| Useful Information | | | |
|---|---|---|---|
| $\hbar c$ | $m_e\ c^2$ | 1 eV | c |
| 197 eV nm | $0.511 \times 10^6$ eV | $1.6 \times 10^{-19}$ J | $3 \times 10^8$ m/s |

## Math Facts

$$e^{i\theta} = \cos\theta + \ i\ \sin\theta \quad , \quad |A+B|^2 = |A|^2 + 2\Re\{A^* B\} + |B|^2$$

$$\int_o^\infty dx\ x^n\ e^{-x} = n! \qquad \int_0^\infty dr\ r^2 \int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\phi \ \equiv \ \int_0^\infty dr\ r^2 \int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi$$

## Time Dependant Perturbation Theory

Final state amplitude: $$c_{i\to f}(t) = \frac{1}{i\hbar}\int_{-\infty}^{t} \langle f|\Delta V(\vec{r},t')|i\rangle \exp\left( i\frac{\left[E_f - E_i\right]t'}{\hbar}\right)\ \text{dt'}$$

## Harmonic Oscillator

$$x\ \varphi_n = \sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n+1}\,\varphi_{n+1} + \sqrt{n}\ \varphi_{n-1}\right);\ \breve{p}\ \varphi_n = i\sqrt{\frac{m\hbar\omega}{2}}\left(\sqrt{n+1}\,\varphi_{n+1} - \sqrt{n}\ \varphi_{n-1}\right)$$

where $\varphi_n$ are the SHO wave functions starting with the ground state $\varphi_0$.

$$\breve{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\ \omega^2\ x^2$$

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| $\varphi_0 = \left(\sqrt{\frac{\beta}{\pi}}\right)^{1/2} \exp\left(-\frac{\beta x^2}{2}\right) \ ; \ \varphi_1 = \left(2\beta\sqrt{\frac{\beta}{\pi}}\right)^{1/2} x\exp\left(-\frac{\beta x^2}{2}\right)$ |
| $\varphi_2 = \left(\frac{1}{2}\sqrt{\frac{\beta}{\pi}}\right)^{1/2}\ 2\beta x^2 - 1\ \exp\left(-\frac{\beta x^2}{2}\right)$ |
| where $\beta = m\omega/\hbar$ and $E_n = (n + \frac{1}{2})\,\hbar\omega$ |

## Gaussian Integrals $I_n = \int_{-\infty}^{+\infty}\ dy\ \ y^n e^{-\beta y^2}$

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| $I_0 = \sqrt{\pi}\beta^{-1/2};\ \ I_2 = \sqrt{\pi}\left(\frac{1}{2}\right)\beta^{-3/2}\ ;\ \ I_4 = \sqrt{\pi}\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\beta^{-5/2}$ |
| $I_6 = \sqrt{\pi}\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\beta^{-7/2}$ |

Hydrogen Atom

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_o}\right)^{3/2} e^{-Zr/a_o} \quad , \quad \psi_{200} = \frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a_o}\right)^{3/2} \left(2 - \frac{Zr}{a_o}\right) e^{-Zr/2a_o}$$

$$\psi_{210} = \frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a_o}\right)^{3/2} \frac{Zr}{a_o} e^{-Zr/2a_o} \cos\theta$$

$$\psi_{21\pm1} = \frac{1}{8\sqrt{\pi}} \left(\frac{Z}{a_o}\right)^{3/2} \frac{Z\ r}{a_o} e^{-Zr/2a_o} \sin\theta \ e^{\pm i\phi}$$

$$\psi_{300} = \frac{1}{81\sqrt{3\pi}} \left(\frac{Z}{a_o}\right)^{3/2} \left(27 - 18\frac{Zr}{a_o} + 2\frac{Z^2 r^2}{a_o^2}\right) e^{-Zr/3a_o}$$

$$a_o = \frac{4\pi\epsilon_o\,\hbar^2}{\mu\ e^2} = \frac{\hbar c}{\alpha\ \mu\ c^2} = 0.0529\ nm \ \text{ for an electron} \quad , \quad E_n = \frac{-13.6\ eV}{n^2}$$

E1 transition: $\Delta\ell = \pm1$ and $\Delta m = 0\ ,\ \pm1$.

Spherical Harmonics

| | | |
|---|---|---|
| $Y_0^{\ 0}$ <br> $\sqrt{\frac{1}{4\pi}}$ | $Y_1^{\ 0}$ <br> $\sqrt{\frac{3}{4\pi}}\ \cos\theta$ | $Y_1^{\ \pm1}$ <br> $\mp\sqrt{\frac{3}{8\pi}}\ e^{\pm i\phi}\ \sin\theta$ |
| $Y_2^{\ 0}$ <br> $\sqrt{\frac{5}{16\pi}}\ \left(3\cos^2\theta - 1\right)$ | $Y_2^{\ \pm1}$ <br> $\mp\sqrt{\frac{15}{8\pi}}\ e^{\pm i\phi}\ \sin\theta\cos\theta$ | $Y_2^{\ \pm2}$ <br> $\sqrt{\frac{15}{32\pi}}\ e^{\pm 2i\phi}\ \sin^2\theta$ |
| $Y_3^{\ 0}$ <br> $\sqrt{\frac{7}{16\pi}}\ (5\ \cos^3\theta - 3\ \cos\theta)$ | $Y_3^{\ \pm1}$ <br> $\mp\sqrt{\frac{21}{64\pi}}\ e^{\pm i\phi}\ \sin\theta\ (5\ \cos^2\theta - 1)$ | $Y_3^{\ \pm2}$ <br> $\sqrt{\frac{105}{32\pi}}\ e^{\pm 2i\phi}\ \sin^2\theta\ \cos\theta$ |

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| $Y_3^{\ \pm3}$ <br> $\mp\sqrt{\frac{35}{64\pi}}\ e^{\pm 3i\phi}\ \sin^3\theta$ |

$$\check{L}^2\ Y_\ell^m = \ell(\ell+1)\hbar^2\ Y_\ell^m \quad , \quad \check{L}_z\ Y_\ell^m = m\ \hbar\ \ Y_\ell^m \ ,\ \text{Parity} = (-1)^\ell$$