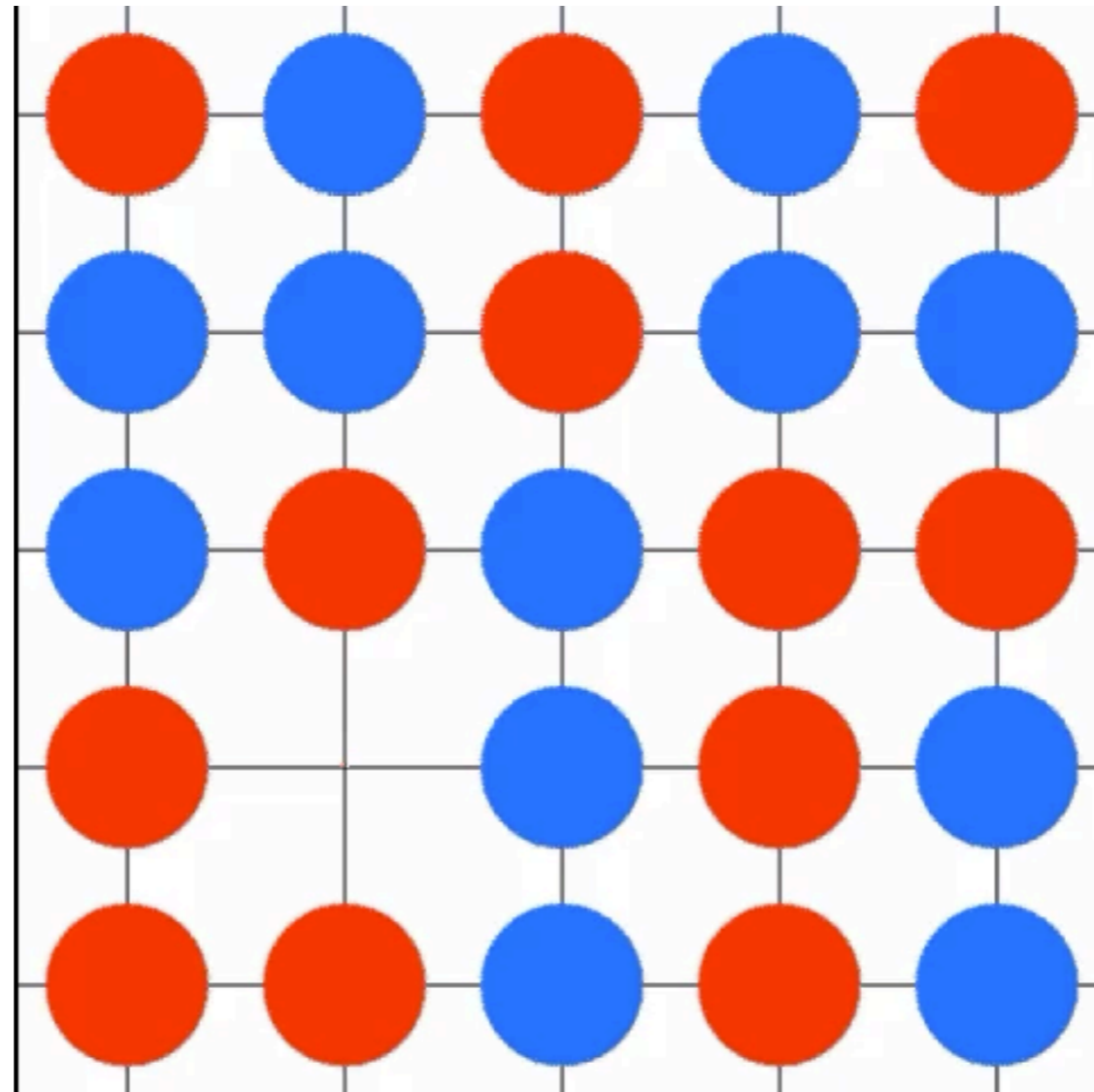
metal



Hubbard
not
necessary
(universality
class)

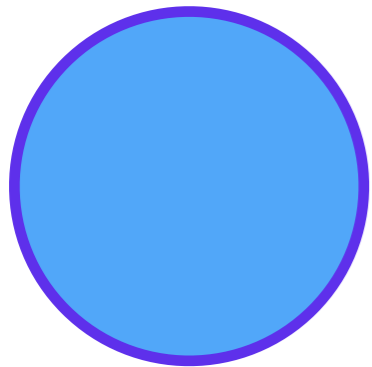
what does the HK model leave out??

$$[H_t, H_U] \neq 0$$

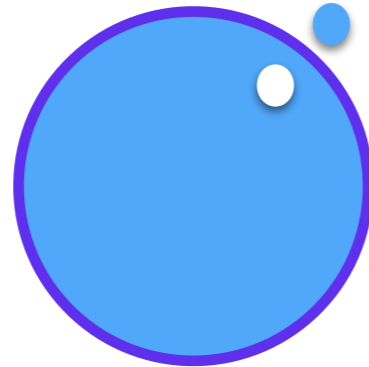


dynamical spectral weight transfer

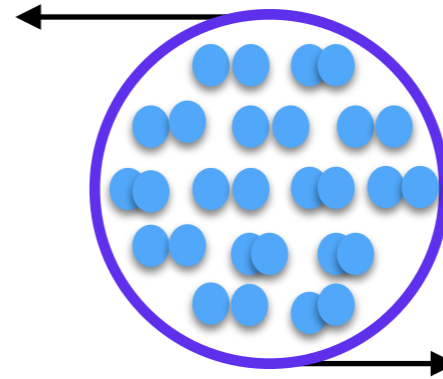
Fermi gas



Fermi liquid



BCS
superconductor



Mottness

2

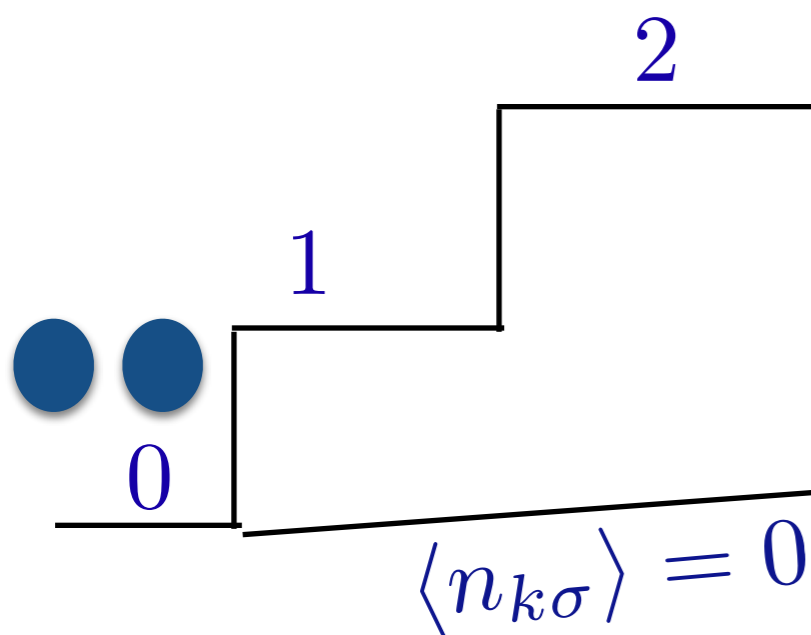
1

0



Superconductivity?

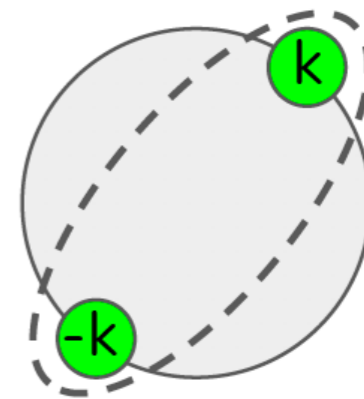
Cooper Instability



$$\langle n_{k\sigma} \rangle = 0$$

$$H = H_{\text{HK}} - gH_p$$

$$|\psi\rangle = \sum_{k \in \Omega_0} \alpha_k b_k^\dagger |\text{GS}\rangle$$

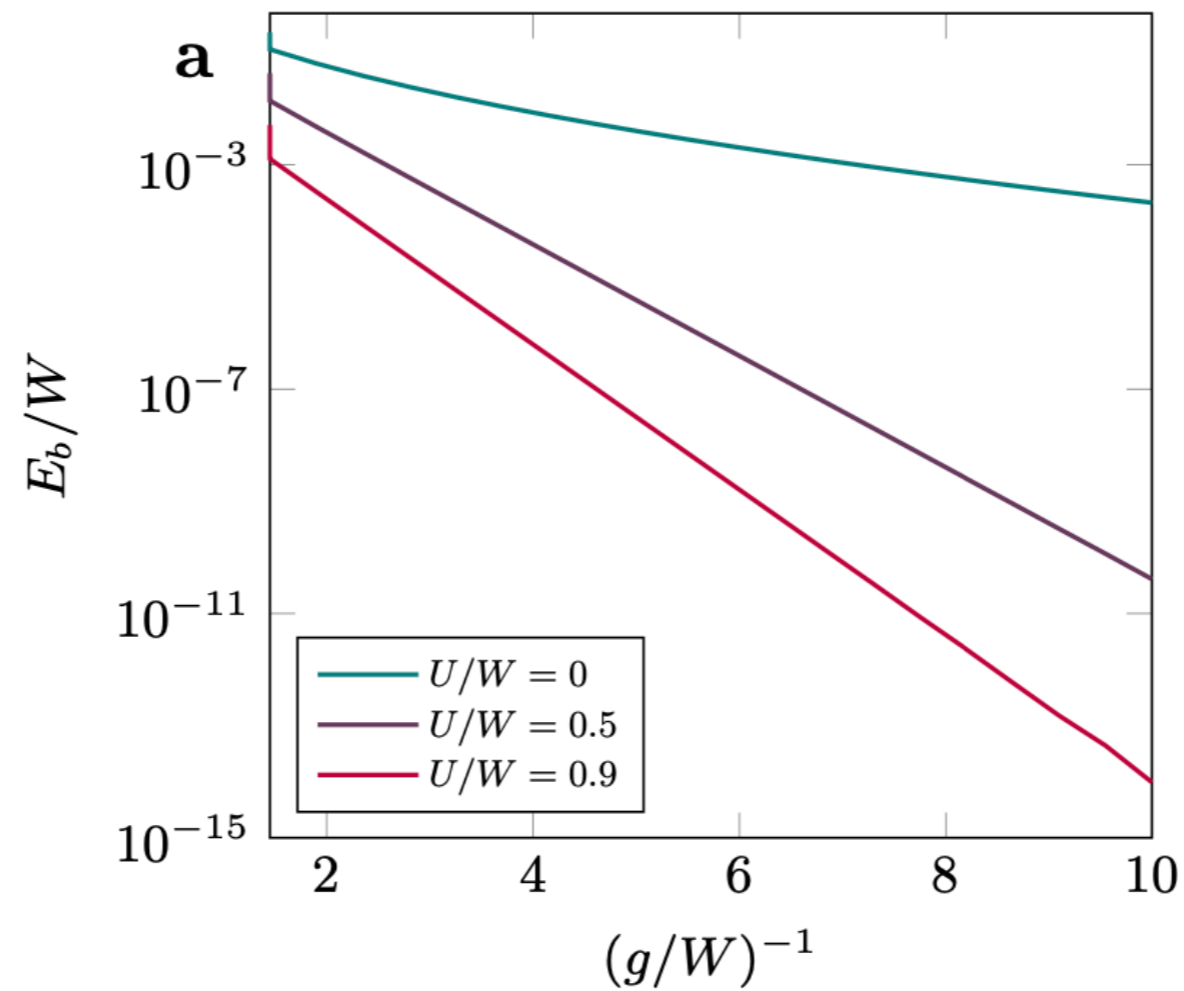


$$= \sum_{k, k'} b_k^\dagger b_{k'}$$

$$E_b = \langle \text{GS} | H | \text{GS} \rangle - \langle \psi | H | \psi \rangle \leq 0$$

Cooper Instability

$$E_b = -E \sim W(1 - (U/W)^2)e^{-\pi W \sqrt{1 - (U/W)^2}/g}$$



Pair Susceptibility

$$\chi(i\nu_n) \equiv \frac{1}{L^d} \int_0^\beta d\tau e^{i\nu_n \tau} \langle T \Delta(\tau) \Delta^\dagger \rangle_g$$

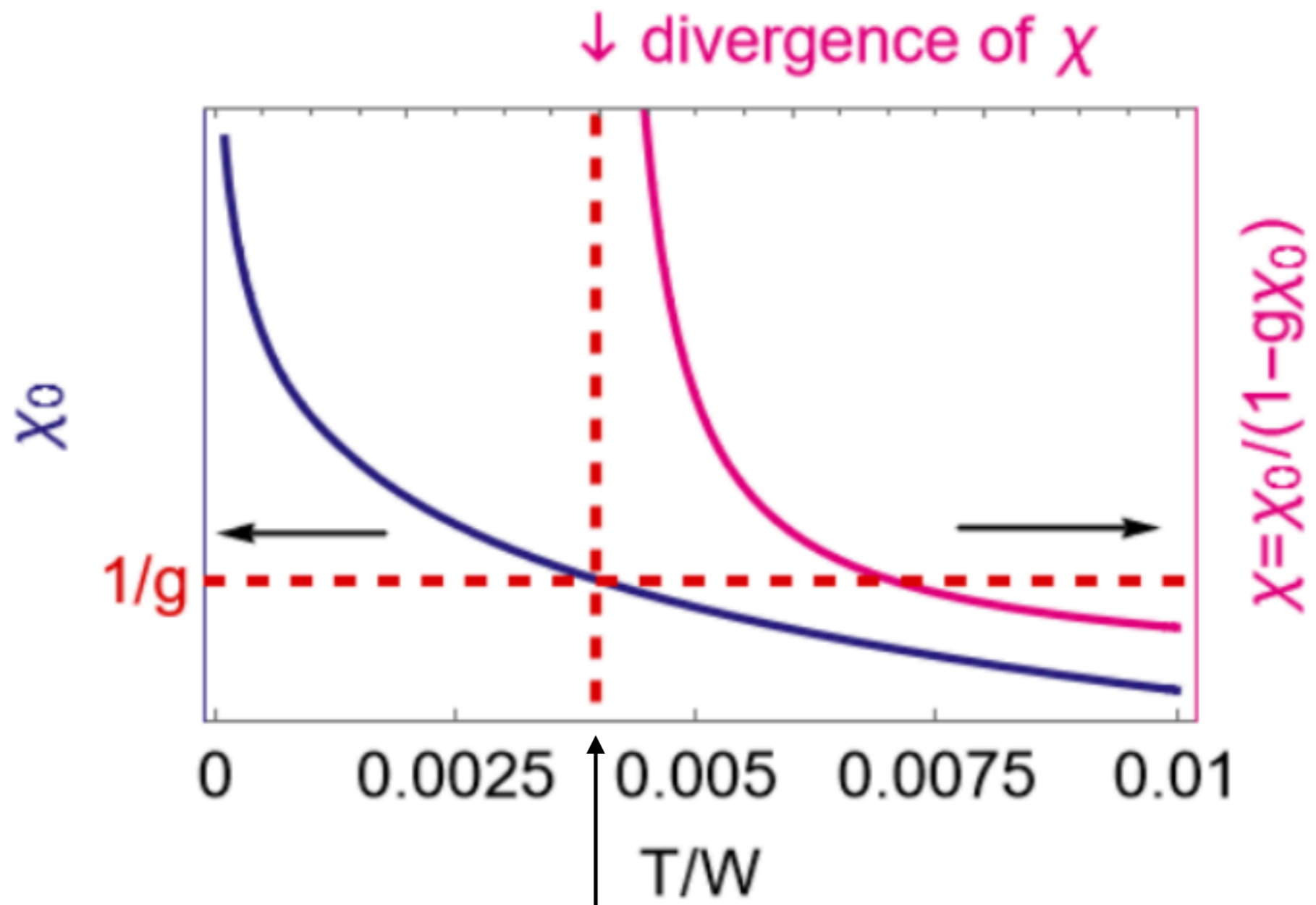


$$= \frac{\chi_0}{1 - g\chi_0}$$



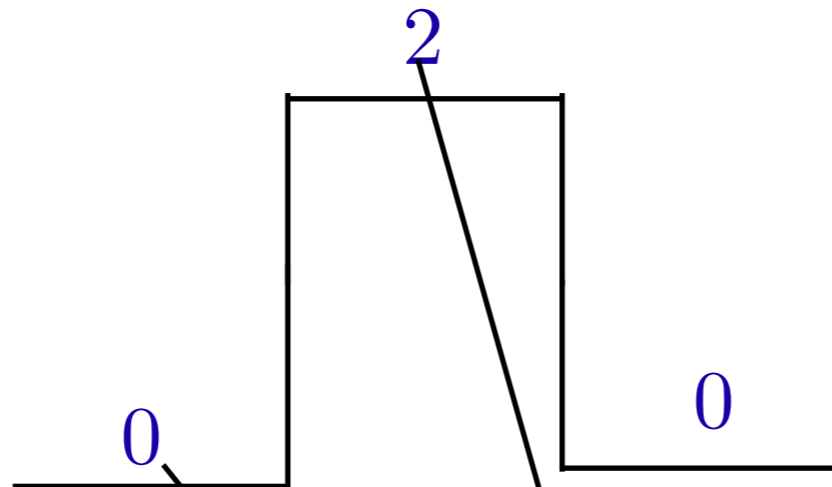
$$g\chi_0 = 1$$

solve for T_c



$$T_c = (W - U)^{4/5} U^{1/5} \frac{e^\gamma}{\pi} e^{-\frac{4}{5} \frac{W}{g}}.$$

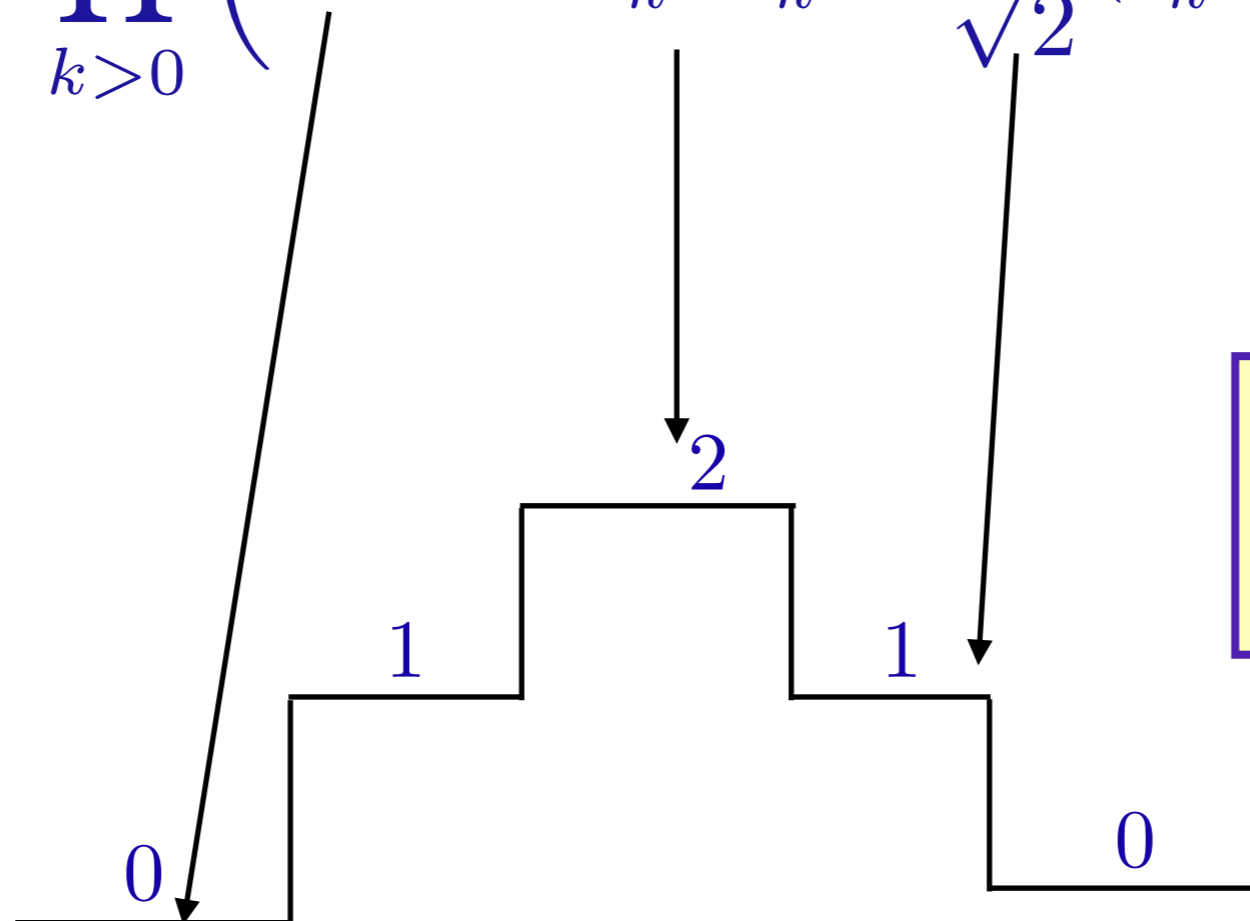
BCS variational wave function



$$|\psi_{\text{BCS}}\rangle = \prod_k (u_k + v_k b_k^\dagger |0\rangle)$$

variational MF wave function

$$|\psi\rangle = \prod_{k>0} \left(x_k + y_k b_k^\dagger b_{-k}^\dagger + \frac{z_k}{\sqrt{2}} (b_k^\dagger + b_{-k}^\dagger) \right) |0\rangle$$



HK
generalization

three variational parameters

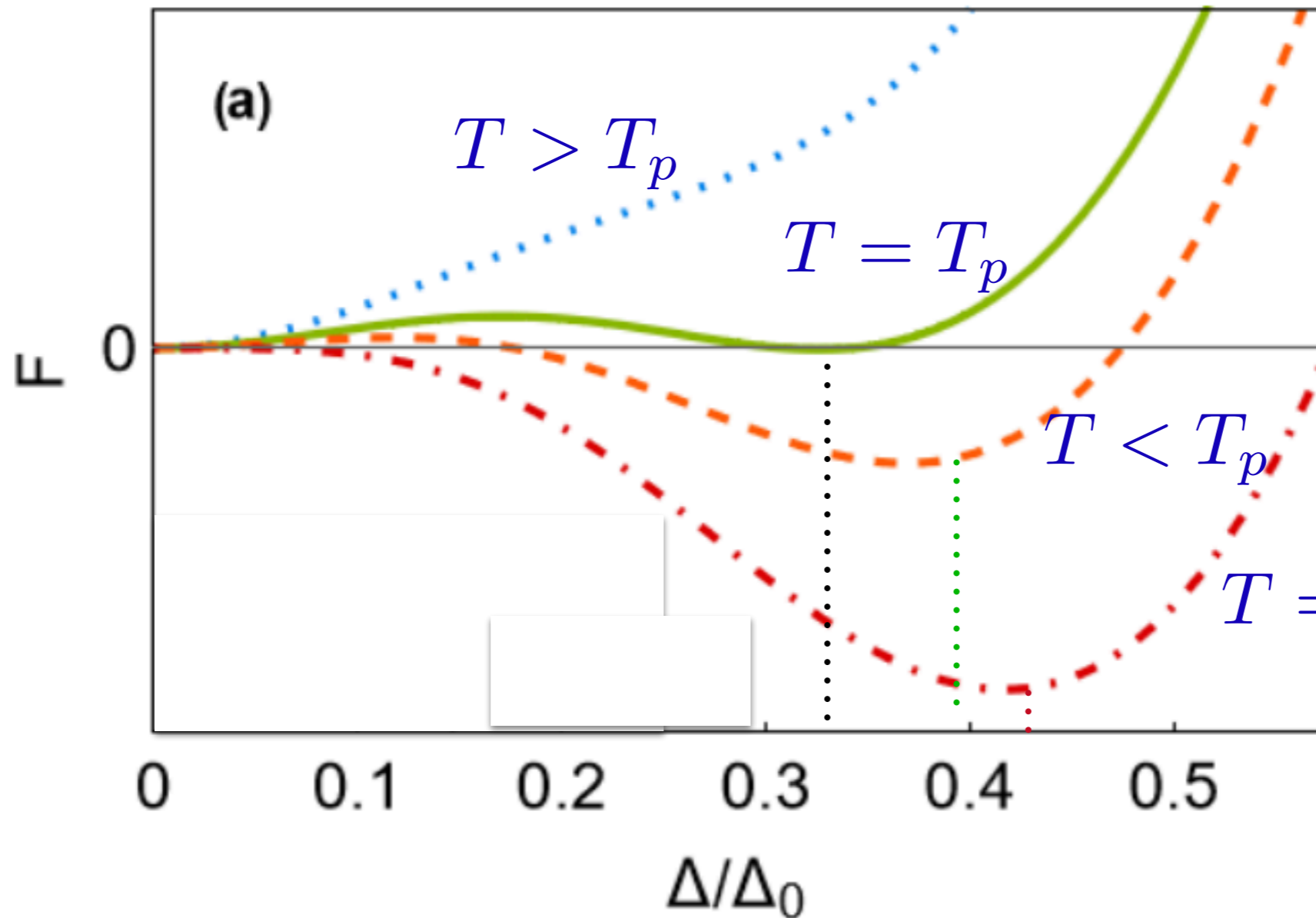
$$|x_k|^2 + |y_k|^2 + |z_k|^2 = 1$$

gap equation


$$\Delta \ll U, W$$

$$\Delta = (W - U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

compute free energy



$t_G \approx 10^{-11}$

MF theory
is accurate!

$$\lim_{g \rightarrow 0} \frac{\Delta}{T_c} \rightarrow \infty$$

Two-stage superconductivity in the Hatsugai-Kohomoto-BCS model

Yu Li,¹ Vivek Mishra,¹ Yi Zhou,^{2,3,4} and Fu-Chun Zhang^{1,4,*}

¹Kavli Institute for Theoretical Sciences, University of Chinese Academy of Sciences, Beijing 100190, China

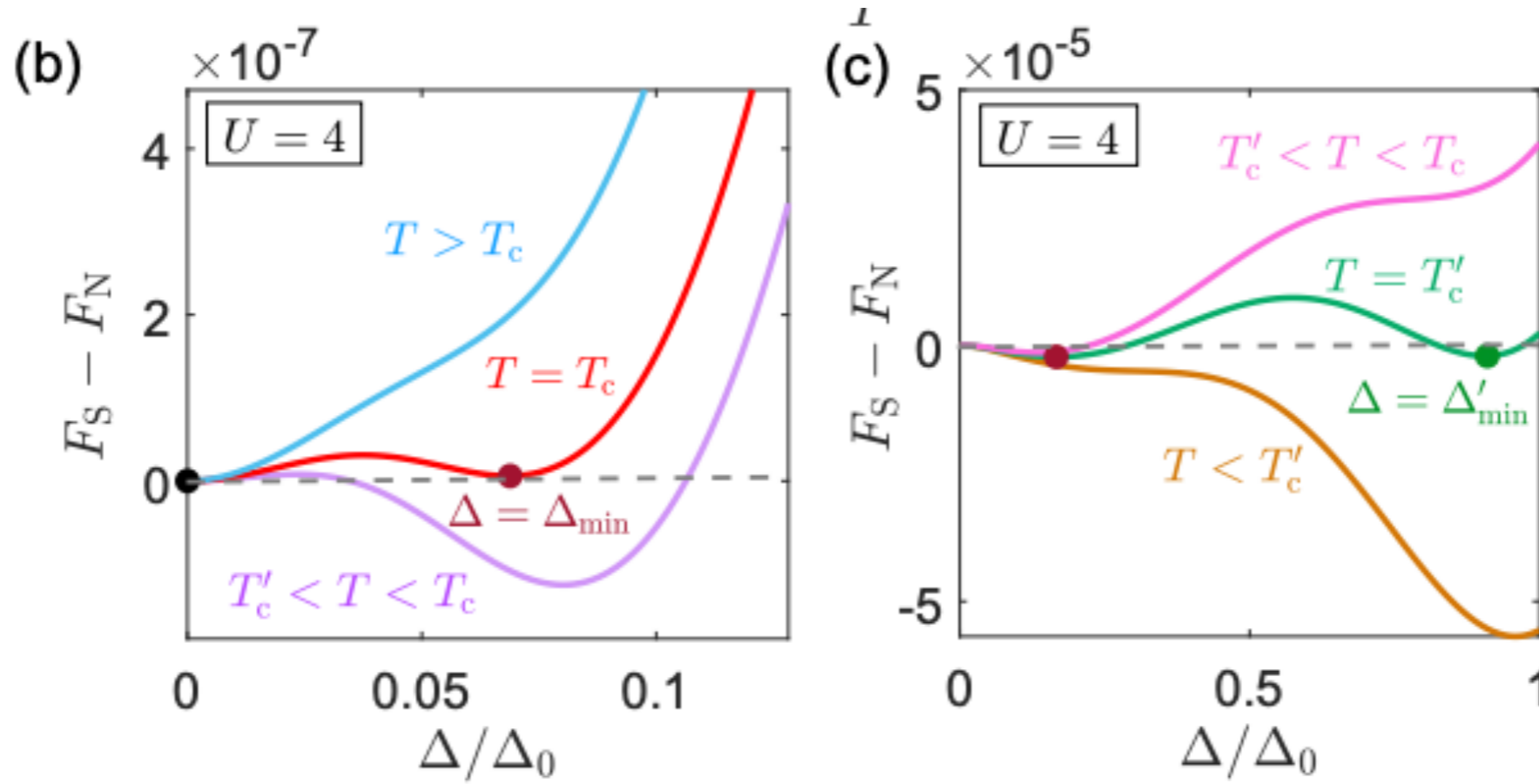
²Beijing National Laboratory for Condensed Matter Physics & Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China

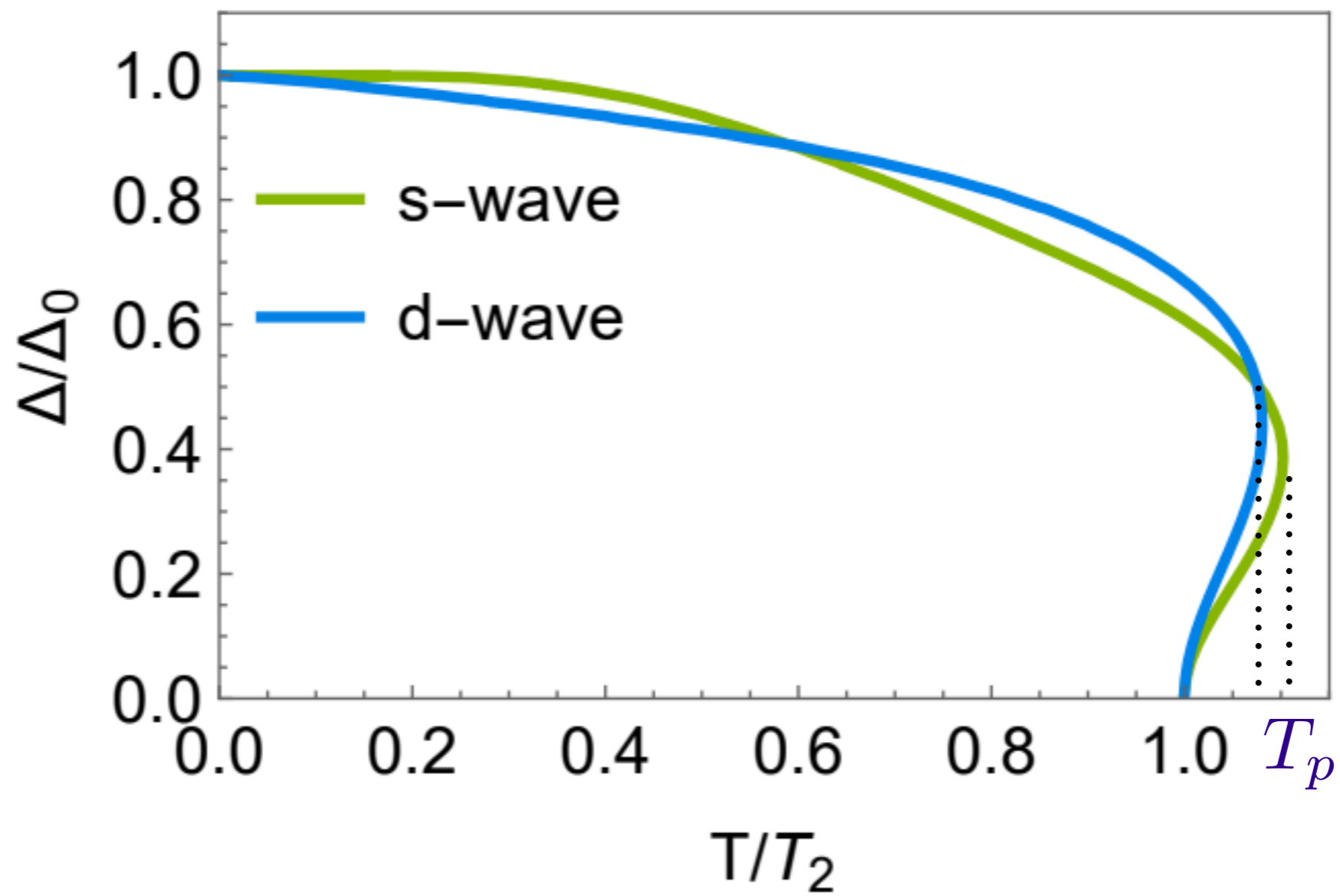
³Songshan Lake Materials Laboratory, Dongguan, Guangdong 523808, China

⁴CAS Center for Excellence in Topological Quantum Computation,
University of Chinese Academy of Sciences, Beijing 100190, China

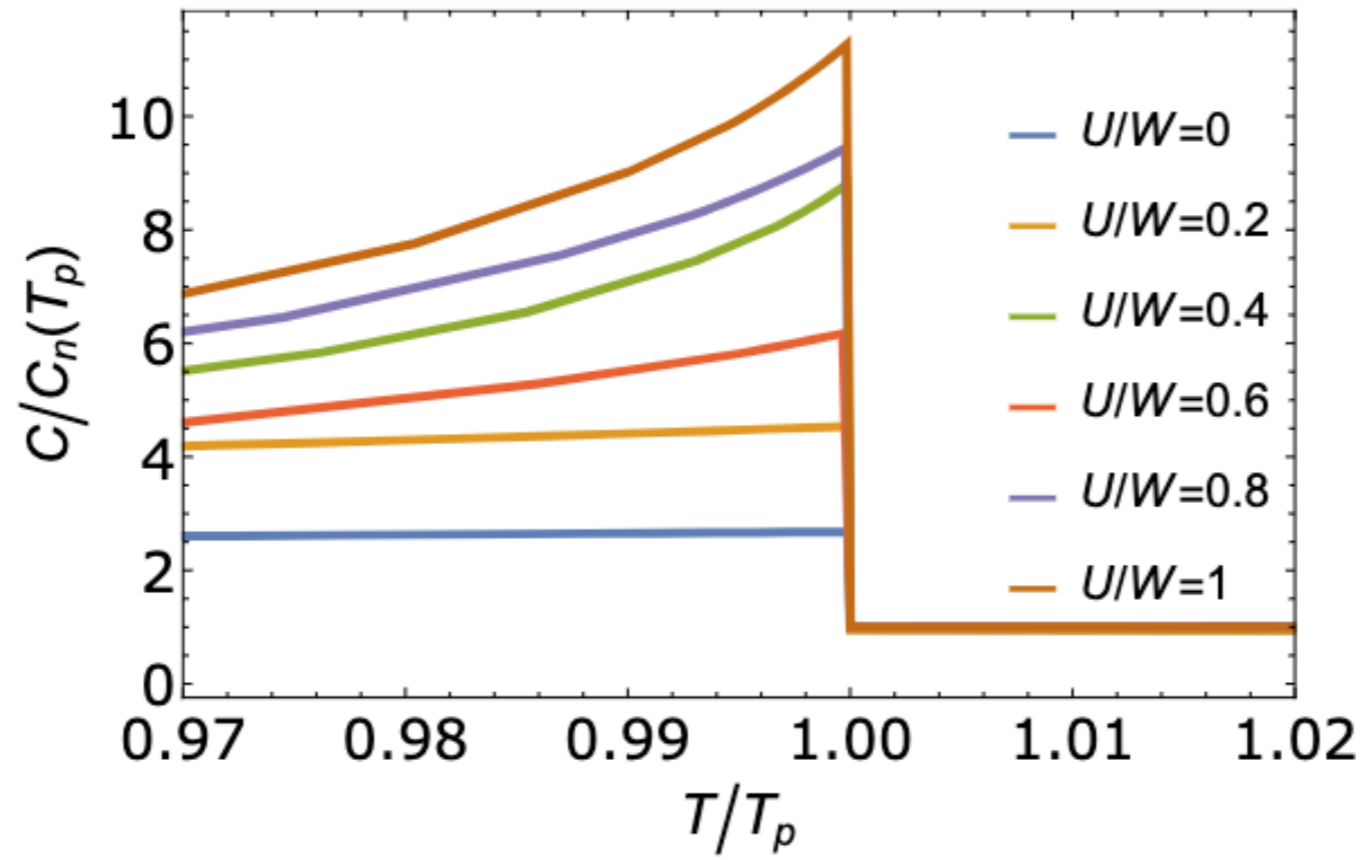
(Dated: July 7, 2022)

<https://arxiv.org/pdf/2207.01904.pdf>



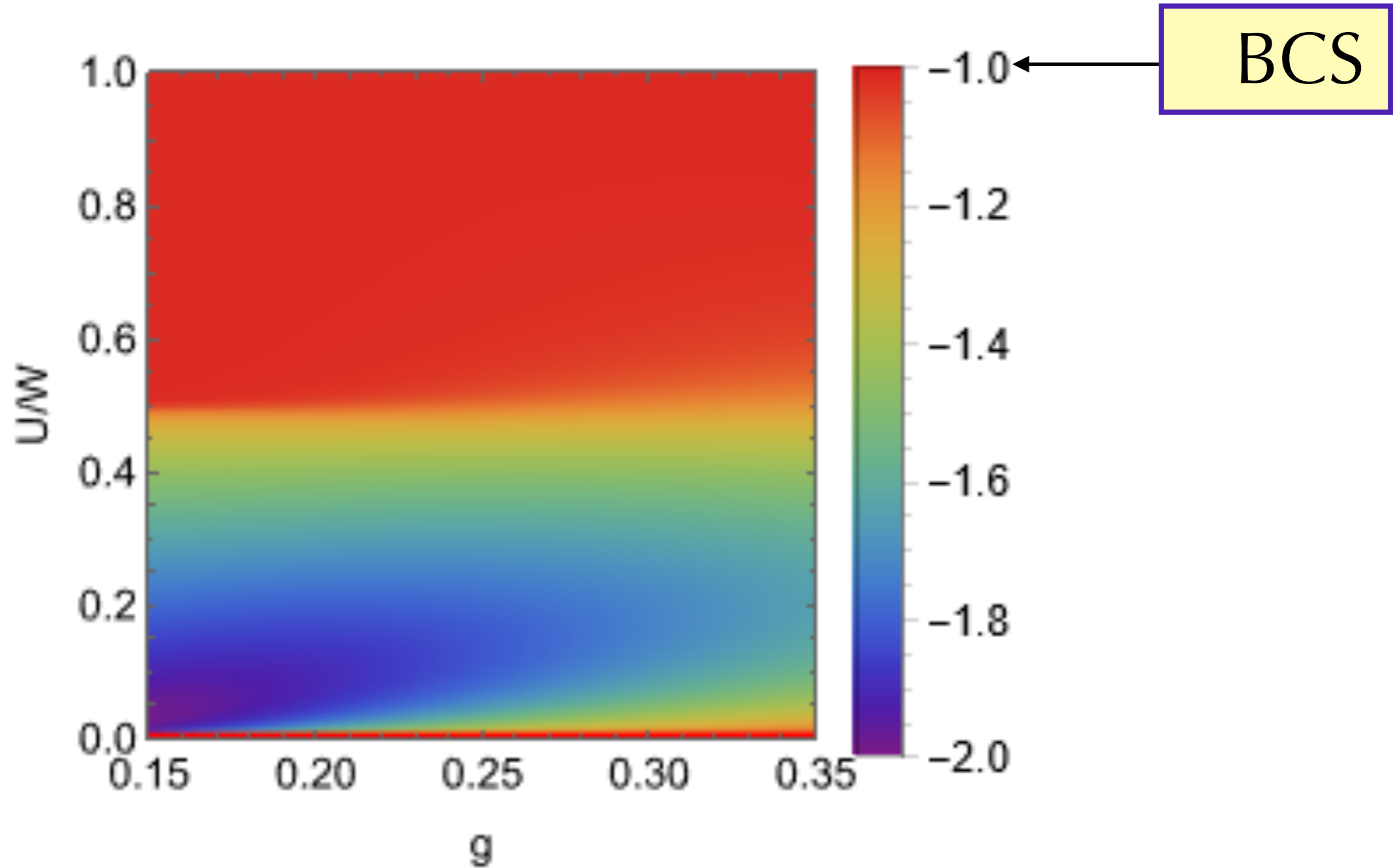


heat capacity jump

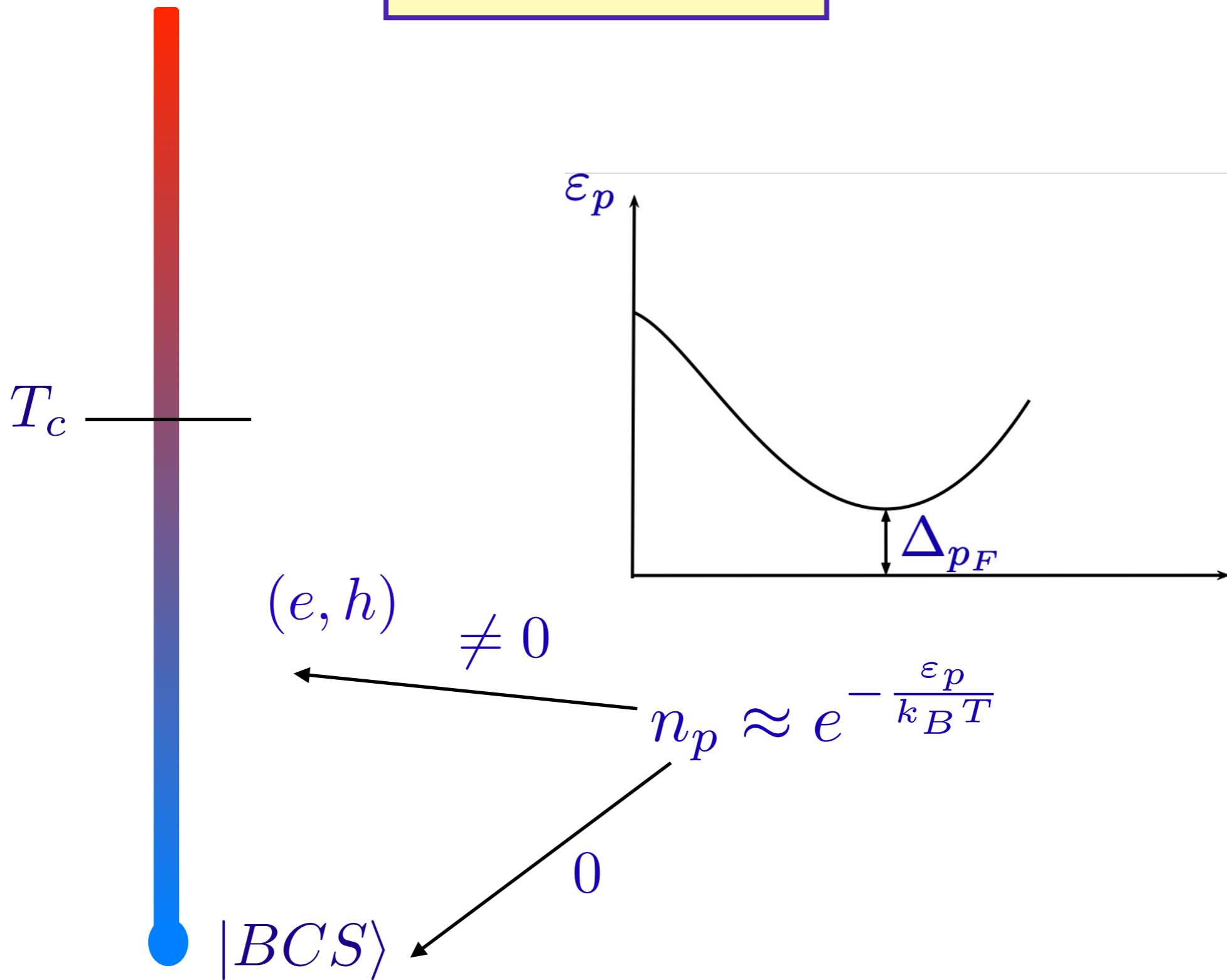


Condensation Energy

$$E_{\text{cond}}/N(0)\Delta^2 < 0$$



excited states



Bogoliubov excitations

$$\gamma_{k\sigma} |\psi_{\text{BCS}}\rangle = 0$$

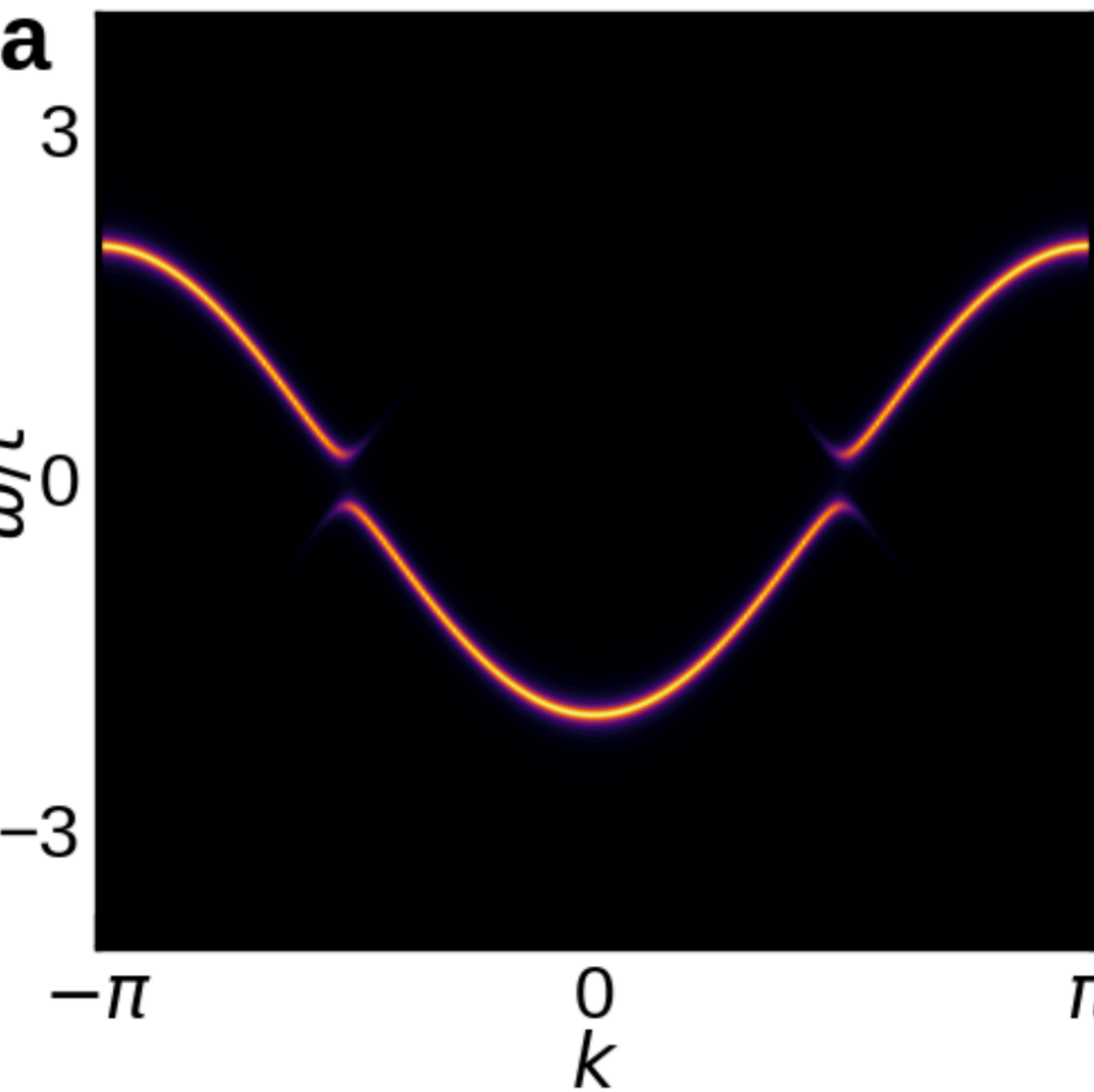
$$\gamma_{k\sigma} = u_k c_{k\sigma} - \sigma v_k c_{-k\bar{\sigma}}^\dagger$$


PYHons excitations

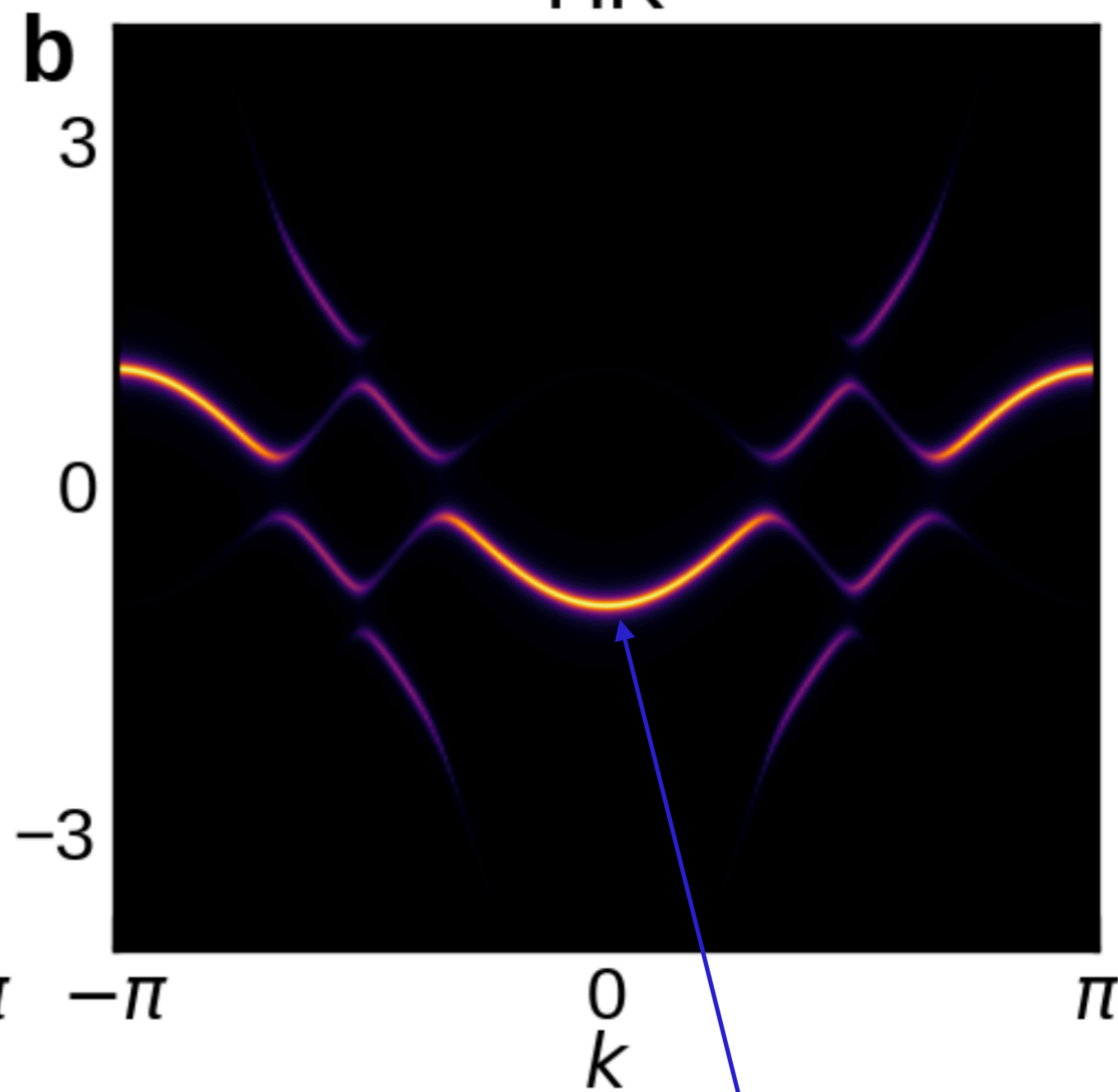
$$\gamma_{k\sigma}^l \propto \sqrt{2} x_k \zeta_{k\sigma}^\dagger - \sigma z_k \zeta_{-k\bar{\sigma}}$$

$$\gamma_{k\sigma}^u \propto z_k \eta_{k\sigma}^\dagger - \sigma \sqrt{2} y_k \eta_{-k\bar{\sigma}}$$

BCS



HK



PYHon band

can we explain the color change?

REPORT

Superconductivity-Induced Transfer of In-Plane Spectral Weight in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$

H. J. A. Molegraaf¹, C. Presura¹, D. van der Marel^{1,*}, P. H. Kes², M. Li²

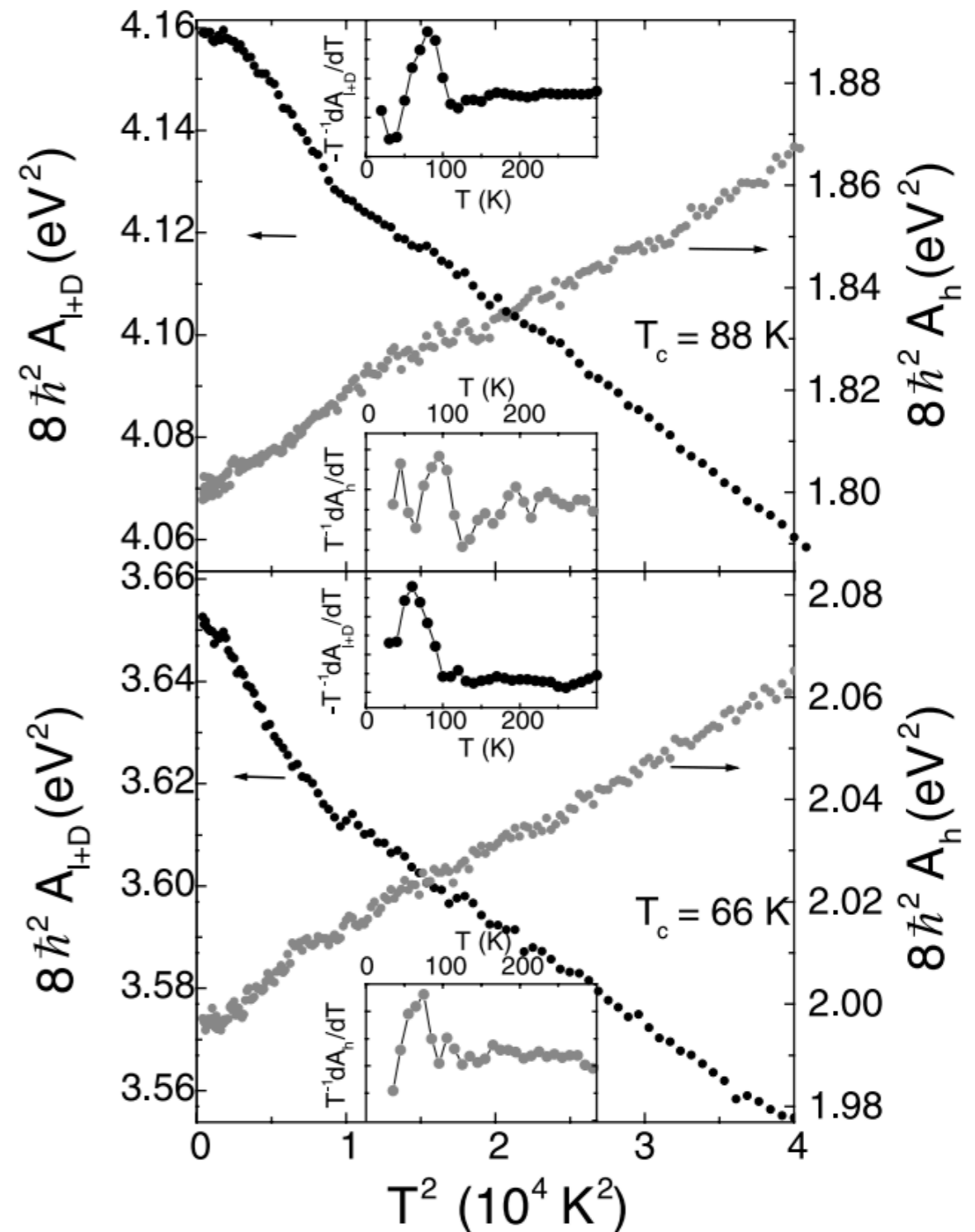
+ See all authors and affiliations

Science 22 Mar 2002:
Vol. 295, Issue 5563, pp. 2239-2241
DOI: 10.1126/science.1069947

$$A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 \text{ cm}^{-1}$$

$$A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 \text{ cm}^{-1}$$

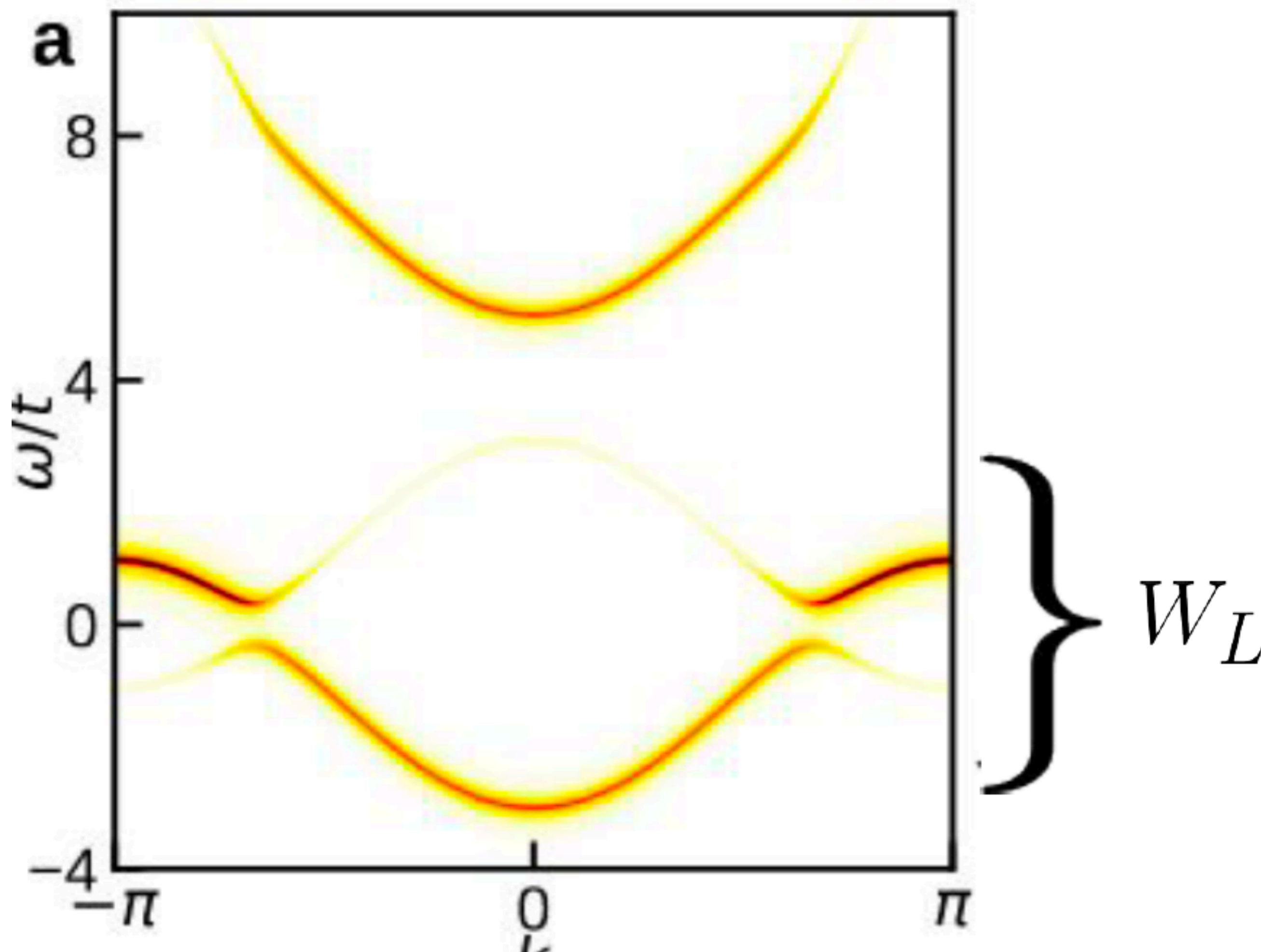
$$\frac{\Delta A_l}{A_l} \propto 3\%$$



condensation energy

Optical data are reported on a spectral weight transfer over a broad frequency range of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$, when this material became superconducting. Using spectroscopic ellipsometry, we observed the removal of a small amount of spectral weight in a broad frequency band from 10^4 cm^{-1} to at least $2 \times 10^4 \text{ cm}^{-1}$, due to the onset of superconductivity. We observed a blue shift of the *ab*-plane plasma frequency when the material became superconducting, indicating that the spectral weight was transferred to the infrared range. Our observations are in agreement with models in which superconductivity is accompanied by an increased charge carrier spectral weight. The measured spectral weight transfer is large enough to account for the condensation energy in these compounds.

UV-IR mixing



why?

$$H = H_{\text{HK}} + H_p$$

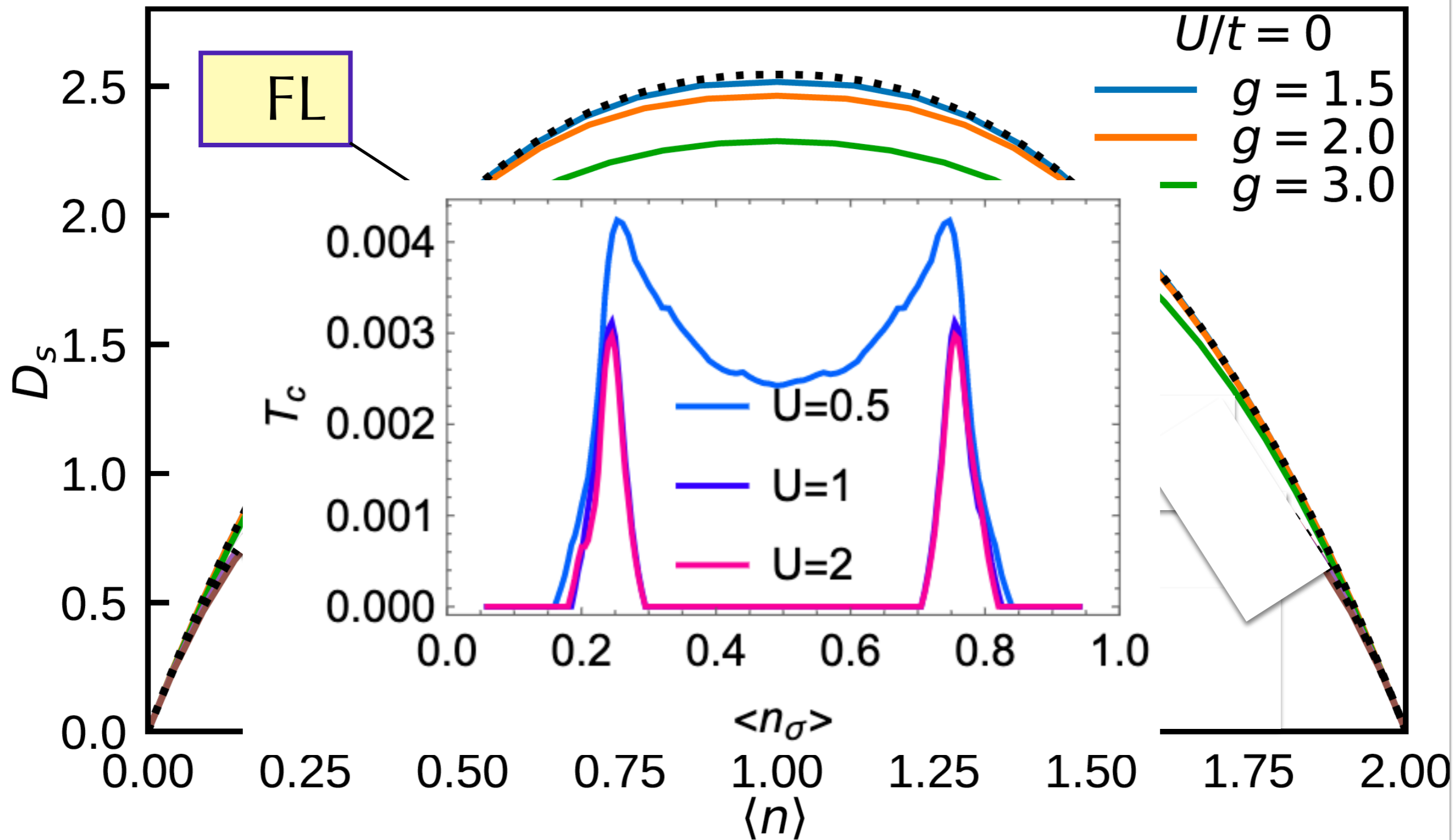
$$[H_{\text{HK}}, H_p] \neq 0$$



dynamical
spectral weight
transfer

Superfluid Density

Mottness-induced suppression



Superconductivity

observable

BCS/FL

$$\chi \rightarrow \infty$$

$$T_c$$

$$\Delta \neq 0$$

$$T_c$$

$$\lim_{g \rightarrow 0} 2\Delta_0/k_B T_c$$

$$3.52$$

quasi – particles

Bogoliubons

t_G (Ginzburg)

$$\approx 10^{-12}$$

$$1/TT_1$$

HS peak

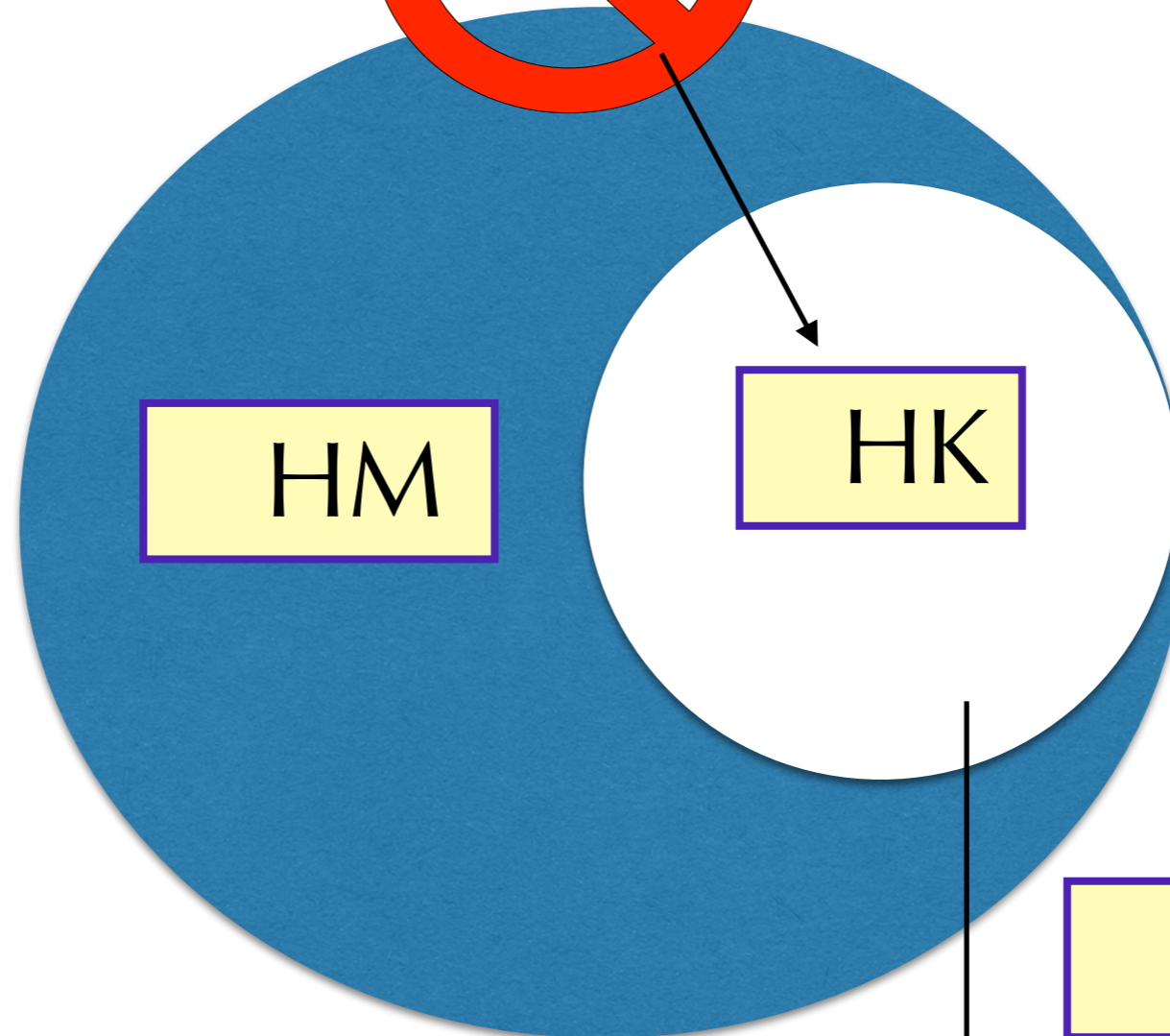
Landau Expansion

$$a = \alpha t, b > 0$$

$$E_{\text{cond}}/N(0)\Delta^2$$

$$-1$$

Mottness



HM

HK

PYHons

violation of Luttinger

non-BCS
superconductivity

Topology + Strong Correlations?

Are Exact Statements Possible?

Haldane + HK model



TI

1/4-filled MI

